Smooth discrete feedback control of walking robots

An intermediate between fully passive and high bandwidth feedback control

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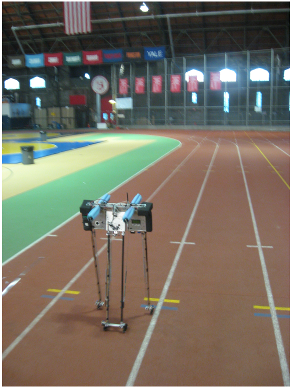

Cornell Ranger walked 14.3 miles (=23 kms.) using only 3 cents worth of electricity!

State of the art in walking robots

High Bandwidth Feedback Control

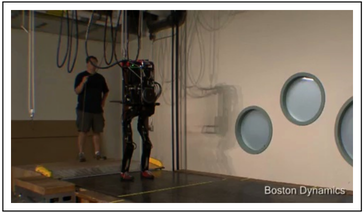

Figure: PETMAN from Boston Dynamics

- Robust, Versatile.
- Energy Inefficient.

Passive Dynamic Walkers

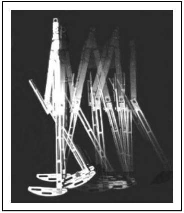

Figure: Passive Kneed Walker

- Energy efficient
- Not Robust.

Is there an intermediate approach?

- can be made close to optimal feedback control.
- can be made robust.
- can be made dumb.

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    - equations of motion
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4. Stabilize the approximate optimal control policy (details ahead)

Stability is easy

Equation of motion

$$\delta x_{n+1} = A\delta x_n + B\delta u_n$$

Output equation

$$\delta z_{n+1} = C\delta x_n + D\delta u_n$$

- n = step number*
- δx_n = differential about nominal value for state vector.
- δu_n = differential about nominal value for control vector.
- δz_n = differential about nominal value for output vector.
- A = Jacobian = $\partial(\delta x_{n+1})/\partial(\delta x_n)$
- B = Sensitivity = $\partial(\delta x_{n+1})/\partial(\delta u_n)$
- C = Jacobian = $\partial(\delta z_{n+1})/\partial(\delta x_n)$
- D = Sensitivity = $\partial(\delta z_{n+1})/\partial(\delta u_n)$

* could be time or state based

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Stability is easy

Output Equation

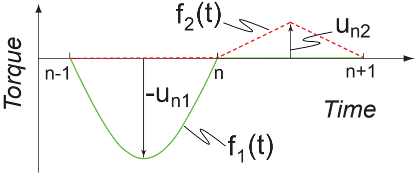
$$\delta z_{n+1} = C\delta x_n + D\delta u_n$$

NOTE:

- δu_n is intermittent.
- δu_n can be made smooth.

e.g.

$$\delta u_n = \begin{bmatrix} u_{n1} f_1(t) \\ u_{n2} f_2(t) \end{bmatrix}$$



Stability is easy

How to use the output equation for control?

For linear control,

$$\delta u_n = K \delta x_n$$

Thus,

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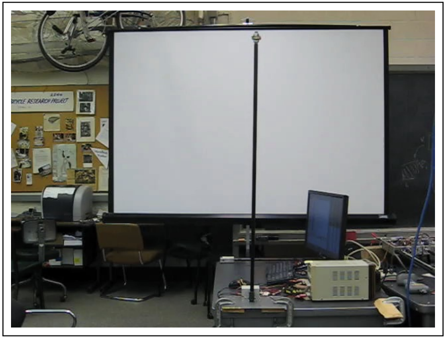
e.g. dead beat control
$(\delta z_{n+1} = 0)$

$$\delta u_n = K\delta x_n = -D^{-1}C\delta x_n$$

Example 1: Inverted Pendulum

Linearization over 1 sec.
i.e. n (seconds) = 1,2,3 ...



Output Equation

$$\overbrace{\delta x_{n+1}}^{state} = C\delta x_n + D\,\underbrace{K\delta x_n}_{\delta u_n}$$

For dead beat control
$\delta x_{n+1} = 0$

$$\delta u_n = K\delta x_n = -D^{-1}C\delta x_n$$

Example 2: Walking Robot

Linearization about upright position.
and n (step number) = 1,2,3 ...

Output Equation

$$\overbrace{\delta E_{n+1}}^{\text{Mech. Energy}} = F\delta x_n + G \underbrace{\overbrace{K\delta x_n}^{\text{Hip Torque}}}_{\delta u_n}$$

For dead beat control $\delta E_{n+1} = 0$

$$\delta u_n = K\delta x_n = -D^{-1}C\delta x_n$$

Walk Statistics

Distance:

14.3 mi (=23 km).

Time:

10 hrs, 40 min, 48 sec

Speed:

1.34 mi/hr (=2.15 km/hr)

No. of Steps:

65,185.

Power:

24.5 watt

Energy:

262 watt hours

COT = Energy/(Distance × Weight):

0.49

Thank You

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