Numerical Optimization

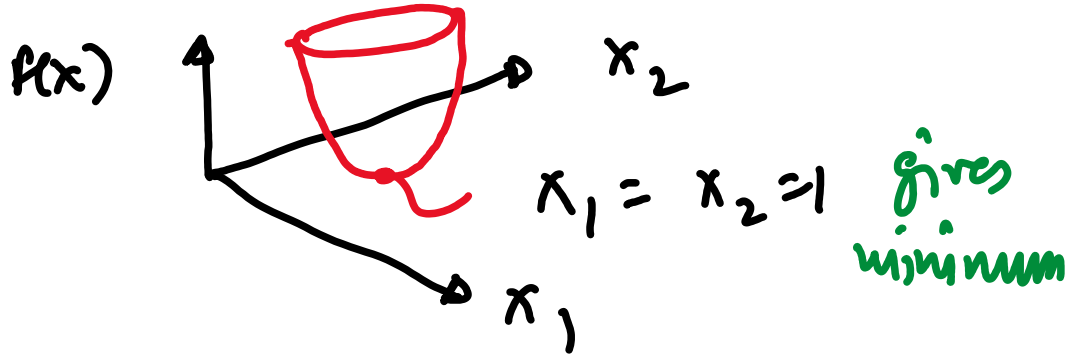
① Unconstrained optimization

$$\min_{x_1, x_2} F(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

↑ minimize this function (cost)

Ways to solve

① Graph of $f(x)$ vs x



② Guess ← domain knowledge

$f(x) \geq 0$ sum of squares

min of $f(x)$ at $f(x) = 0$ ← Guess

$x_2 - x_1^2 = 0$ & $1 - x_1 = 0$

$x_1 = 1 = x_2$

some $x_1 = 1 = x_2$

③ min $f(x)$

$g(x) = \frac{df}{dx} = 0$ (extremum) solve for x^*

root finding (f solve)

$\frac{d^2f}{dx^2} > 0$ min is at x^*

$\frac{d^2f}{dx^2} < 0$ max is at x^*

scipy.minimize → use this for optimization

Constrained optimization

$$\min_{x_1, x_2, x_3, x_4, x_5} f(x) \quad x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2$$

subject to:

$x_1 + x_2 + x_3 = 5$ — Linear equality constrained

$x_3^2 + x_4 = 5$ — Non linear equality

$x_1 \geq 0.3$ } Bounds

$x_3 \leq 5$ }

$0.3 \leq x_1 < \infty$

$-\infty < x_3 \leq 5$

$x_4^2 + x_5^2 \leq 5$ — Non linear inequality

$-\infty < x_2, x_4, x_5 < \infty$

$\rightarrow x_4^2 + x_5^2 - 5 \leq 0$

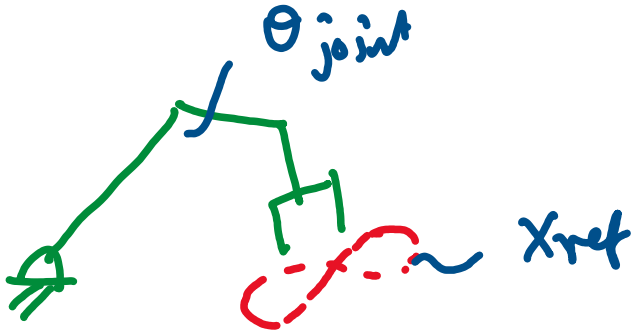
$\rightarrow \{5 - x_4^2 - x_5^2\} \geq 0$

Linear

$$x_1 + x_2 + x_3 = 5$$

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \underbrace{5}_{b}$$

① Inverse kinematics



$\theta_{joint} = fsolve(X_{ref})$

6×1 $\quad$ 4×1

$\theta_{joint} = minimize(X_{ref})$

7×1 $\quad$ 4×1

7 variables

6 constraints (equation)

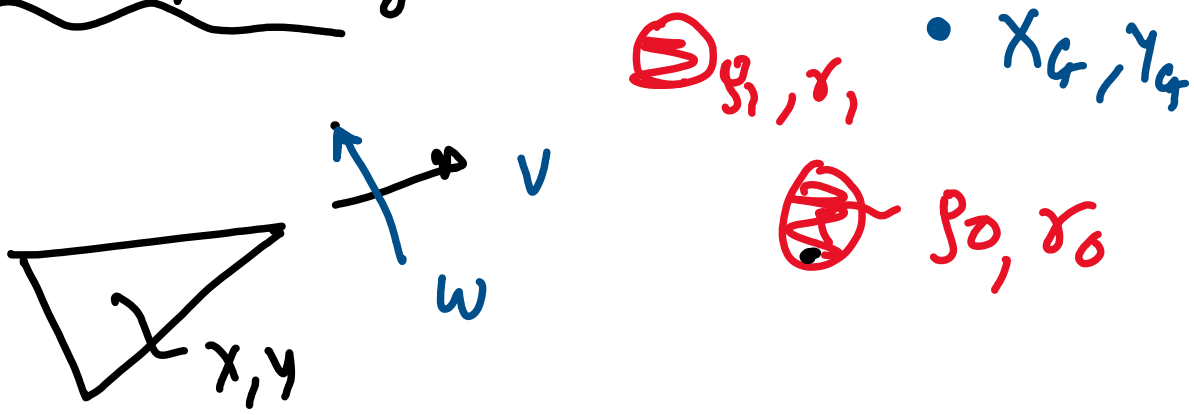
$\min_{\theta} \text{Cost}$ $\quad \Sigma(X - X_{ref})^2$ $\quad$ Cost $\Sigma(\theta - \theta_{avg})^2$

$X = F(\theta)$ { forward kinematics }

$\theta_{min} \le \theta \le \theta_{max}$ Bound

$X - X_{ref} = 0$ Equality constraint

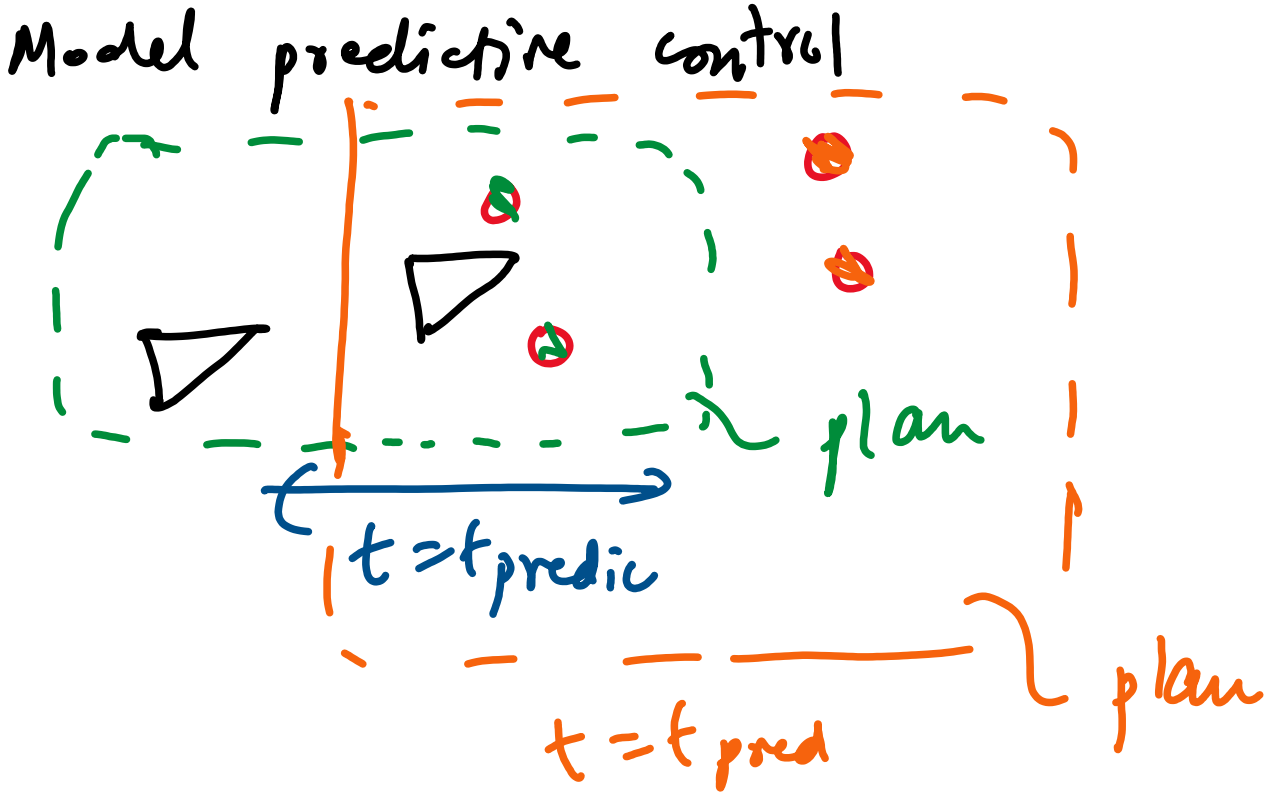
② Motion planning



$$\min_{v,w} \text{Cost.} \quad \sum_{\text{time}} (x - x_G)^2 + (y - y_G)^2 - \varepsilon \sum_{\text{time}} \sum_{\text{all obstacles}} \left[(x - x_i)^2 + (y - y_i)^2 \right]$$

$$\left.\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= w \end{aligned}\right\} \text{ integrate to compute } x, y, \theta$$

Constraints $(x - x_i)^2 + (y - y_i)^2 - r_i^2 \geq 0$


Model predictive control
plan
t = t predic
plan
t = t pred