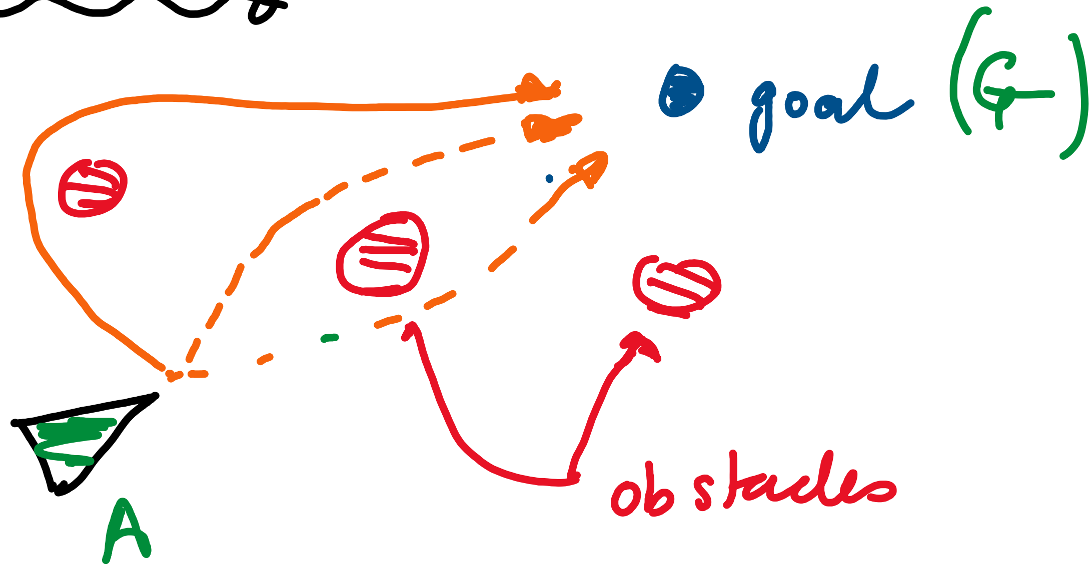
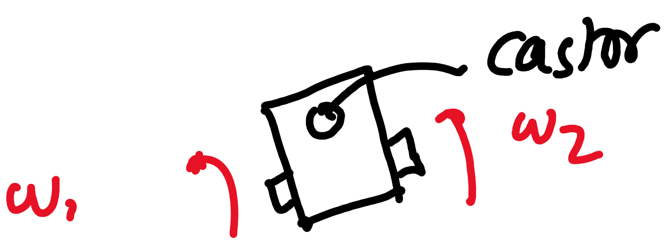
Motion planning



① Mobile robot : Legged

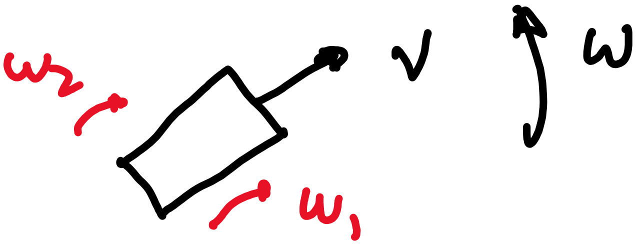
Command : xdot_ref , ydot_ref, psidot_ref

v_x v_y $\dot{\psi}$



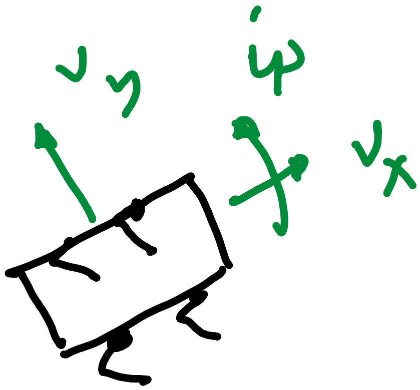
straight: spin w_1 / w_2 at the same speed / direction

turn : spin w_1 / w_2 at different speeds



$w_1, w_2 \longrightarrow v, w$

$\left.\begin{array}{cc}\text{linear speed}, & \text{angular speed}\end{array}\right]$ commands

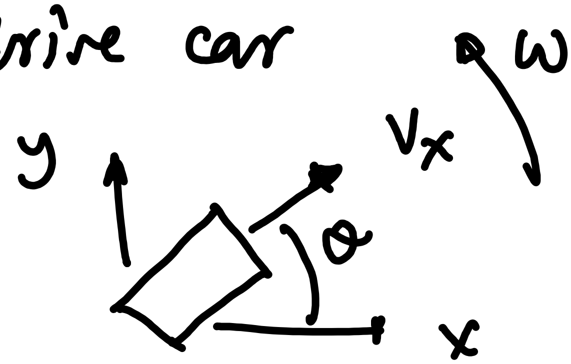


$v_x = v$

$\dot{\psi} = w$

$v_y = 0$

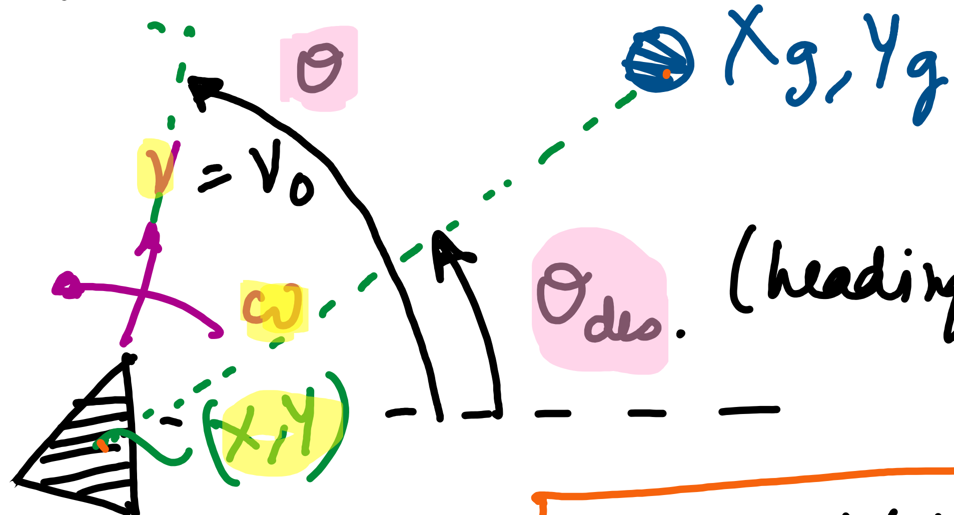
Equations of a diff drive car



$$\boxed{\begin{aligned}\dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= w\end{aligned}} \longrightarrow \text{legged robot}$$

① Go-to-goal & Obstacle avoidance

⑨ Go-to-goal

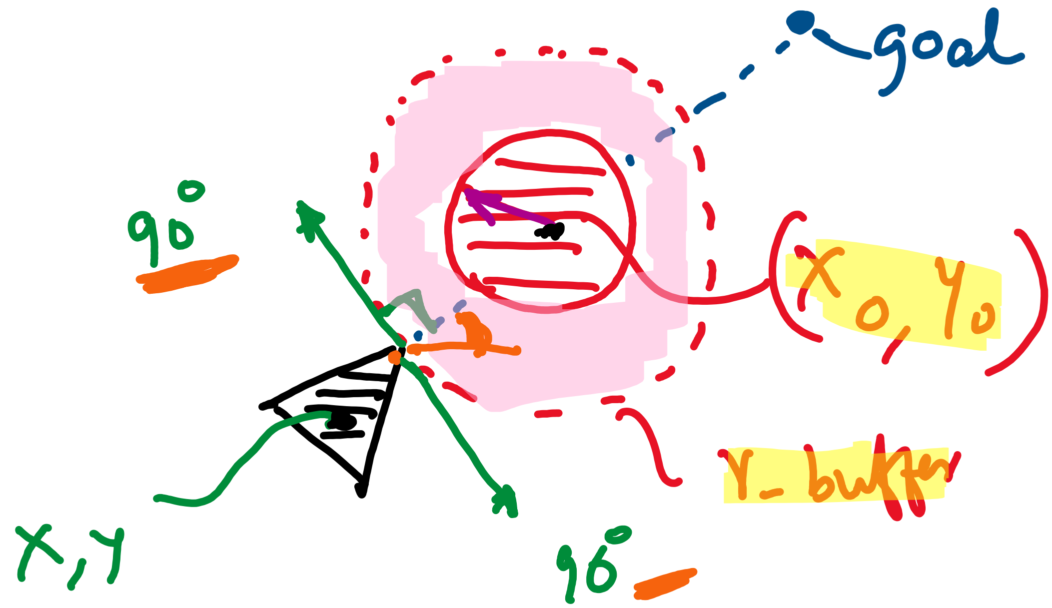


Diff. drive car

$$\omega = K(\theta_{des} - \theta)$$

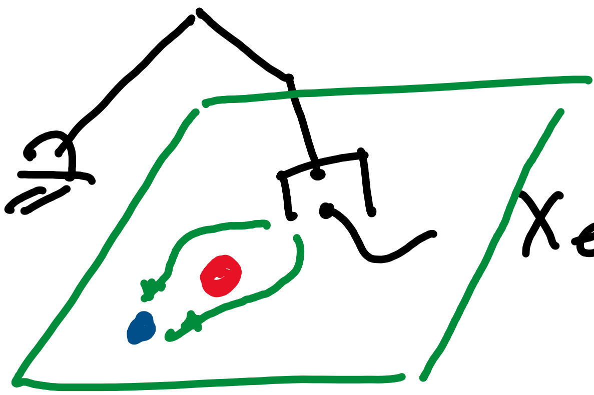
$$\boxed{\theta_{des} = \tan^{-1}\left(\frac{y - y_G}{x - x_G}\right)}$$

$v = v_0$ (user set)



$$\text{if } \left(\sqrt{(x-x_0)^2+(y-y_0)^2} - r_{buffer} < 0\right):$$

$$\theta_{des} = \underbrace{\tan^{-1}\left(\frac{y-y_0}{x-x_0}\right)}_{\text{heads toward the obstacle}} \pm \frac{\pi}{2}$$



X_e, θ_e

Impedance control: F_x, F_y

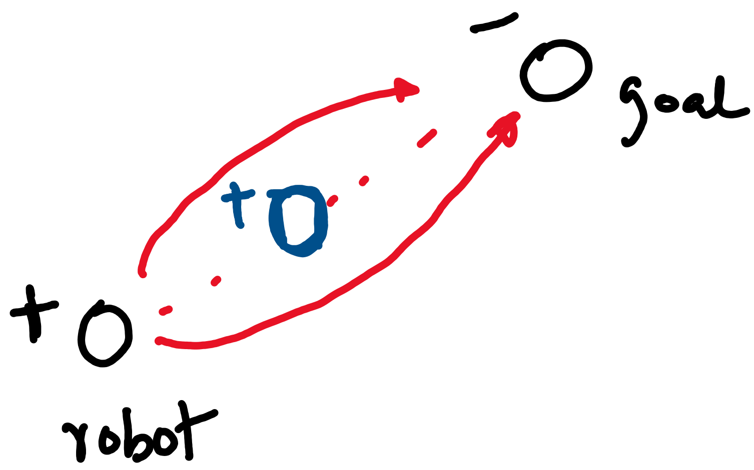
$$F_x^2 + F_y^2 = F_0 = \text{constant}$$

$$\theta_{des} = \frac{F_y}{F_x} = \tan^{-1}\left(\frac{y - y_G}{x - x_G}\right)$$

$$\left.\begin{aligned} F_x &= F_0 \cos\theta_{des} \\ F_y &= F_0 \sin\theta_{des} \end{aligned}\right\} \checkmark$$

$$\tau = \;-\;-\;-\; + J^T \begin{pmatrix} F_x \\ F_y \end{pmatrix}$$

② Potential fields



$\underline{U_{att}}$ - attractive field

$\underline{U_{rep}}$ - repulsive field

$U = U_{att} + U_{rep}$.

$\underline{F} = -\nabla U(q) \rightsquigarrow$ jacobian

(a) Attractive potential field U_{att}

(i) Conic potential field

goal

• $p(q_g)$

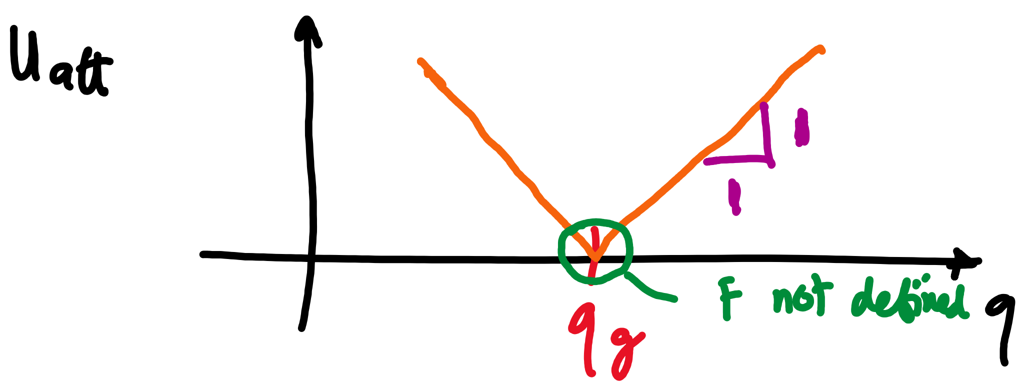
• $p(q)$ degrees of freedom

position

norm

$$U_{att} = \| p(q) - p(q_g) \|$$

$$= \sqrt{(p(q) - p(q_g)) \cdot (p(q) - p(q_g))}$$



$$F = -\nabla U_{att} = \frac{(p(q) - p(q_g))}{\| p(q) - p(q_g) \|}$$

$$U_{att} = \sqrt{x(q)\cdot x(q)} = \| x(q) \|$$

$$F = -\frac{\partial U_{att}}{\partial q} = \_\_\_\_\_\_$$

$$\boxed{\frac{d}{dx} x^n = n\, x^{n-1}}$$

$$U = \left((x - x_g)^2 + (y - y_G)^2 \right)^{1/2}$$

$$\frac{\partial U}{\partial (x, y)} = \frac{1}{2}\left[(x - x_g)^2 + (y - y_G)^2 \right]^{\frac{1}{2} - 1} \cdots$$

$$\left[2\underline{(x - x_G)},\ 2(y - y_G) \right]$$

$$= \left[\frac{\cancel{2}(x - x_G)}{\cancel{2}\sqrt{(x - x_g)^2 + (y - y_G)^2}},\ \frac{\cancel{2}(y - y_G)}{\cancel{2}\sqrt{(x - x_G)^2 + (y - y_G)^2}} \right]$$

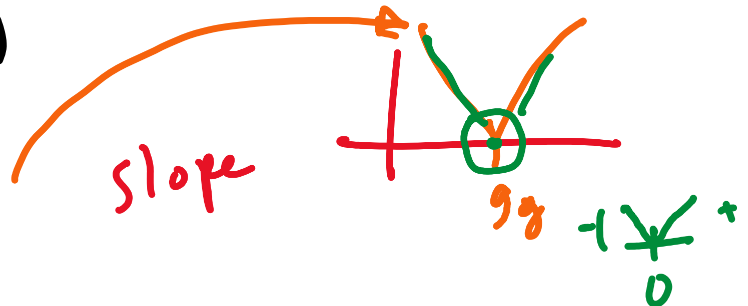
$$F = -\frac{\partial U}{\partial q}$$

$$= -\left[\frac{(x - x_G)}{\sqrt{(x - x_g)^2 + (y - y_G)^2}},\ \frac{(y - y_G)}{\sqrt{(x - x_G)^2 + (y - y_G)^2}} \right]$$

$$F = -\frac{\text{vector}}{\|\text{vector}\|}$$

$$= \frac{p(q) - p(q_g)}{\| p(q) - p(q_g) \|}$$

NOTE: $\|F\| = 1$

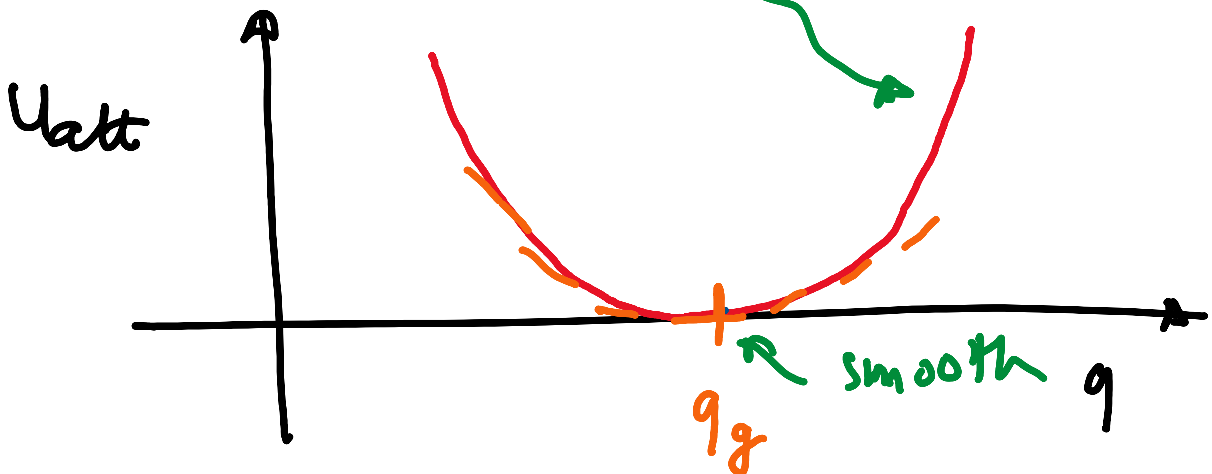


At $q = q_g$ F is not defined

$$F = \begin{cases} -\nabla U_{att} & q \neq q_g \\ 0 & q = q_g \end{cases}$$

(ii) Parabolic potential

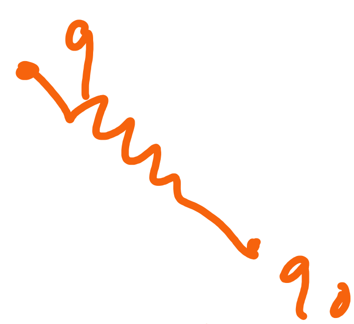
$$U_{att} = \frac{1}{2}\,\xi\,\|p(q) - p(q_g)\|^2$$



$$F = -\frac{\partial U}{\partial q} = -\frac{\partial}{\partial q}\frac{\xi}{2}\left[\overbrace{\left(p(q) - p(q_g)\right)}^{x}\cdot\overbrace{\left(p(q) - p(q_g)\right)}^{x}\right]$$

$$= -\frac{\xi}{\cancel{2}}\,\cancel{2}\,\underbrace{\left[p(q) - p(q_g)\right]}_{2x}$$

$$F = -\underset{\text{constant}}{\xi}\,\underbrace{\left[p(q) - p(q_g)\right]}_{q \qquad q_0}$$



$$F = -k(q - q_0)$$

Conic:

Constant force away from q_g

F is not defined at q_g

Parabolic

F is defined at q_g continuous

F is proportional to distance from q_g

Combine Conic & Parabolic

Choose a distance d from q_g

If robot is at a distance $\geq d$
use conic potential

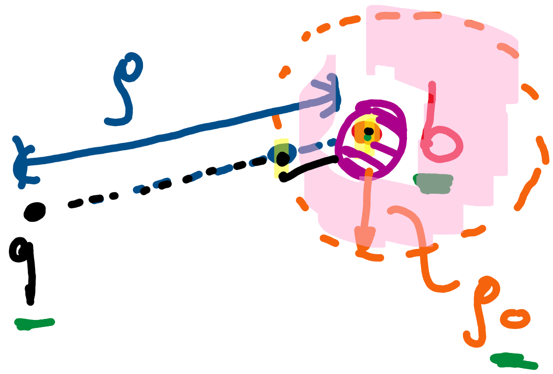
If robot is at a distance $< d$
use parabolic potential.

(iii) Combined conic / parabolic field

$$U_{att} = \begin{cases} \frac{1}{2}\xi \| p(q) - p(q_g) \|^2 & \| p(q) - p(q_g) \| \le d \\ d\xi \| p(q) - p(q_g) \| - \underbrace{\frac{1}{2}\xi d^2}_{\text{constant}} & \| p(q) - p(q_g) \| > d \end{cases}$$

$$\boxed{F_{att} = \begin{cases} -\xi \| p(q) - p(q_g) \| & \| p(q) - p(q_g) \| \le d \\ -d\xi \dfrac{p(q) - p(q_g)}{\| p(q) - p(q_g) \|} & \| p(q) - p(q_g) \| > d \end{cases}}$$

ⓑ Repulsive field



$\rho(q) \to \rho_0$

$U \to \infty$

$$\rho(q) = \| p(q) - b^0 \|$$

$$U_{rep} \begin{cases} 0 & \rho(q) > \rho_0 \\ \frac{\eta}{2}\left\{ \frac{1}{\rho(q)} - \frac{1}{\rho_0} \right\}^2 & \rho(q) \leq \rho_0 \end{cases}$$

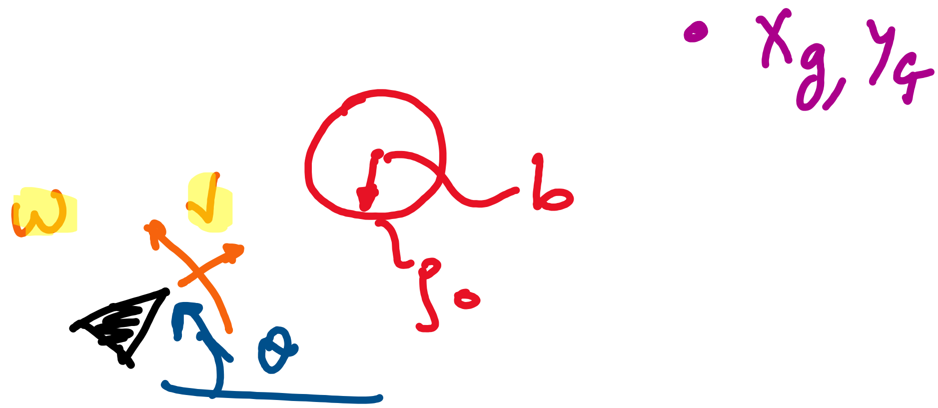
$$F_{rep} = -\partial U_{ref}$$

$$F_{rep} \begin{cases} 0 & \rho(q) > \rho_0 \\ \eta \left\{ \frac{1}{\rho(q)} - \frac{1}{\rho_0} \right\} \frac{1}{\rho^2(q)} \nabla\rho & \rho(q) \leq \rho_0 \end{cases}$$

$$\nabla\rho = \frac{p(q) - b}{\| p(q) - b \|} \quad \{\text{Unit vector}\}$$

$$\| p(q) - b \|$$

(i) Motion planning of a car



$$U = U_{att} + U_{rep}$$

$$F = -\partial U = -\partial U_{att} - \partial U_{rep}$$

$$\underline{F} = F_{att} + F_{rep}$$

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2 controls : v, ω

$$v = v_0 \quad \text{(nominal speed)}$$

$$\theta_{des} = \tan^{-1}\left(\frac{F_y}{F_x}\right) \qquad F = F_x \hat{i} + F_y \hat{j}$$

$$\omega = K(\theta_{des} - \theta)$$

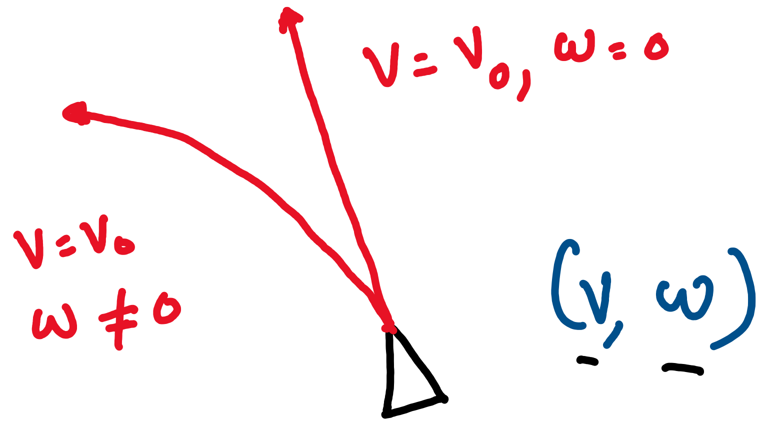
user chosen constant

② manipulator

f_x, f_y

$\tau = J^T F$

③ Dynamic Window Approach



- (v, ω) pair gives a curve
- Over time t_h (prediction horizon)

  set v, ω pairs

  compute v, ω pairs that avoid obstacles

  compute v, ω pairs that get to the goal

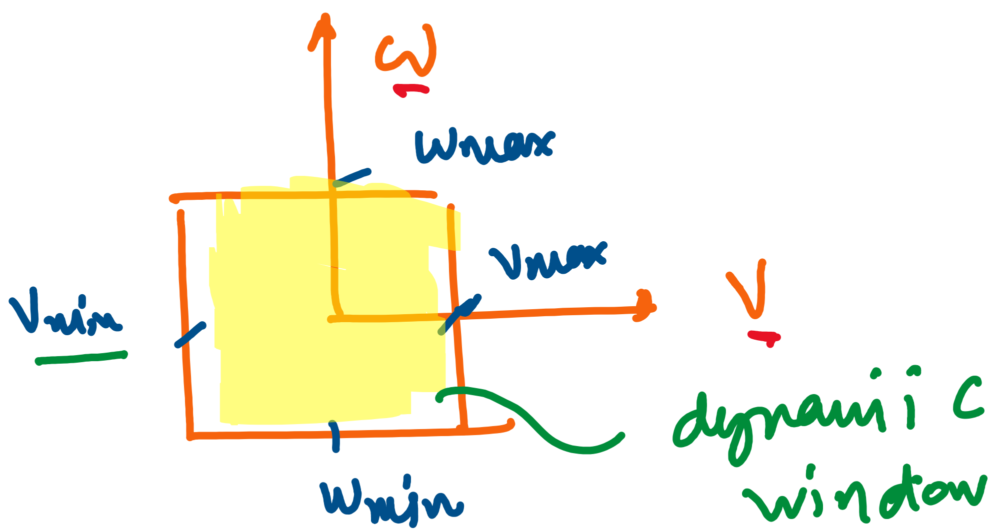
  define a cost for getting to a goal / avoiding obstacles — user defines

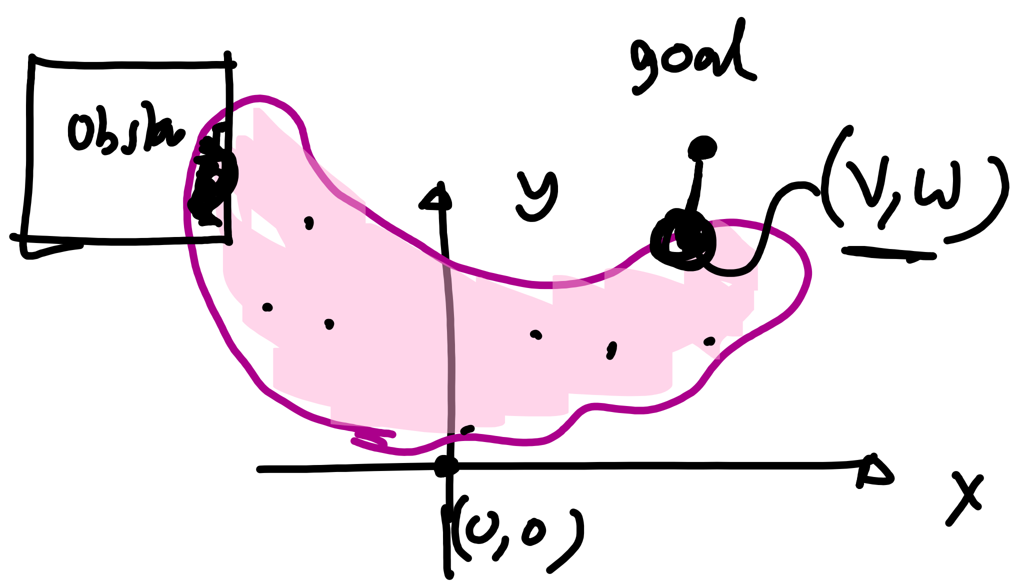
  find v, ω that minimizes this cost.

(i) How to choose v, ω pairs:

$$v \equiv (\underbrace{v_0 - a_{min}\,dt}_{v_{min}};\ \underbrace{v_0 + a_{max}\,dt}_{v_{max}})$$

$$\omega \equiv (\underbrace{\omega_0 - \alpha_{min}\,dt}_{\omega_{min}};\ \underbrace{\omega_0 + \alpha_{max}\,dt}_{\omega_{max}})$$



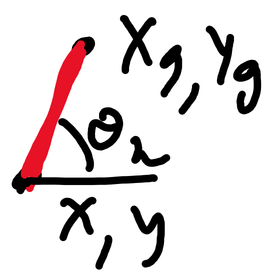


(ii) Choosing a cost:

$$\text{cost} = \left.\begin{aligned} &c_1\,(\text{cost\_to\_goal}) + \\ &c_2\,(\text{cost\_obstacle}) + \\ &c_3\,(\text{cost\_speed}) \end{aligned}\right\}\ \text{can add more}$$

c_1, c_2, c_3 — user chosen constants.

$$\text{cost\_to\_goal} = \sqrt{(x-x_g)^2+(y-y_G)^2}$$



$$= \tan^{-1}\left(\frac{y-y_G}{x-x_G}\right) = \theta_h$$

$$\text{cost\_obstacle} = \begin{cases} 0 & \text{no obstacle} \\ \infty & \text{there is an obstacle} \end{cases}$$

$$= \frac{1}{\sqrt{(x-x_{obs})^2+(y-y_{obs})^2}}$$

$\sqrt{(x-x_{obs})^2 + (y-y_{obs})^2}$

cost_speed $= (V_{max} - V)^2$ favors driving fast

These are heuristics. You can add more, modify, or remove some of them

(iii) Simulate the system over t_h and compute the cost for each (v, w)

Choose (v_0, w_0) corresponding to the minimum cost.