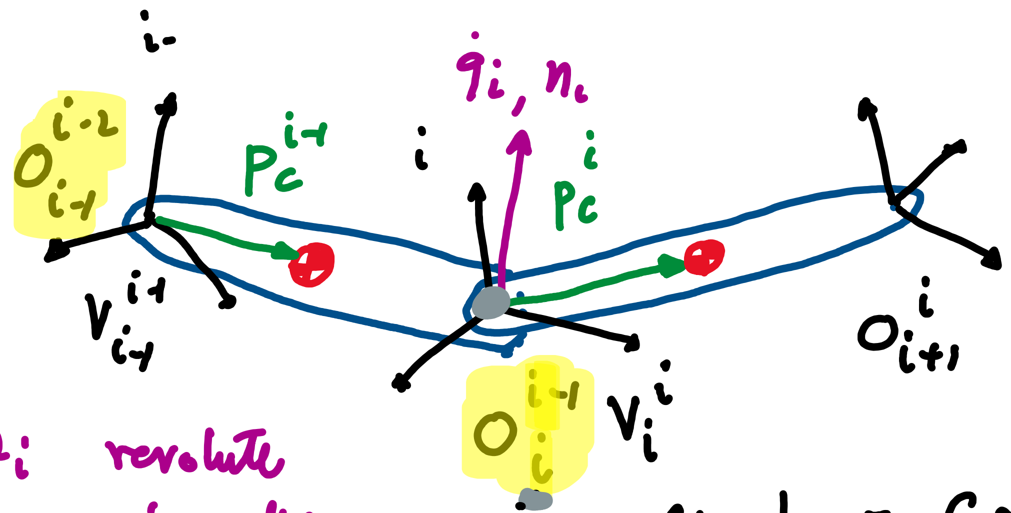
② Revolute / Prismatic Joint



$$\dot{q}_i = \begin{cases} \dot{\theta}_i & \text{revolute} \\ \dot{d}_i & \text{prismatic} \end{cases}$$

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Joint i is revolute

$$\omega^i_i = (R^{i-1}_i)^T \omega^{i-1}_{i-1} + \dot{\theta}_i \hat{n}_i$$

$$v^i_i = (R^{i-1}_i)^T \left[v^{i-1}_{i-1} + \omega^{i-1}_{i-1} \times O^{i-1}_i \right]$$

Joint i is prismatic

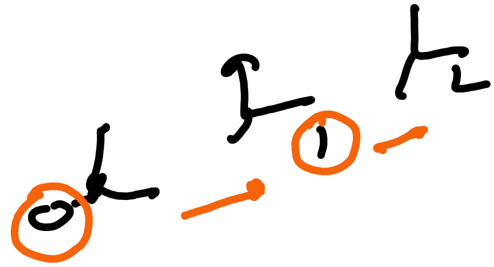
$$\omega^i_i = (R^{i-1}_i)^T \omega^{i-1}_{i-1}$$

$$v^i_i = (R^{i-1}_i)^T \left[v^{i-1}_i + \omega^{i-1}_{i-1} \times O^{i-1}_i \right] + \dot{d}_i \hat{n}_i$$

For both:

$$v^i_{i,com} = v^i_i + \omega^i_i \times p^i_c$$

How to use the formula



$$\omega_0 = v_0 = [0, 0, 0]$$

Now ω_i, v_i may be computed using the recursive formula

To compute global velocities,

$$v_i^0 = R_i^0 \, v_i^i$$

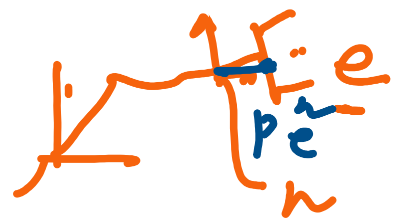
$$\omega_i^0 = R_i^0 \, \omega_i^i$$

where R_i^0 is in $H_i^0 = \begin{bmatrix} R_i^0 & O_i^0 \\ 0 & 1 \end{bmatrix}$

If $i = n$ is the last link then the end-effector velocity is

$$v_{\underline{e}}^{n} = v_{\underline{n}}^{n} + \underline{\omega}_n^n \times p_{\underline{e}}^{n}$$

end-eff. pos$^{\underline{n}}$ in frame n



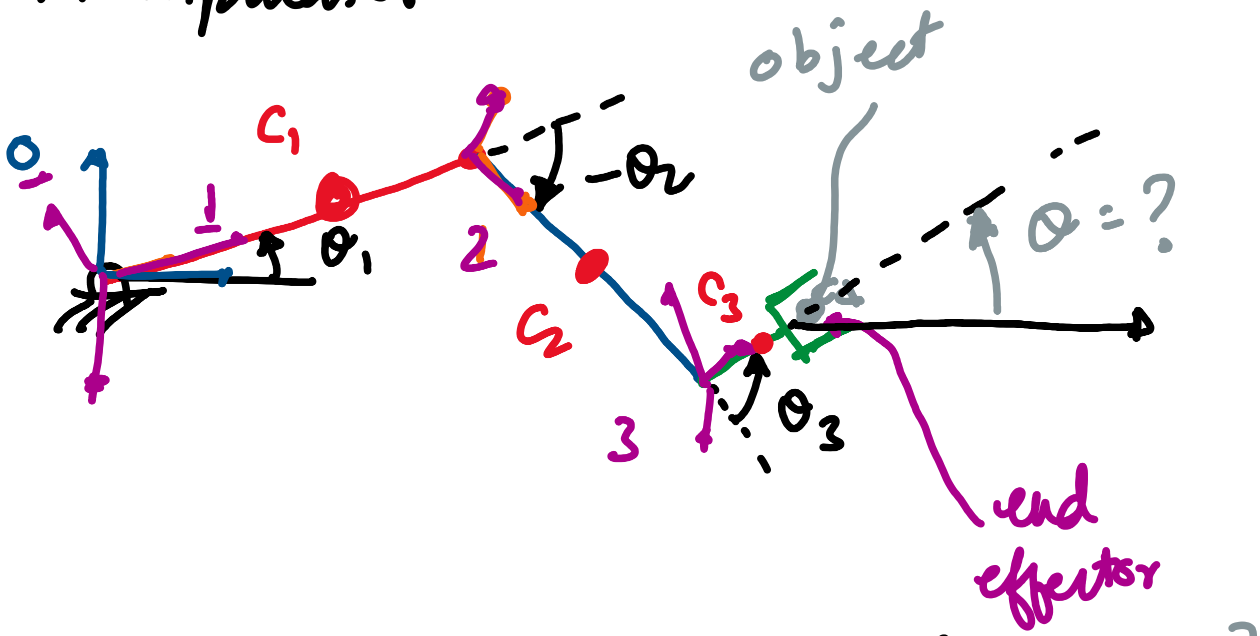
$$\underset{\sim}{\omega_e^n} = \underset{\sim}{\omega_n^n}$$

Global velocities are

$$\underset{\sim}{v_e^0} = \underset{\sim}{R_n^0}\, v_e^n$$

$$\underset{\sim}{\omega_e^0} = \underset{\sim}{R_n^0}\, \omega_e^n$$

Linear / Angular velocity for planar manipulator



$(x_e\ y_e) = ?$

Compute

V_1^0, V_2^0, V_3^0
$\omega_1^0, \omega_2^0, \omega_3^0$ } Lin / Ang vel of frames

$V_{c_1}^0, V_{c_2}^0, V_{c_3}^0$ — Lin vel of center of mass

V_e^0, ω_e^0 — Lin / Ang vel of end-effector

We will use these formulae

$$\omega_i^i = (R_i^{i-1})^T \omega_{i-1}^{i-1} + \dot{\theta}_i \hat{n}_i$$

$$v_i^i = (R_i^{i-1})^T \left[v_{i-1}^{i-1} + \omega_{i-1}^{i-1} \times o_i^{i-1} \right]$$

$$\omega_0^0 = v_0^0 = [0, 0, 0]$$

$$\omega_1^1 = (R_1^0)^T \omega_0^0 + \dot{\theta}_1 \hat{k}$$

$$v_1^1 = (R_1^0)^T \left[v_0^0 + \omega_0^0 \times o_1^0 \right]$$

$$\omega_2^2 = (R_2^1)^T \omega_1^1 + \dot{\theta}_2 \hat{k}$$

$$v_2^2 = (R_2^1)^T \left[v_1^1 + \omega_1^1 \times o_2^1 \right]$$

$$\omega_3^3 =$$

$$v_3^3 =$$

$$v_1^0 = R_1^0 v_1^1 \qquad \omega_1^0 = R_1^0 \omega_1^1$$

$$v_2^0 = R_2^0 v_2^2 \qquad \omega_2^0 = R_2^0 \omega_2^2$$

$$v_3^0 = \qquad \omega_3^0 =$$

$$v_{i,com}^i = v_i^i + \omega_i^i \times p_c^i$$

$$v_{c_1}^1 = v_1^1 + \omega_1^1 \times p_c^1 \Rightarrow v_{c_1}^0 = R_1^0 v_{c_1}^1$$

$$v_{c_2}^2 = v_2^2 + \omega_2^2 \times p_c^2 \Rightarrow v_{c_2}^0 = R_2^0 v_{c_2}^2$$

$$v_{c_3}^3 = \text{- - -} \Rightarrow v_{c_3}^0 =$$

$$v_{\underline{e}}^n = v_{\underline{n}}^n + \omega_{\underline{n}}^n \times p_{\underline{e}}^n$$

$$\omega_e^n = \omega_n^n$$

$$v_e^3 = v_3^3 + \omega_3^3 \times p_e^n \Rightarrow v_e^0 = R_3^0 v_e^3$$

$$\omega_e^3 = \omega_3^3 \Rightarrow \omega_e^0 = R_3^0 \omega_3^3$$