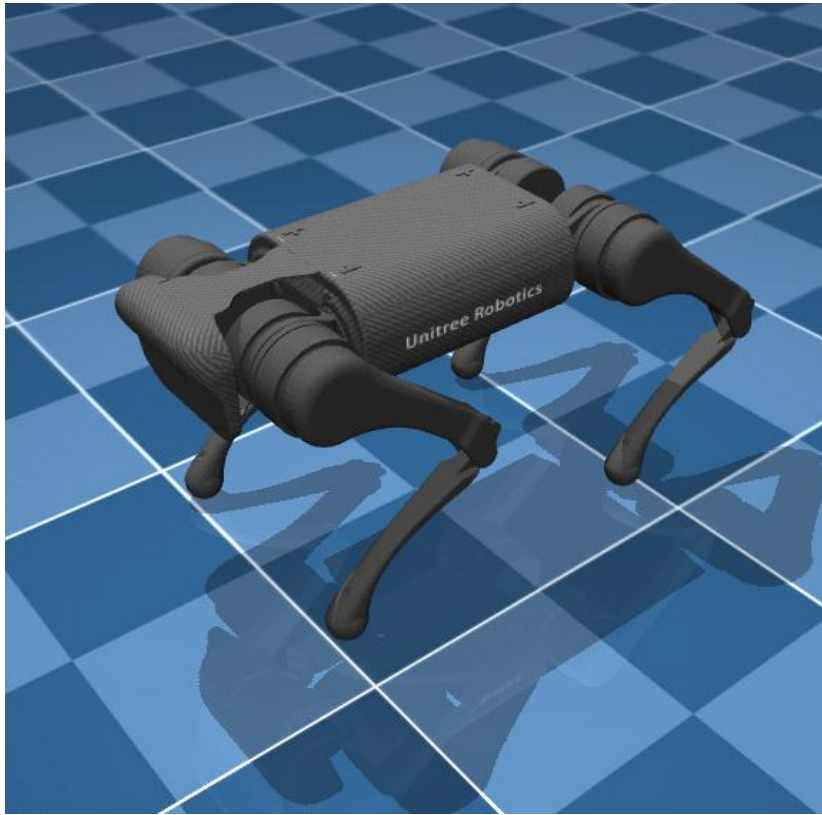
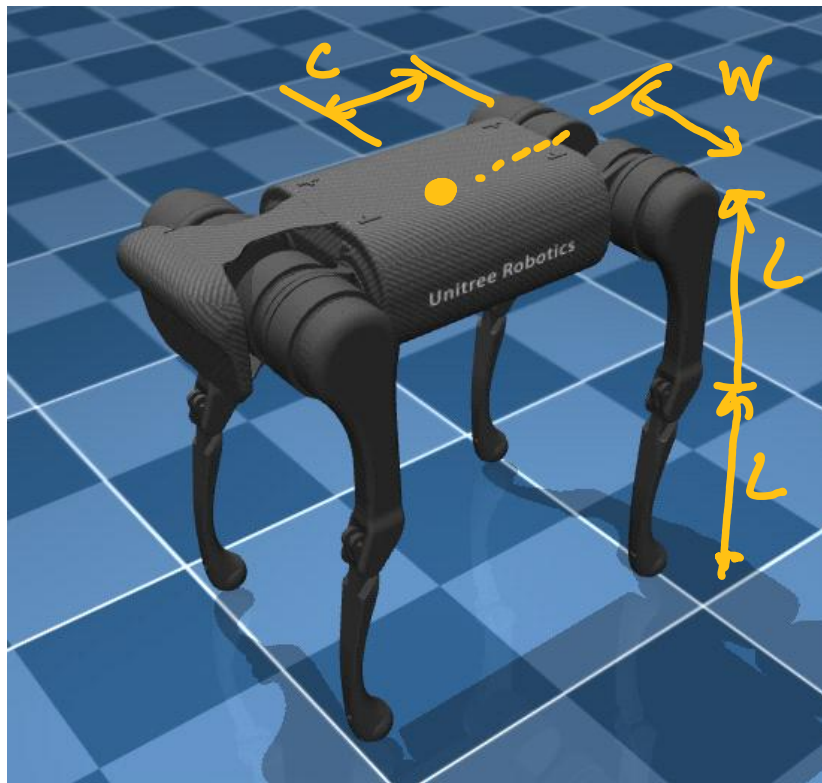


Unitree A1 (load key)



Unitree A1 (zero position)

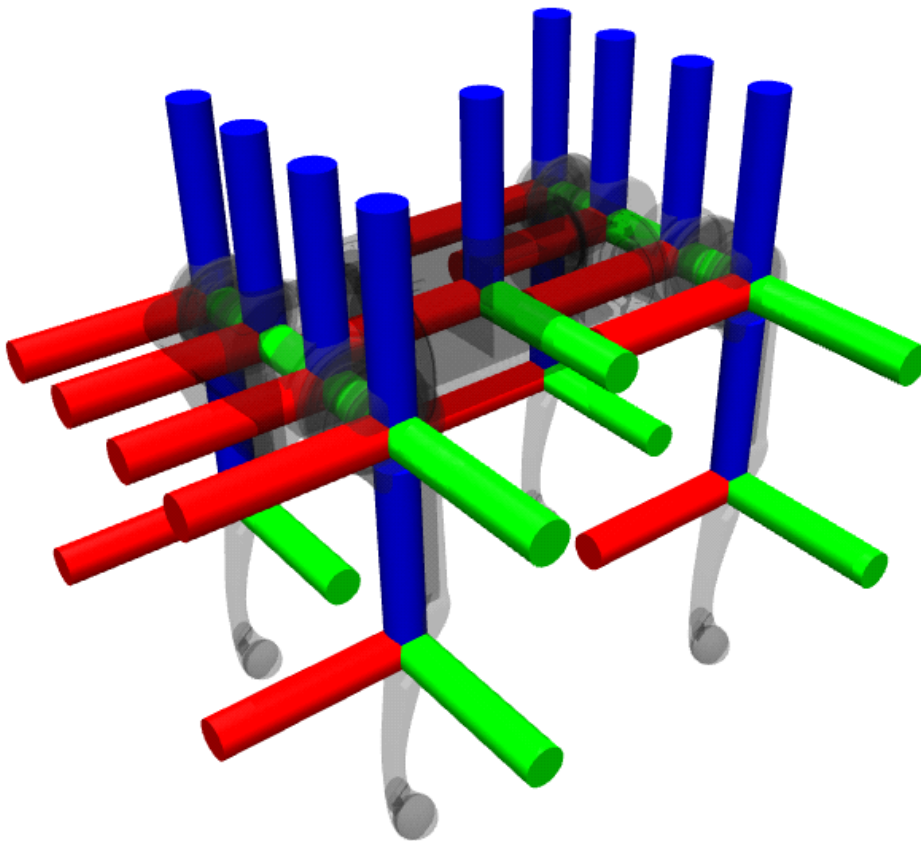
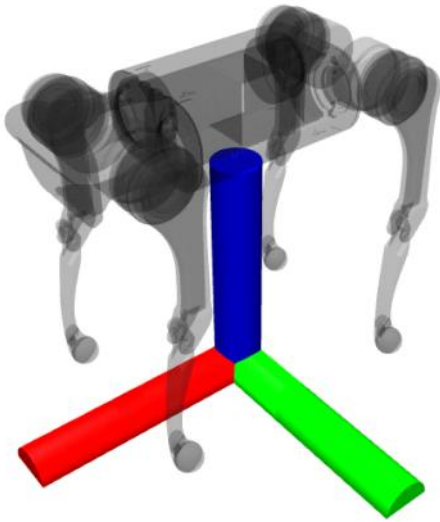


$$L = 0.2$$

$$W = 0.047$$

$$C = 0.183$$

Unitree A1 (frames)



Unitree A1 (joints)

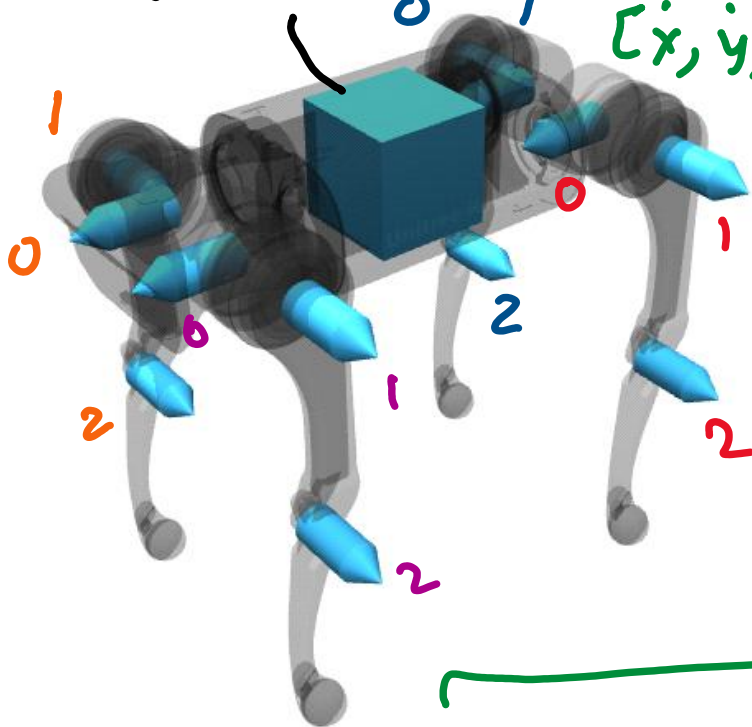
7 dofs

free joint $[x, y, z, \theta_0, \theta_x, \theta_y, \theta_z]$

$[\dot{x}, \dot{y}, \dot{z}, \dot{\omega}_{bx}, \dot{\omega}_{by}, \dot{\omega}_{bz}]$
6 velocity

revolute

$\theta, \dot{\theta} \times 12$



xml defined this way

→ $q_{pos} = 7$
tree
FR $\theta_0, \theta_1, \theta_2$ FL $\theta_0, \theta_1, \theta_2$ RR $\theta_0, \theta_1, \theta_2$ RL $\theta_0, \theta_1, \theta_2$
3 3 3 3

→ $q_{vel} = 6$
free
 $\dot{\theta}_0, \dot{\theta}_1, \dot{\theta}_2, \dots$

0 FR - front right 2 RR - Rear Right
1 FL - front left 3 RL - Rear left

leg-no = 0, 1, 2, 3
FR, FL, RR, RL

Trot gait: 0, 3
1, 2.

H, FL, RL, RL

1/2.

Unitree A1 Trotting Code Flow

1) Kinematic Control

✓ State Machine()
✓ Cartesian_Traj()
✓ Joint_Traj()
Mj_forward

x, y, z legs } ✓
IK 0

2) Dynamic Control

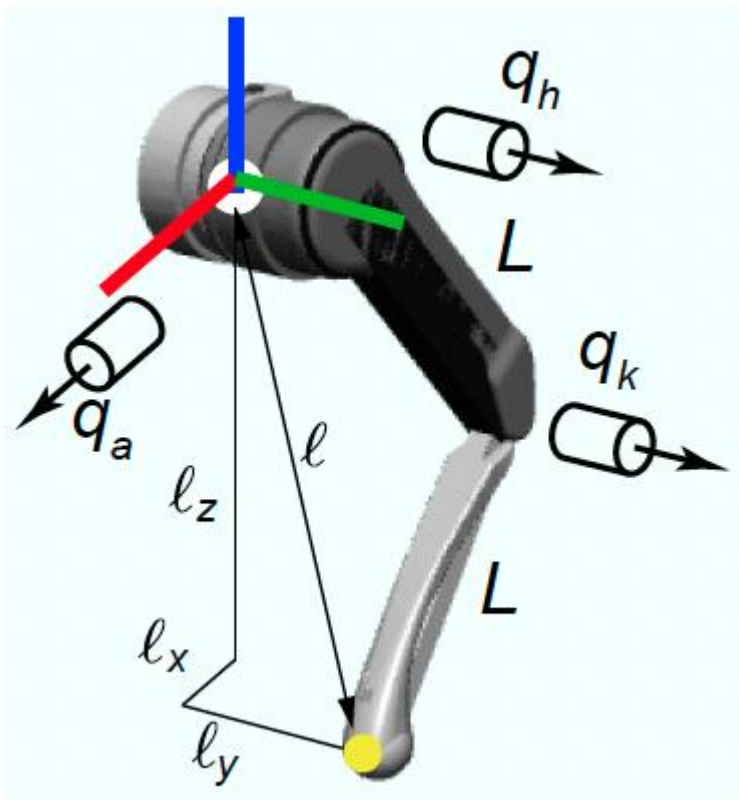
✓ State Machine()
✓ Cartesian_Traj()
✓ Joint_Traj()
Joint_Control() •
Mj_step • *dynamic*

3) High Level Control

State Machine()
Cartesian_Traj()
Joint_Traj()
Joint_Control()
High_Level_Control()
Mj_step

set \dot{x} -ref (\dot{x})
 \dot{y} -ref (\dot{y})
 $\dot{\psi}$ -ref ($\dot{\psi}$)

Unitree A1 (Analytic Inverse Kinematics)

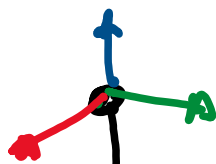


HW5

yellow dot is
below the white
dot when

$$q_a = q_h = q_k = 0$$

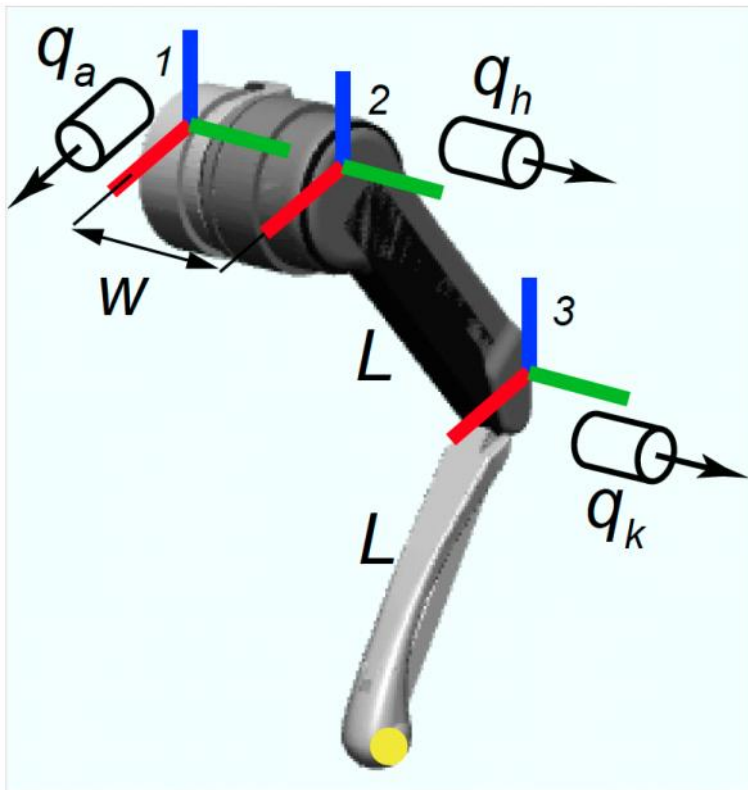
inverse kinematic - analytic()



$$q_a = q_h = q_k = 0$$

hcd { (0, 0, l_{z0})
user defined
-ive value

Unitree A1 Leg (Kinematics/Jacobian)



$W \neq 0$

$W \neq 0$

forward_kinematics_leg()
jacobian_end_eff_leg()

State machine (4 legs)

4 finite state machine (one for each leg)

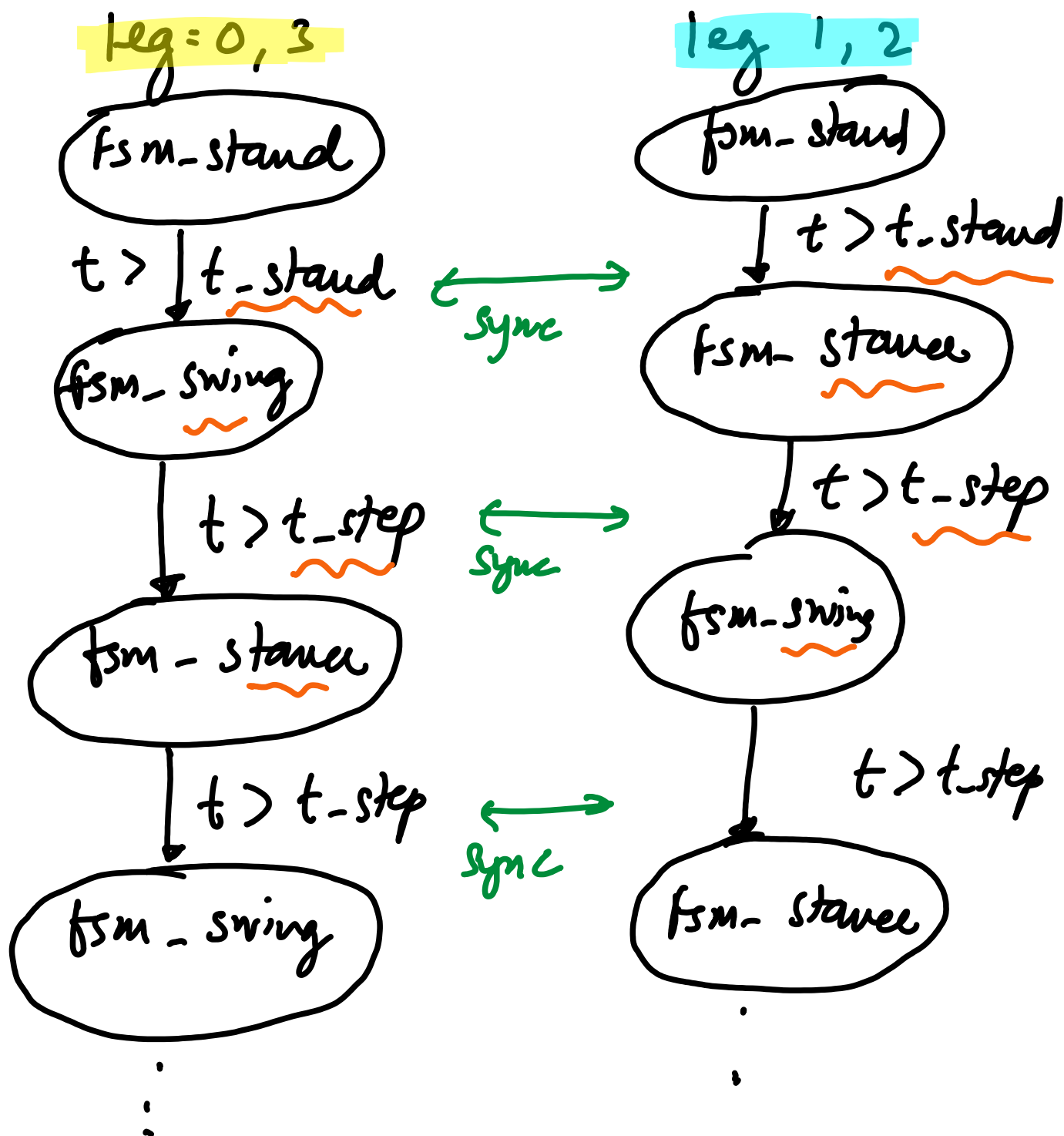
fsm [leg-no] leg-no = 0, 1, 2, 3
FR, FL, RL, RL

fsm-stand \rightarrow state where all legs are on the ground.

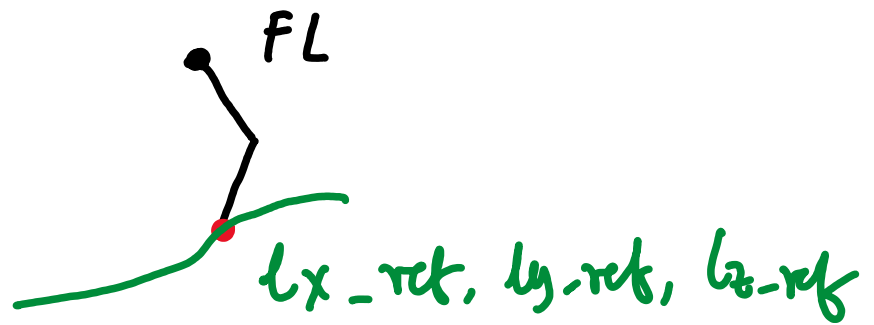
fsm [0:3) = fsm-stand {initialization}

fsm-swing : — leg is in the air

fsm-stance — less than 4 legs are on the ground

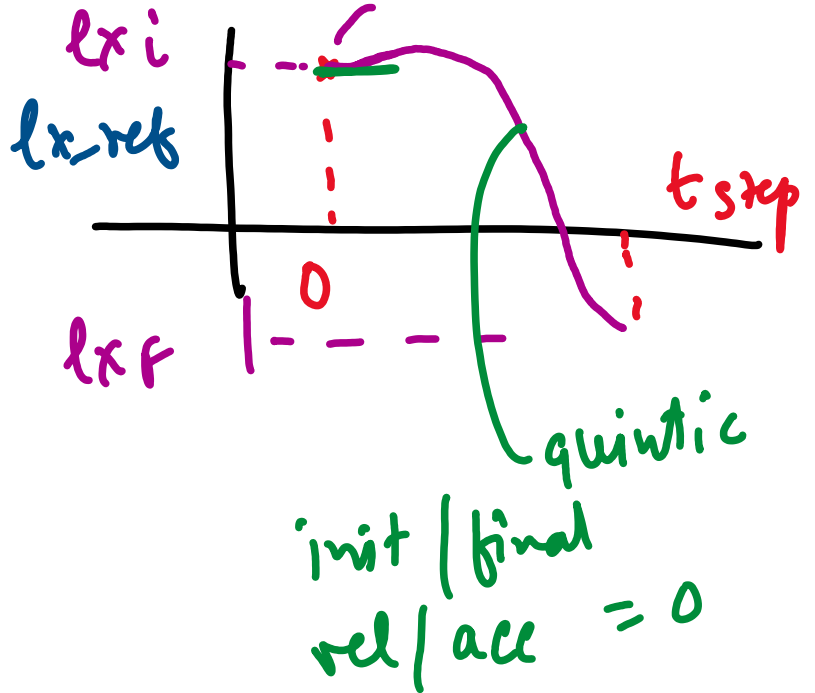


cartesian_traj



quintic-polynomial

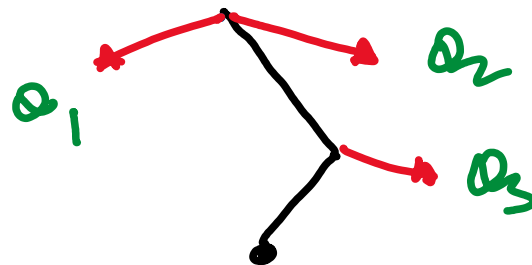
②



③ l_{y-ref}

① l_{z-ref}

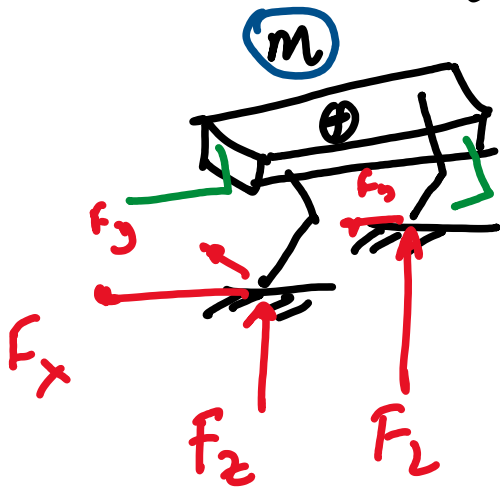
joint_traj()



l_{x-ref} , l_{y-ref} , l_{z-ref}

analytic_solution

① Simple gravity compensation in stance



↓ g

feet are massless

$$m = m_{\text{trunk}} + 4 m_{\text{leg}}$$

all mass is lumped in the trunk

$$\sum F_z = 0$$

$$F_z + F_z = mg$$

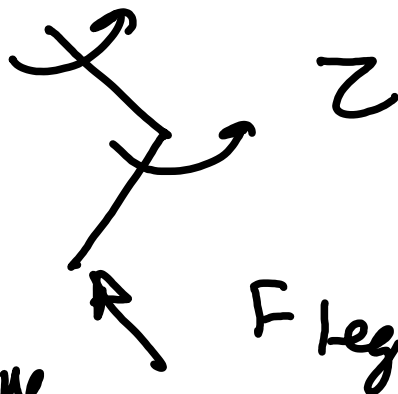
$$F_z = 0.5 mg$$

$$\sum F_x = \sum F_y = 0$$

Force due to gravity $F_{\text{leg}} = [0, 0, 0.5 mg]$

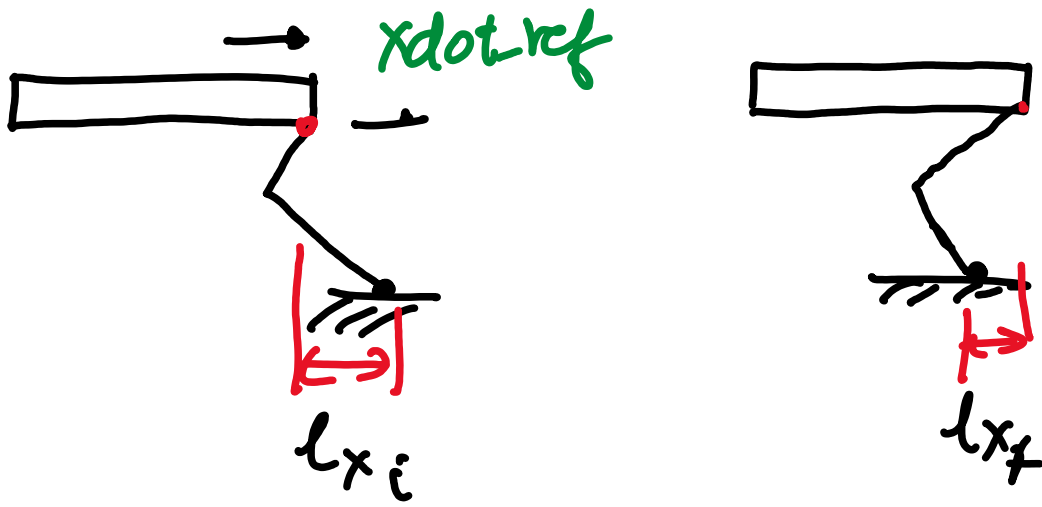
$$\tau = -J^T F_{\text{leg}}$$

↑ gravity torque



'gravity torque' \ 'leg

state-machine (l_{x-i} , l_{x-f})



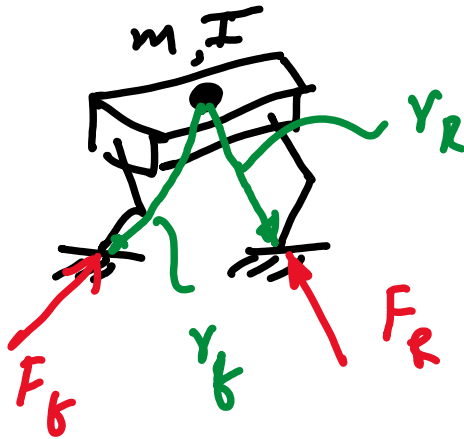
$$l_{x_i} = 0.5 (x_{dot-ref}) (t_{-step})$$

$$l_{x_f} = -0.5 (x_{dot-ref}) (t_{-step})$$

cartesian_traj

l_{x-ref} , $l_{x-dot-ref}$ = quintic-poly.

② Stance leg control for trotting



$$\underline{F_L} + \underline{F_R} - m \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} = m \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix}$$

$$\gamma_L \times \underline{F_L} + \gamma_R \times \underline{F_R} = I \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} + \omega \times (I\omega)$$

$$m \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} f_{x0} \\ f_{y0} \\ f_{z0} \end{bmatrix} = \begin{bmatrix} k_x (\dot{x}_{ref} - \dot{x}) \\ k_y (\dot{y}_{ref} - \dot{y}) \\ k_z (-\dot{z}) - k_z (z - z_{ref}) \end{bmatrix}$$

$$I \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = \begin{bmatrix} M_{x0} \\ M_{y0} \\ M_{z0} \end{bmatrix} = \begin{bmatrix} -k_1 \dot{\phi} - k_2 \dot{\omega}_x \\ -k_3 \dot{\theta} - k_4 \dot{\omega}_y \\ k_\psi (\dot{\psi}_{ref} - \dot{\omega}_z) \end{bmatrix}$$

$b_{6 \times 1}$

b 6x1

$$F_L + F_R$$

$$r_L \times F_L + r_R \times F_R$$

$$(S_{r_L}) F_L + (S_{r_R}) F_R$$

$$= \begin{bmatrix} I_{3 \times 3} & I_{3 \times 3} \\ S_{r_L} & S_{r_R} \end{bmatrix} \begin{bmatrix} F_L \\ F_R \end{bmatrix}$$

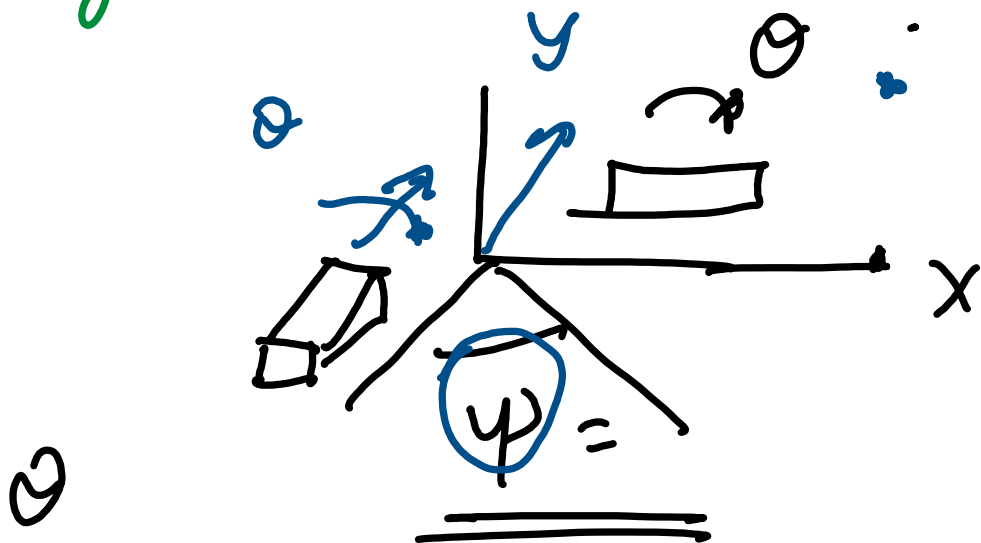
$\underbrace{\hspace{10em}}_{A_{6 \times 6}} \quad \underbrace{\hspace{5em}}_{F_{6 \times 1}}$

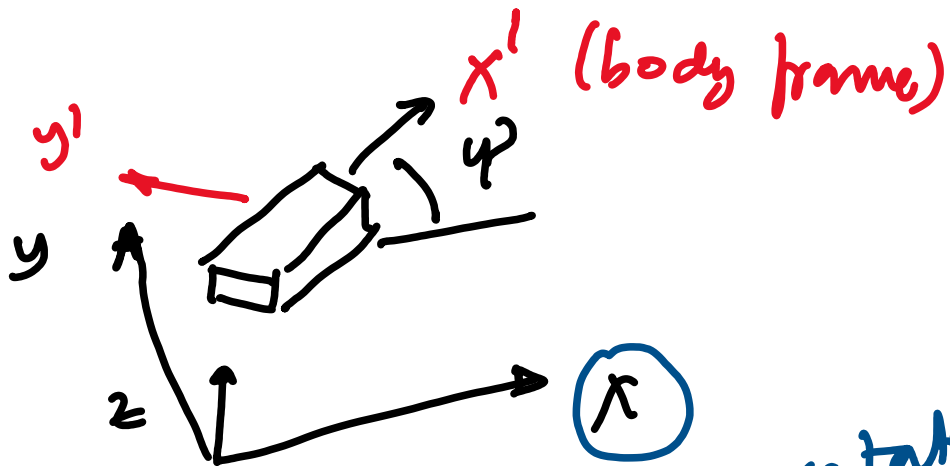
$$A_{6 \times 6} F_{6 \times 1} = b_{6 \times 1} = \begin{bmatrix} L_{x0} \\ F_{y0} \\ F_{z0} + \text{neg} \\ M_{x0} \\ M_{y0} \\ M_{z0} \end{bmatrix}$$

$$F = A^T b$$

↖ use pseudo-inverse

↑ forces from the stance leg.





$R =$ rotation matrix of the trunk

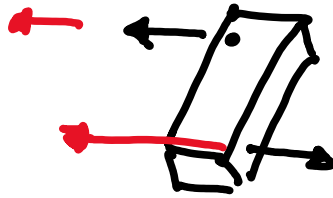
$R_z(\psi)$

$$R_b = R_z^T R$$

euler $-b =$ mat2euler

$$V_{\text{body}} = R_z^T \cdot \underbrace{V_{\text{world}}}_{\text{data.gvel}}$$

Turning (ψ)



— sideways

+
— turn

