Using D-H convention for forward kinematics

This involves 3 steps

1. Assigning co-ordinate frames
2. Generating a table for θ, d, α, a for each link.
3. Applying D-H formulae to do the forward kinematics.

I will give an algorithm for 1. The proceed with problems.

How to assign frames for D-H convention analysis?

Steps

(i) Assign z_i along axis of actuation. Do this for each link.

(ii) Assign the base frame $o_0 x_0 y_0 z_0$. We have already assigned z_0 in earlier step Choose x_0 arbitrarily. Based on direction of z_0 & x_0, assign y_0 using right hand rule.

(iii) Now assign next frame $o_i x_i y_i z_i$. z_i is already attached in (i). We attach x_i based on relation between z_{i-1} & z_i. There are cases.

(a) z_{i-1} & z_i are not coplanar: In this case, there is a unique shortest distance segment that is perpendicular to z_{i-1} & z_i. Choose this as x_i axis. Further, the point where x_i intersects z_i is origin o_i. y_i can be found by using right hand rule

(b) z_{i-1} is parallel to z_i: In this case, there are infinitely many perp-endiculars bet^n z_{i-1} & z_i. Choose o_i anywhere along z_i. x_i is chosen such that it it passes through o_i. To make equations simple, choose o_i such that x_i passes through o_{i-1}. This makes $d_i = 0$. y_i is chosen using right hand rule As z_{i-1} is $\parallel$ to z_i, $\alpha_i = 0$

(c) z_{i-1} intersect z_i: x_i is chosen to be normal to the plane formed by z_{i-1} & z_i. o_i is arbitrary. y_i is obtained after z_i & x_i are set up using right hand rule. Note that $a_i = 0$.

We apply the above procedure to frame 0 to n-1

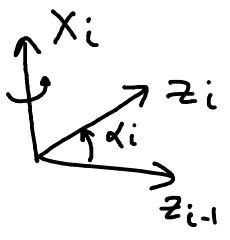
(iii) End-effector Frame: Finally, we need to find the position & orientation of frame n n which attach the end of link n. This is where the tool will attach to the manipulator. Choose z_n to be the same as z_{n-1}. Now, using how z_{n-1} & z_n are oriented wrt each other & info (ii), we can attach x_n & complete y_n using right hand rule.

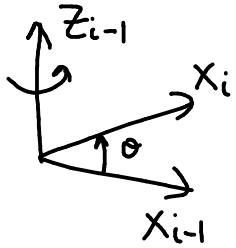
② Once the co-ordinate axis are set, we identify $\alpha_i, a_i, d_i, \theta_i$ in a table as follows

Link	a_i	α_i	d_i	θ_i
1				
2				
.				
.				
n				

Here is a cheat sheet which will help you populate the above table

a_i ⟶ distance bet^n z_{i-1} & z_i along x_i
α_i ⟶ angle bet^n z_{i-1} & z_i along x_i
d_i ⟶ distance bet^n x_{i-1} & x_i along z_{i-1}
θ_i ⟶ angle bet^n x_{i-1} & x_i along z_{i-1}





③ Finally, we can use transformation in D-H convention

$$A_i^{i-1} = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If the tool is attached to origin of frame n, then position of end effector, p, in base frame can be computed in 2 steps

Find A_n^0:

$$A_n^0 = A_1^0 A_2^1 A_3^2 \dots A_n^{n-1} = \begin{bmatrix} R_n^0 & O_n^0 \\ 0 & 1 \end{bmatrix}$$

Then:

Position of end effector: O_n^0 } ANSWER
Orientation of end effector: R_n^0 }