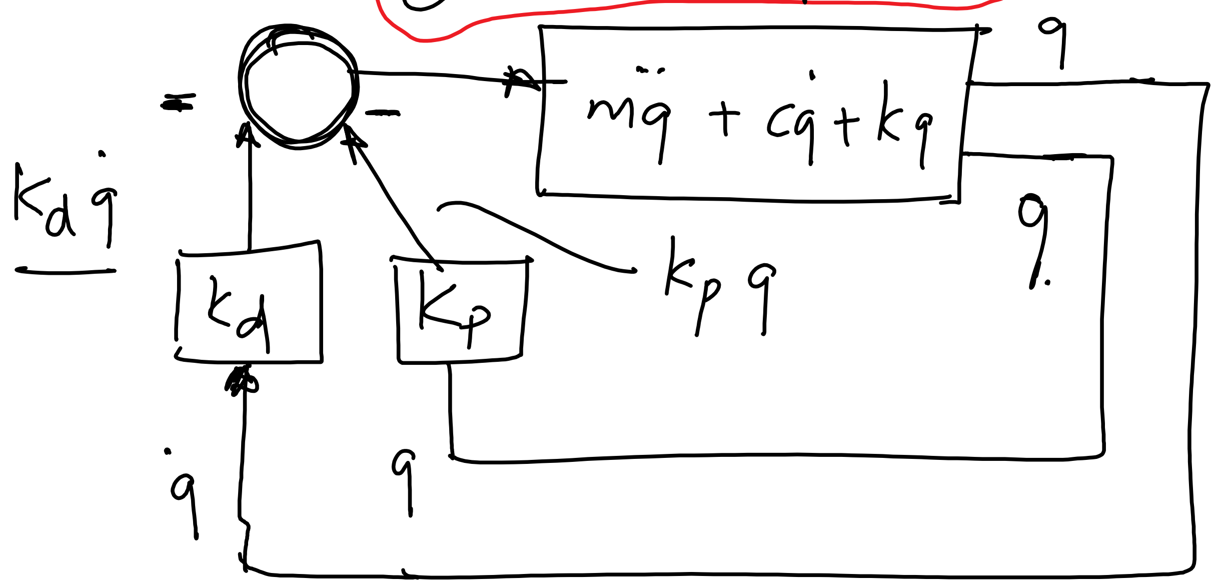
Block diagram

$$m\ddot{q} + c\dot{q} + kq = \tau$$ ✓

$$\boxed{\tau = -k_d\dot{q} - k_p q}$$



more information

Consider a 2-dof system

$$M\ddot{q} + C\dot{q} + Kq = \tau$$

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

$$\tau = -K_p q - K_d \dot{q}$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = -\begin{bmatrix} K_{p11} & K_{p12} \\ K_{p21} & K_{p22} \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} - \begin{bmatrix} K_{d11} & K_{d12} \\ K_{d21} & K_{d22} \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

We can find K_p's & K_d's such that the system is critically damped but the calculations are too long and complicated.

We will reduce the system complexity first and then design 2 independent single degree of freedom controllers

i.e. $\tau_1 = -K_{p_1} q_1 - K_{d_1} \dot{q}_1$

$\tau_2 = -K_{p_2} q_2 - K_{d_2} \dot{q}_2$

Control partitioning

System dynamics $M\ddot{q} + C\dot{q} + Gq = \tau$ (1)

Choose $\boxed{\tau = \hat{M}(-K_p q - K_d \dot{q}) + \hat{C}\dot{q} + \hat{G}q}$ (2)

$\hat{M}$, $\hat{C}$, $\hat{G}$ are estimates of M, C, and G respective

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Put (2) in (1)

$$M\ddot{q} + C\dot{q} + Gq = \hat{M}(-\underline{\underline{K_p}}\, q - \underline{\underline{K_d}}\, \dot{q}) + \hat{C}\dot{q} + \hat{G}q$$

$$\boxed{\hat{M}^{-1}\left(M\ddot{q} + (C-\hat{C})\dot{q} + (G-\hat{G})q \right) + K_p q + K_d \dot{q} = 0}$$

$$\underline{\underline{\hat{M}}}^{-1}\left(M\ddot{q} + (C-\hat{C})\dot{q} + (G-\hat{G})q\right) + K_p q + K_d \dot{q} = 0$$

Assume perfect model $M = \hat{M}$, $\dot{C} = \hat{C}$, $G = \hat{G}$

$$\Rightarrow \underbrace{M^{-1}(M}\ddot{q} + \cancel{(C-C)}\dot{q} + \cancel{(G-G)}q + K_p q + K_d \dot{q} = 0$$

$$\Rightarrow \ddot{q} + K_p q + K_d \dot{q} = 0$$

$$\Rightarrow \ddot{q} + K_d \dot{q} + K_p q = 0 \quad \text{— I}$$

Recap: Lec 17	$m\ddot{q} + (c+K_d)\dot{q} + (k+K_p)q = 0$ $K_d = -c + 2\sqrt{m(k+K_p)}$ — II

Comparing I with II

$$m = 1; \quad c = 0; \quad k = 0$$

$$K_d = -0 + 2\sqrt{(1)(0+K_p)}$$

$$\boxed{K_d = +2\sqrt{K_p}}$$ — To get the system to be critically damped.

$$\boxed{K_d = + 2\sqrt{K_p}}$$

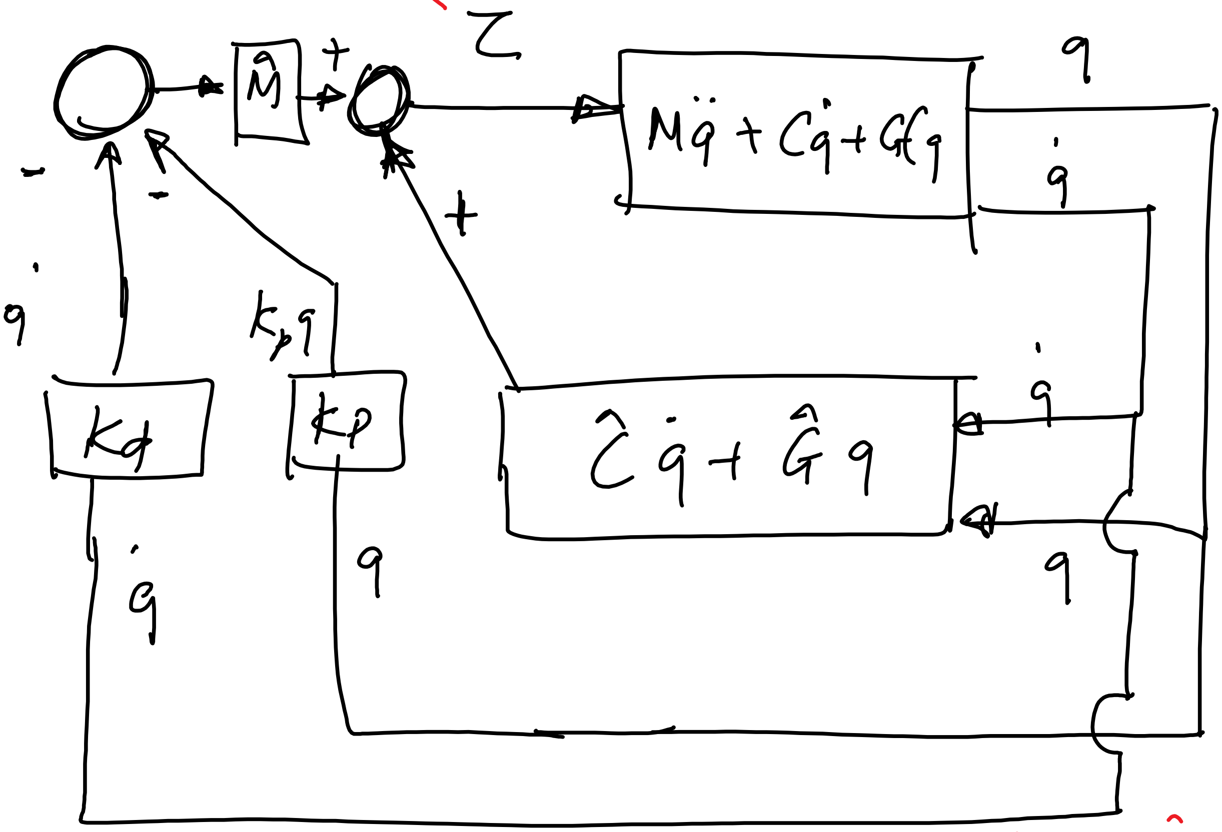
— To get the system to be critically damped.

Block diagram

$C(q,\dot q)$, $G(q)$

$$M\ddot q + C\dot q + Kq = \tau$$

$$\tau = \hat M(-k_p q - k_d \dot q) + \hat C \dot q + \hat G q$$

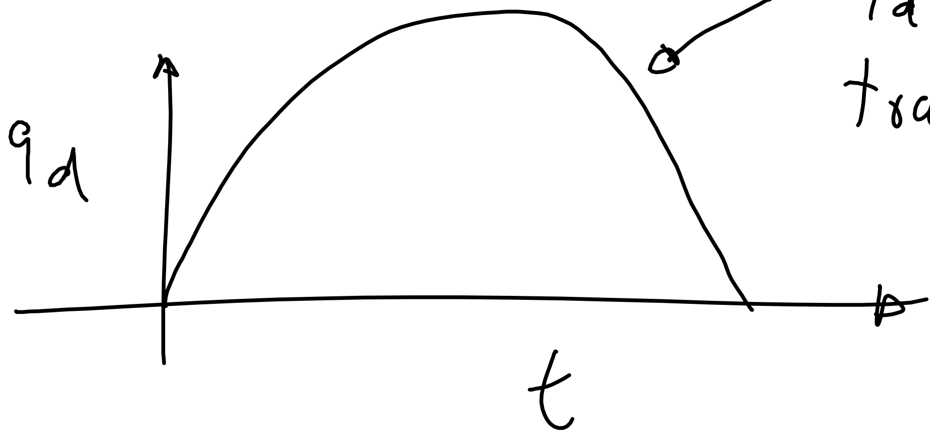


Set-point control $\to$ Set-point is 0.

$$\tau = -k_p(q - 0) - k_d \dot q$$

set point

Trajectory tracking



$q_d(t)$
track this trajectory

$$\tau = \hat{M}\left(-k_p(q - q_d) - k_d(\dot{q} - \dot{q}_d)\right) + \hat{C}\dot{q} + \hat{G}$$

set point is a function of time

Draw the block diagram for the above controller.