Integral control

Consider the system

$$M\ddot{q} + C\dot{q} + Gq = \tau + \tau_{dist} \quad \text{--- (1)}$$

disturbance torque (unknown)

$$\tau = M(-k_p \underset{(q-0)}{q} - k_d \dot{q}) + C\dot{q} + Gq \quad \text{--- (2)}$$

(control law partitioning)

Put (2) in (1)

$$\Rightarrow \quad M\ddot{q} + \cancel{C\dot{q}} + \cancel{Gq} = M(-k_p q - k_d \dot{q}) + \cancel{C\dot{q}} + \cancel{Gq} + \tau_{dist}$$

$$\Rightarrow \quad M(\ddot{q} + k_d \dot{q} + k_p q) = \tau_{dist}$$

Steady state $\rightarrow \dot{q} = \ddot{q} = 0$

$$\Rightarrow \quad M(k_p q) = \tau_{dist}$$

$$q = \frac{\tau_{dist}}{M k_p} \quad \text{(steady state error)}$$

Change the controller

$$\tau = M\left(-K_p q - K_d \dot{q} - \underbrace{K_i \int q\, dt}_{\text{added integral term}}\right) + C\dot{q} + Gq$$

Put in system equation (1)

$$M\ddot{q} + \cancel{C\dot{q}} + \cancel{Gq} = M\left(-K_p q - K_d \dot{q} - K_i \int q\, dt\right) + \cancel{C\dot{q}} + \cancel{Gq} + \tau_{dist}$$

$$\Rightarrow M\left(\ddot{q} + K_p q + K_d \dot{q} + K_i \int \underline{q\, dt}\right) = \tau_{dist}$$

q

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Diff the expression above once

$$\Rightarrow M\left(\dddot{q} + K_p \dot{q} + K_d \ddot{q} + K_i q\right) = \dot{\tau}_{dist}$$

Steady state $\dot{q} = \ddot{q} = \dddot{q} = \dot{\tau}_{dist} = 0$

$$\Rightarrow M K_i q = 0 \quad \Rightarrow q = 0$$

no steady state error.

Implementation in ode4 or ode45

$$m\ddot{q} + c\dot{q} + Gq = \tau + \tau_{dist}$$

original rhs (p-d controller)

$$X = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} \dot{q} \\ \dfrac{(\tau + \tau_{dist} - c\dot{q} - Gq)}{m} \end{bmatrix}$$

modify rhs (p-i-d control)

$$X = \begin{bmatrix} q \\ \dot{q} \\ \underline{\underline{\int q\,dt}} \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} \dot{q} \\ (\tau + \tau_{dist} - c\dot{q} - Gq)/m \\ q \end{bmatrix}$$