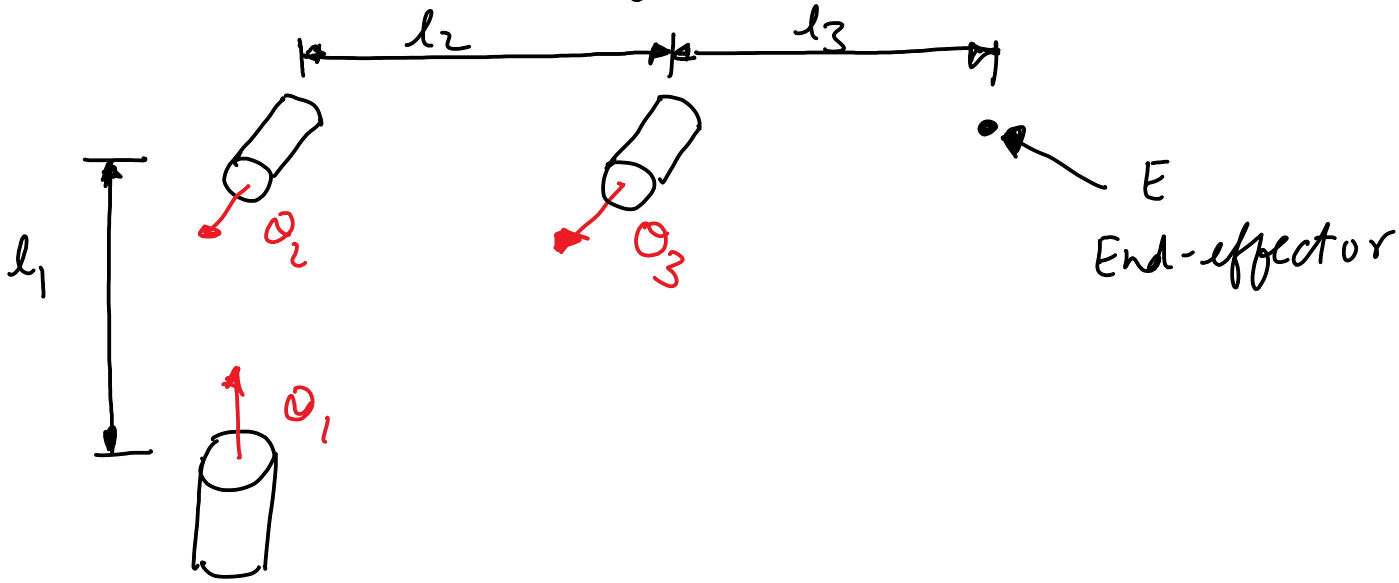
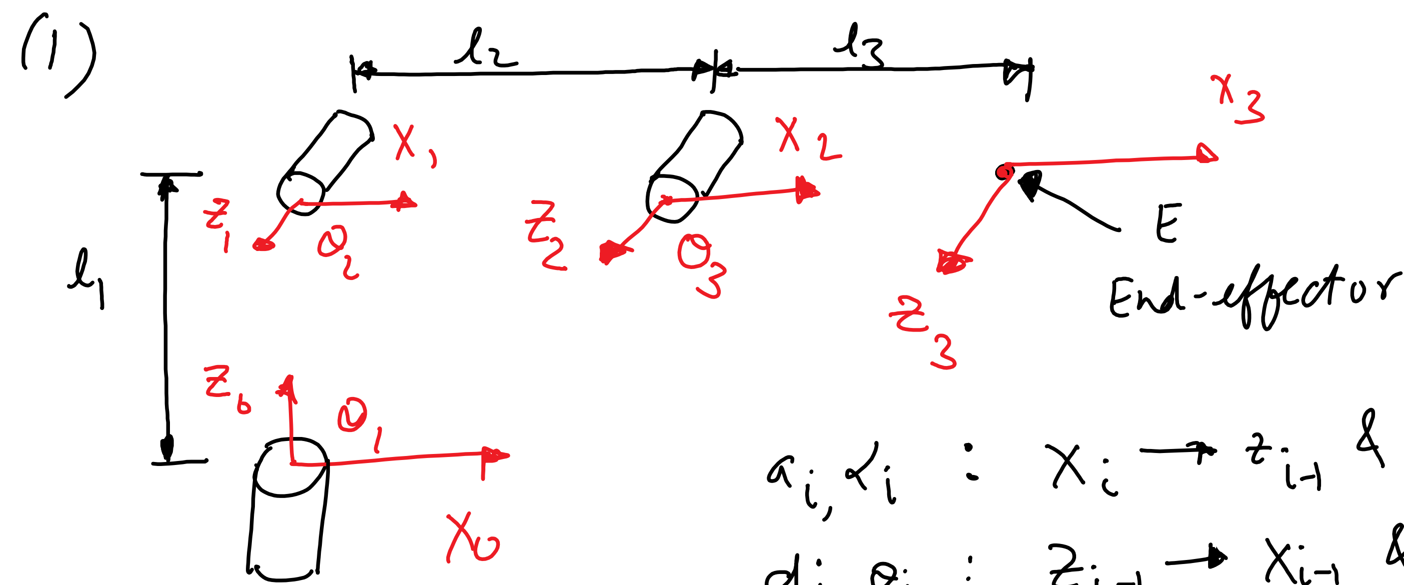
Denavit Hartenberg (continued)



Q. Find the position and orientation of the end-effector E.

(1)



a_i, α_i : $x_i \rightarrow z_{i-1}$ & z_i

d_i, θ_i : $z_{i-1} \rightarrow x_{i-1}$ & x_i

(2)

Link i	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	l_1	θ_1
2	l_2	0	0	θ_2
3	l_3	0	0	θ_3

Use MATLAB symbolics to find these.

(3)
$$A_1^0 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2^1 = \begin{bmatrix} c_2 & -s_2 & 0 & l_2 c_2 \\ s_2 & c_2 & 0 & l_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} c_3 & -s_3 & 0 & l_3 c_3 \\ s_3 & c_3 & 0 & l_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3^0 = A_1^0 A_2^1 A_3^2 = \begin{bmatrix} R_3^0 & d_3^0 \\ 0 & 1 \end{bmatrix}$$

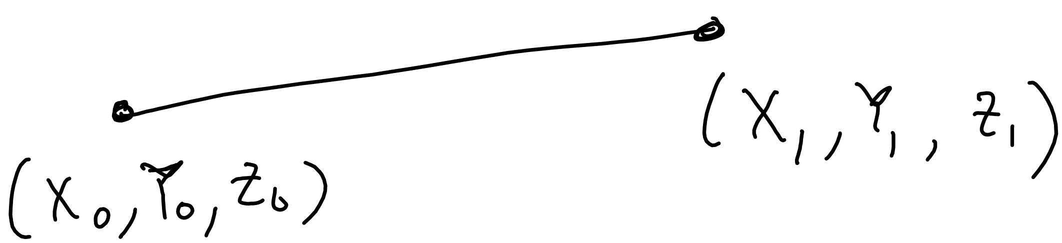
$c_1 = \cos\theta_1$
$s_1 = \sin\theta_1$ and so on

$\boxed{\begin{array}{l} E \text{ position } d_3^0 \\ E \text{ orientation } R_3^0 \end{array}}$

Drawing the manipulator using MATLAB

We will use MATLAB command LINE

$$\text{line}\left([X_0 \;\; X_1], [Y_0 \;\; Y_1], [Z_0 \;\; Z_1]\right)$$



Recipe

1. Attach co-ordinate frames
2. Find D-H table
3. Compute $A_1^0, A_2^0, A_3^0, \ldots$
4. Find the co-ordinates of the joints

joint 0 : $[0;0;0]$

joint 1 : $A_1^0(1{:}3, 4) \rightarrow \begin{bmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{bmatrix}$

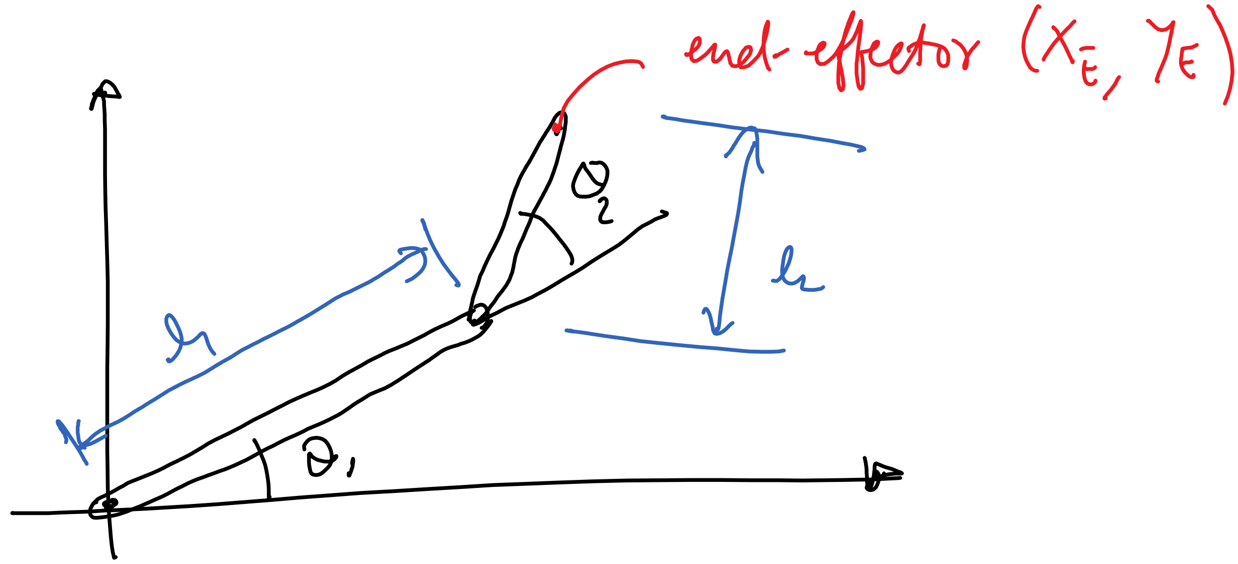
joint 2 : $A_2^0(1{:}3, 4)$

$\vdots$

column 4

5. Use line command to join the joints together.

Inverse kinematics



We have derived the formula for E

$$\left.\begin{aligned} X_E &= l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ Y_E &= l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \end{aligned}\right\} - (I)$$

Forward kinematics

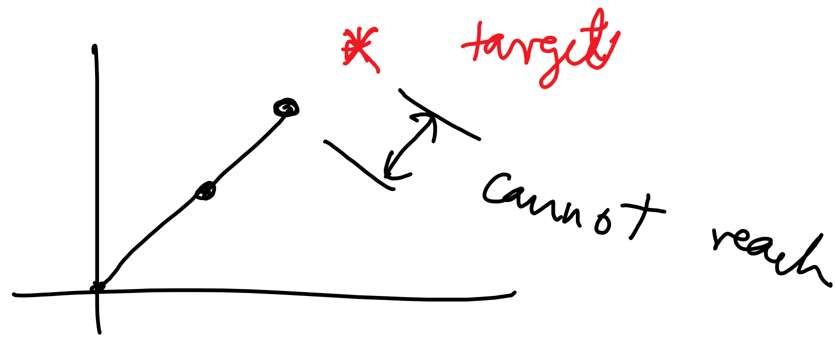
Given θ_1 and θ_2 and we find X_E, Y_E using (I)

Inverse kinematics:

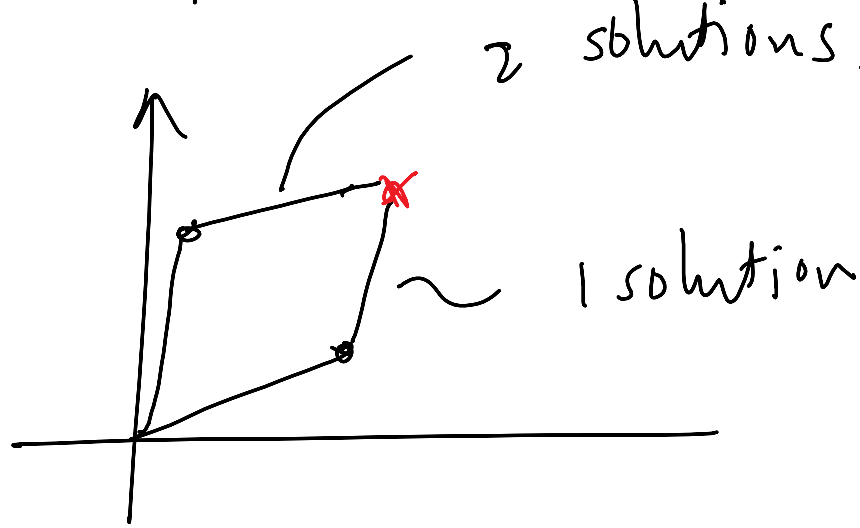
Given X_E, Y_E, we need to find θ_1 and θ_2

Issues with IK

- no solution (e.g. out of bounds)



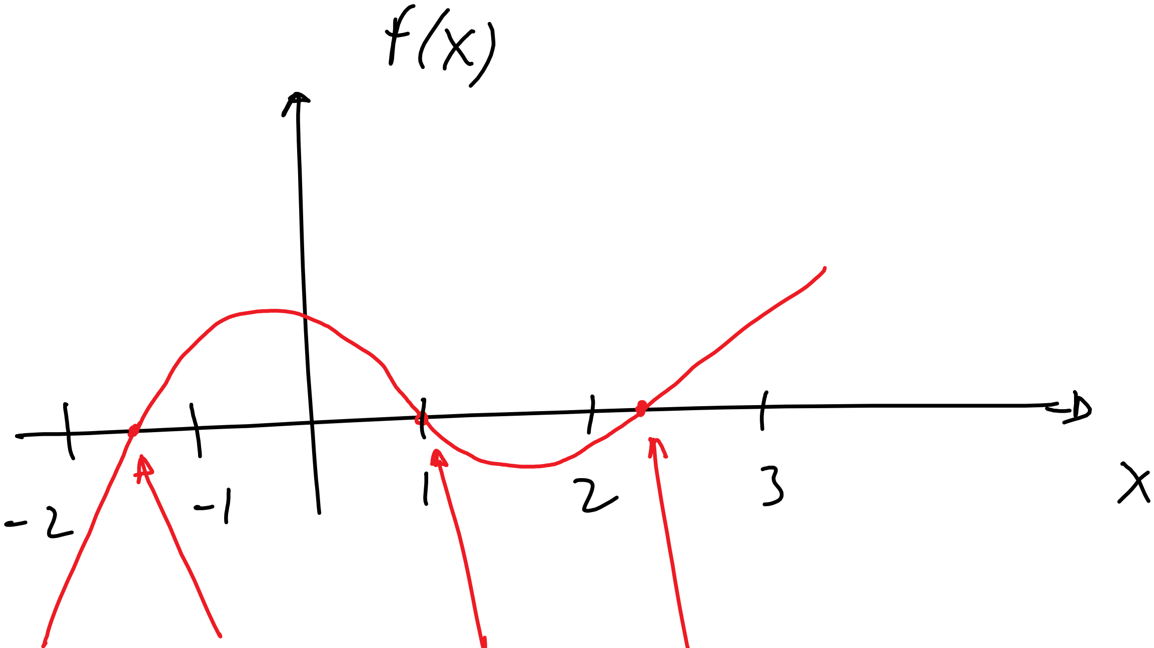
- multiple solutions



Solving non linear equations in MATLAB

e.g. $f(x) = x^3 - 2x^2 - 2x + 3 = 0$

Find x.



3 roots of the equation

fsolve to solve for the roots

$$[x_f, fval, exitflag] = fsolve('f', x_0)$$

Solution — value at x_f — converged or not — $f(x)=0$ — initial guess