Denavit Hartenberg convention

4 transformations

$$A_i^{i-1} = Rot_{z,\theta_i}\, Trans_{z,d_i}\, Trans_{x,a_i}\, Rot_{x,\alpha_i}$$

$$A_i^{i-1} = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{A_i^{i-1} = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$c\theta_i = \cos\theta_i \qquad s\theta_i = \sin\theta_i$

$c\alpha_i = \cos\alpha_i \qquad s\alpha_i = \sin\alpha_i$

Ref. Spong pg. 78

Normally, one needs 6 numbers (3 trans + 3 rotation) to describe a rigid body but D-H uses only 4. Why?

— Axis x_i is perpendicular to z_{i-1}
— Axis x_i intersect the axis z_{i-1}

2 conditions
$6-2$
$=4$

D-H uses only 4. Why?

- Axis x_i is perpendicular to z_{i-1}
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2 conditions

$6-2$

$=4$

Using D-H convention for forward kinematics

This involves 3 steps
1. Assigning co-ordinate frames
2. Generating a table for θ, d, α, a for each link.
3. Applying D-H formulae to do the forward kinematics.

I will give an algorithm for ①. The proceed with problems.

How to assign frames for D-H convention analysis?

Steps

(i) Assign z_i along axis of actuation. Do this for each link.

(ii) Assign the base frame $o_0 x_0 y_0 z_0$. We have already assigned z_0 in earlier step. Choose x_0 arbitrarily. Based on direction of z_0 & x_0, assign y_0 using right hand rule.

(iii) Now assign next frame $o_i x_i y_i z_i$. z_i is already attached in (i). We attach x_i based on relation between z_{i-1} & z_i. There are cases.

(a) z_{i-1} & z_i are not coplanar: In this case, there is a unique shortest distance segment that is perpendicular to z_{i-1} & z_i. Choose this as x_i axis. Further, the point where x_i intersects z_i is origin o_i. y_i can be found by using right hand rule

(b) z_{i-1} is parallel to z_i: In this case, there are infinitely many perpendiculars bet^n z_{i-1} & z_i. Choose o_i anywhere along z_i. x_i is chosen such that it it passes through o_i. To make equations simple, choose o_i such that x_i passes through o_{i-1}. This makes $d_i = 0$. y_i is chosen using right hand rule
As z_{i-1} is $\parallel$ to z_i, $\alpha_i = 0$

(c) z_{i-1} intersect z_i: x_i is chosen to be normal to the plane formed by z_{i-1} & z_i. o_i is arbitrary. y_i is obtained after z_i & x_i are set up using right hand rule. Note that $a_i = 0$.

We apply the above procedure to frame 0 to $n-1$

(iii) End-effector Frame: Finally, we need to find the position & orientation of frame n ~~n which attach~~ the end of link n. This is where the tool will attach to the manipulator. Choose z_n to be the same as z_{n-1}. Now, using how z_{n-1} & z_n are oriented wrt each other & info (ii), we can attach x_n & complete y_n using right hand rule.

② Once the co-ordinate axis are set, we identify $\alpha_i, a_i, d_i, \theta_i$ in a table as follows

Link	a_i	α_i	d_i	θ_i
1				
2				
.				
.				
n				

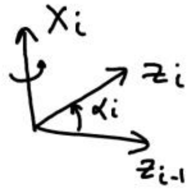
Here is a cheat sheet which will help you populate the above table

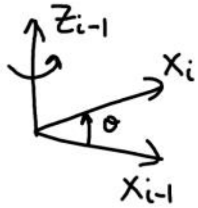
$a_i \longrightarrow$ distance bet^n z_{i-1} & z_i along x_i

$\alpha_i \longrightarrow$ angle bet^n z_{i-1} & z_i along x_i

$d_i \longrightarrow$ distance bet^n x_{i-1} & x_i along z_{i-1}

$\theta_i \longrightarrow$ angle bet^n x_{i-1} & x_i along z_{i-1}





③ Finally, we can use transformation in D-H convention

$$A_i^{i-1} = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If the tool is attached to origin of frame n, then position of end effector, p, in base frame can be computed in 2 steps

Find A_n^0:

$$A_n^0 = A_1^0 A_2^1 A_3^2 \cdots A_n^{n-1} = \begin{bmatrix} R_n^0 & O_n^0 \\ 0 & 1 \end{bmatrix}$$

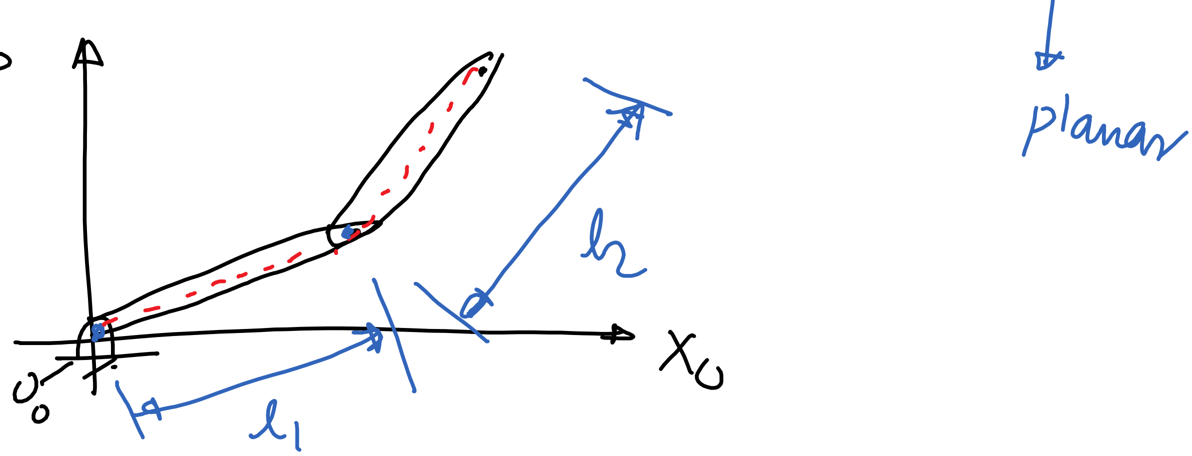
Then:

Position of end effector: O_n^0

Orientation of end effector: R_n^0

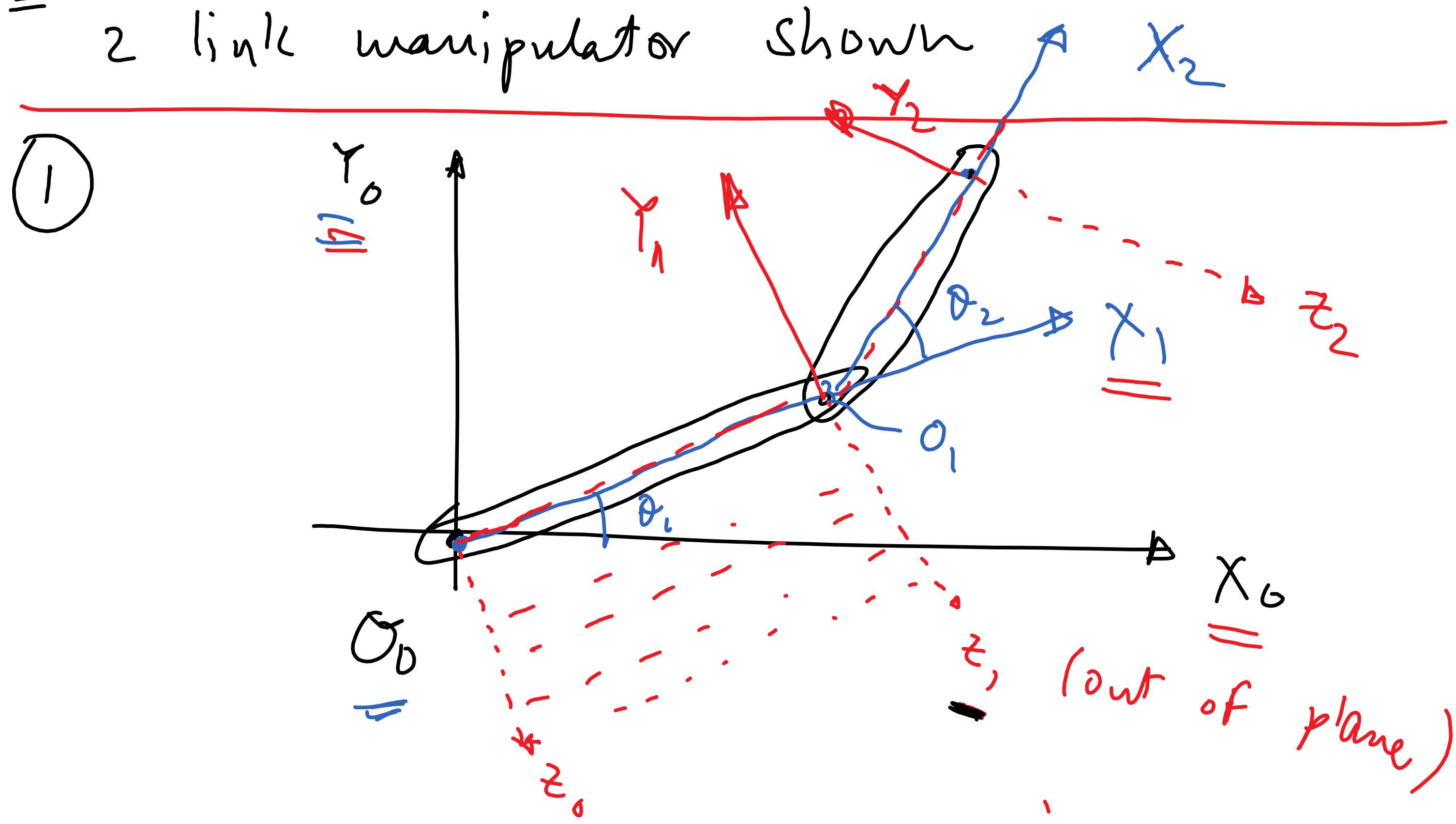
$\Big\}$ ANSWER

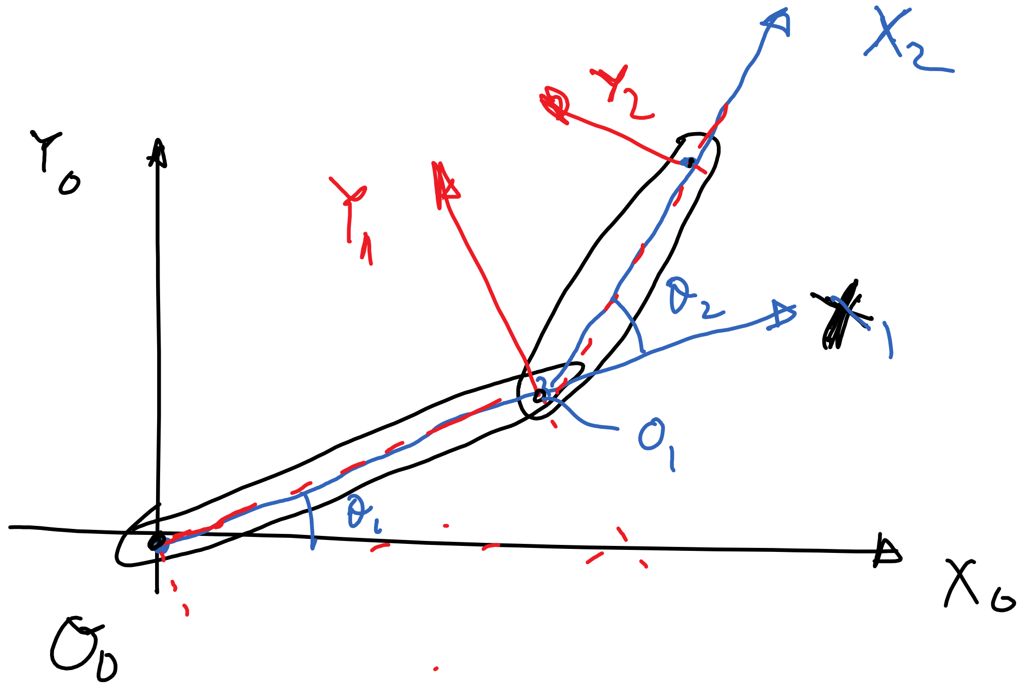
Prob 1 : Forward Kinematics of 2link manipulator



Two degrees of freedom manipulator (θ_1, θ_2)

Q- Derive the forward kinematics of the 2 link manipulator shown

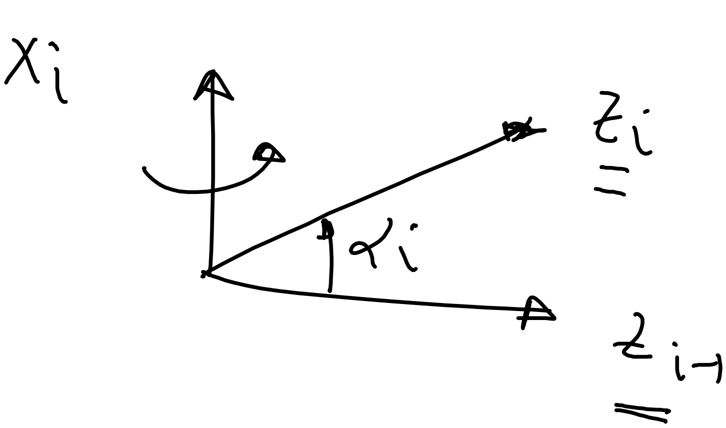


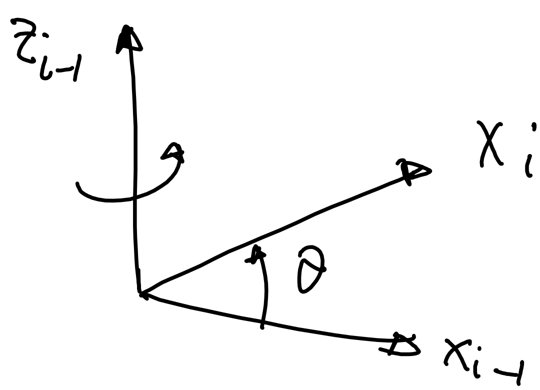


2

Link	a_i	α_i	d_i	θ_i
1	l_1	0	0	θ_1
2	l_2	0	0	θ_2

Check





③ $$A_1^0 = \begin{bmatrix} C_1 & -S_1 & 0 & l_1 C_1 \\ S_1 & C_1 & 0 & l_1 S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{aligned} C_1 &= \cos\theta_1 \\ S_1 &= \sin\theta_1 \end{aligned}$$

$$A_2^1 = \begin{bmatrix} C_2 & -S_2 & 0 & l_2 C_2 \\ S_2 & C_2 & 0 & l_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{aligned} C_2 &= \cos\theta_2 \\ S_2 &= \sin\theta_2 \end{aligned}$$

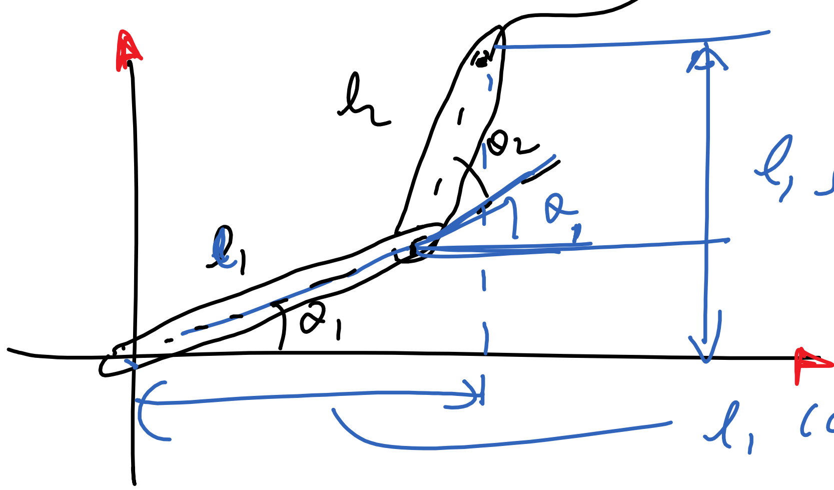
$$A_2^0 = A_2^1 A_1^0 = \begin{bmatrix} \begin{pmatrix} C_{12} & -S_{12} & 0 \\ S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} & \begin{pmatrix} l_1 C_1 + l_2 C_{12} \\ l_1 S_1 + l_2 S_{12} \\ 0 \end{pmatrix} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}$$

R_2^0 $\qquad$ d_2^0

$C_{12} = \cos(\theta_1 + \theta_2)$

$S_{12} = \sin(\theta_1 + \theta_2)$

$$E = d_2^0 = \begin{bmatrix} l_1 C_1 + l_2 C_{12} \\ l_1 S_1 + l_2 S_{12} \\ 0 \end{bmatrix}$$



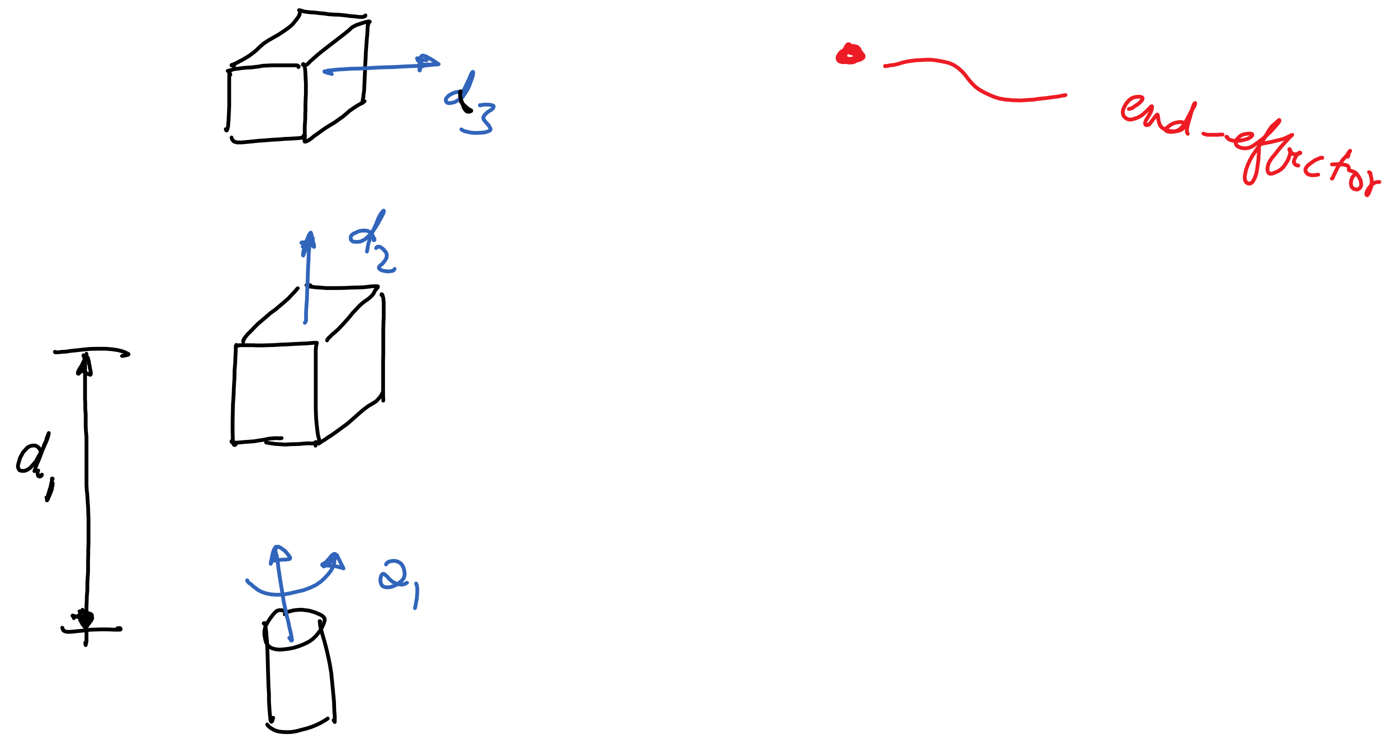
$l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)$

same

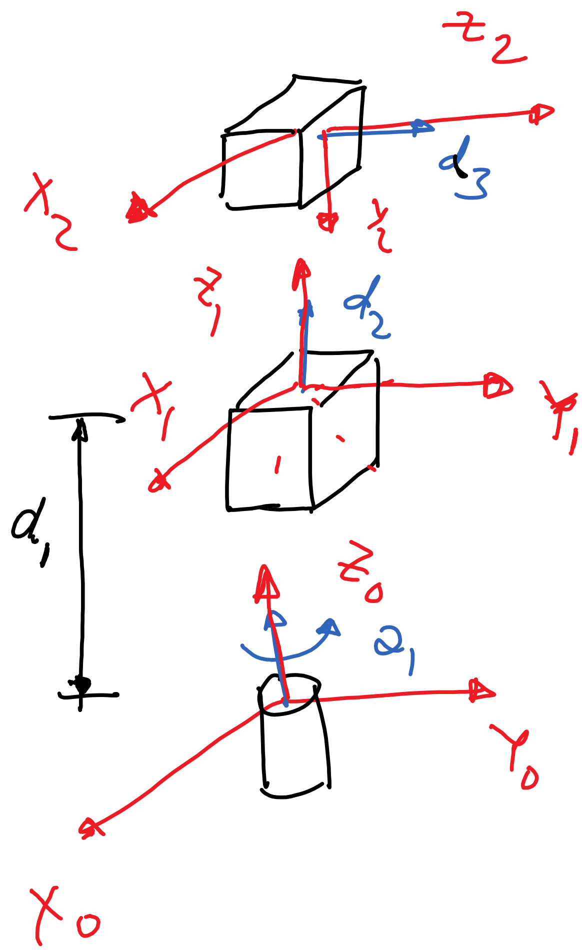
same

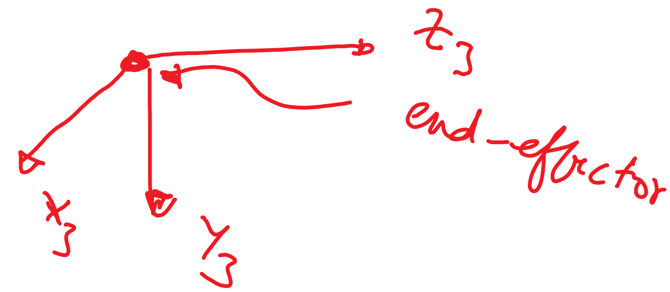
$l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$

Find the position and orientation of the end-effector for the manipulator shown below.



1





a_i, α_i : $X_i \longrightarrow z_{i-1}$ to z_i

d_i, θ_i : $z_{i-1} \longrightarrow x_{i-1}$ to x_i

2

Link i	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1
2	0	$-\pi/2$	d_2	0
3	0	0	d_3	0

Check

$$A_1^0 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^0 = A_1^0 A_2^1 A_3^2 = \begin{bmatrix} c_1 & 0 & -s_1 & -s_1 d_3 \\ s_1 & 0 & c_1 & c_1 d_3 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$c_1 = \cos\theta_1$

$s_1 = \sin\theta_1$

$$= \begin{bmatrix} R_3^0 & d_3^0 \\ 0 & 1 \end{bmatrix}$$

$$\boxed{\text{orientation } R_3^0 = \begin{pmatrix} c_1 & 0 & -s_1 \\ s_1 & 0 & c_1 \\ 0 & -1 & 0 \end{pmatrix} \quad d_3^0 = \begin{pmatrix} -s_1 d_3 \\ c_1 d_3 \\ d_1 + d_2 \end{pmatrix}}$$

$$\begin{bmatrix} 0 & -1 & 0 \end{bmatrix} \qquad d_1 + d_2 /$$