Computing J_v

$$\underline{\underline{v_n^0}} = \underline{\underline{\dot{O}_n^0}} = \sum_{i=1}^{n} \frac{\partial O_n}{\underline{\underline{\partial q_i}}} \dot{q}_i \quad \text{(By chain rule)}$$

$$= \underbrace{\begin{bmatrix} \frac{\partial O_n}{\partial q_1} & \frac{\partial O_n}{\partial q_2} & \cdots & \frac{\partial O_n}{\partial q_n} \end{bmatrix}}_{J_v} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{pmatrix}$$

$$v_n^0 = J_v \dot{q}$$

$\dot{q}_1 = 1, \dot{q}_2 = \dot{q}_3 = \dots \dot{q}_n = 0 \qquad v_n^0 = \frac{\partial O_n}{\partial q_1} = J_{v_1}$

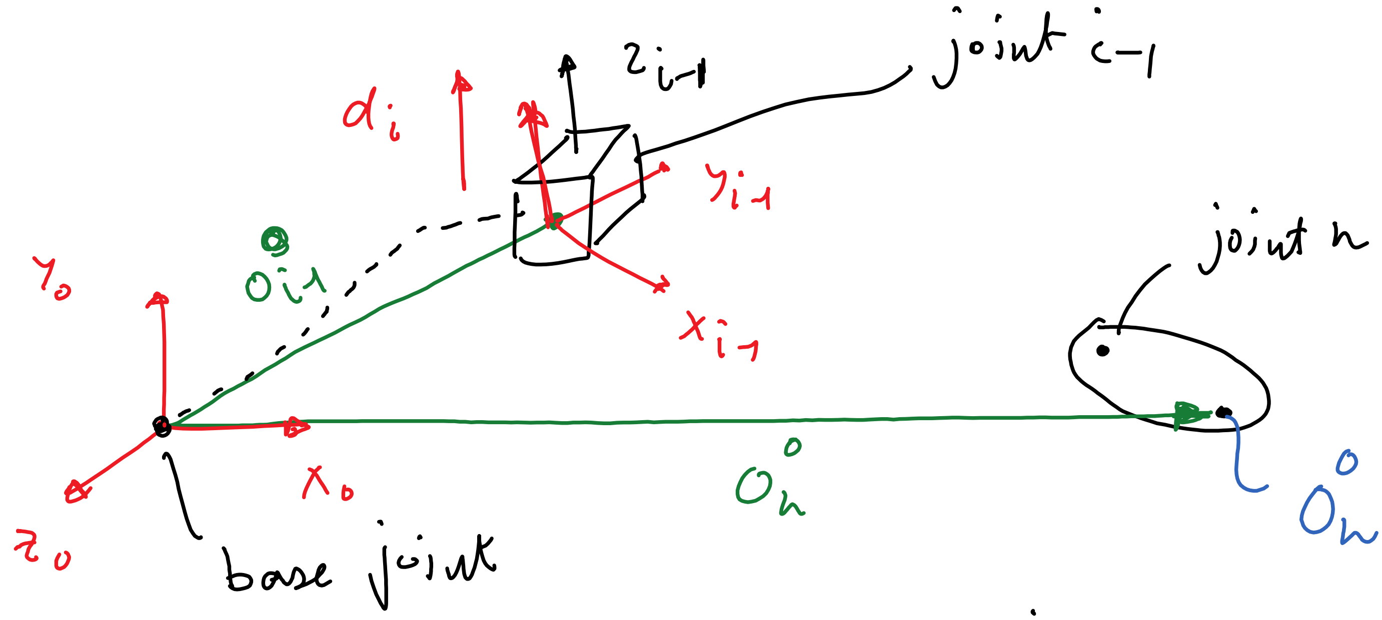
We observe that $v_n^0 = J_{v_i}$ when $\dot{q}_i = 1$ and every other joint rate is set to zero

---

We will derive a formula for J_v for
(a) prismatic joint (b) revolute joint

---

a) <u>Prismatic joint</u>



$$\boxed{J_{v_i} = \left(\frac{\partial O_n}{\partial d_i}\right) = V_n^0 = \dot{O}_n^0 \qquad \dot{O}_n = (1)\, z_{i-1}}$$

{ when $\dot{d}_i = 1$ and other dofs have zero speed }

unit velocity $\dot{d}_i = 1$

$$\dot{O}_n = \dot{d}_i \, z_{i-1} = \dot{d}_i \left(R_{i-1}^0 k\right)$$

if not equal to 1

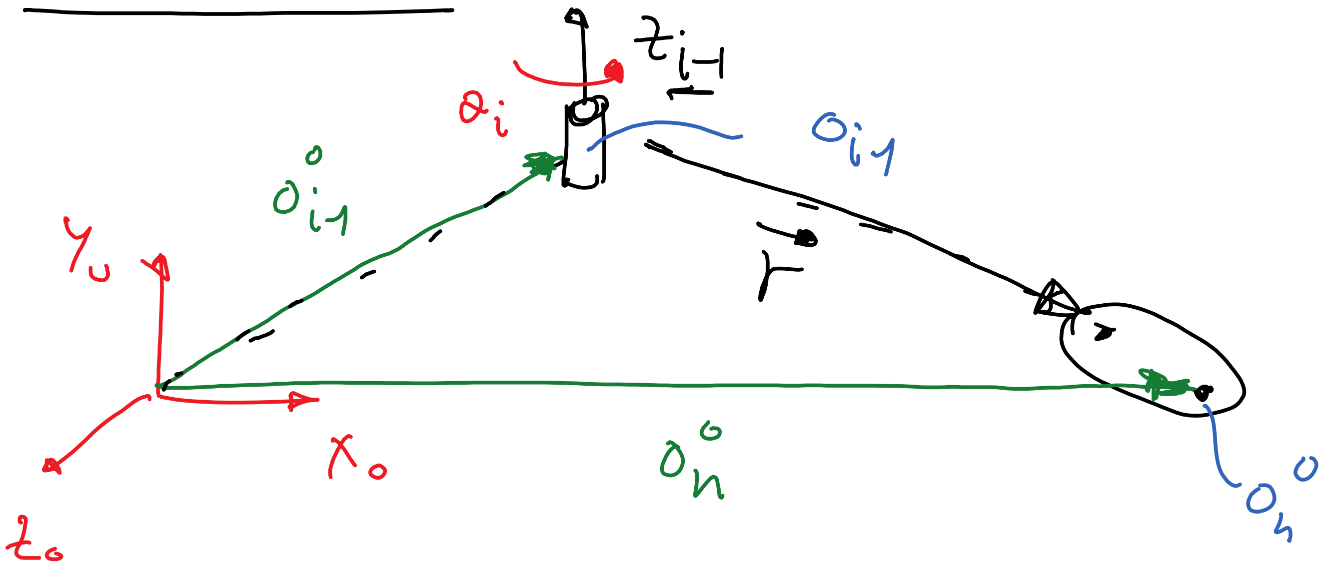
$$R_{i-1}^0 k = z_{i-1}$$

from these 2 equations

$$J_{v_i} = z_{i-1} = R_{i-1}^0 k$$

$$\boxed{J_{v_i} = R_{i-1}^0 k}$$ — Prismatic

(b) Revolute joint



Set $\underline{\dot{\theta}_i = 1}$ and all other rates equal to zero.

$$J_{v_i} = \frac{\partial o_i}{\partial \theta_i} = V_n^0 = \dot{o}_n^0$$

$$= \vec{\omega} \times \vec{r}$$

$$= \underbrace{\dot{\theta}_i}_{=1} z_{i-1} \times \left(o_n^0 - o_{i-1}^0\right)$$

$$J_{v_i} = z_{i-1} \times \left(o_n^0 - o_{i-1}^0\right)$$

$$\boxed{J_{v_i} = R_{i-1}^0 \hat{k} \times \left(o_n^0 - o_{i-1}^0\right)}$$ — Revolute joint

$$J_{v_i} = R^0_{i-1}\, k \times (o_n - o_{i-1})$$ — Revolute joint

Summary

$$J_{v_i} = \begin{cases} R^0_{i-1}\hat{k} \times (o^0_n - o^0_{i-1}) & \text{Revolute} \\ R^0_{i-1}\hat{k} & \text{Prismatic} \end{cases}$$

$$J_{\omega_i} = \begin{cases} R^0_{i-1}\hat{k} & \text{Revolute} \\ 0 & \text{Prismatic} \end{cases}$$

$$\hat{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

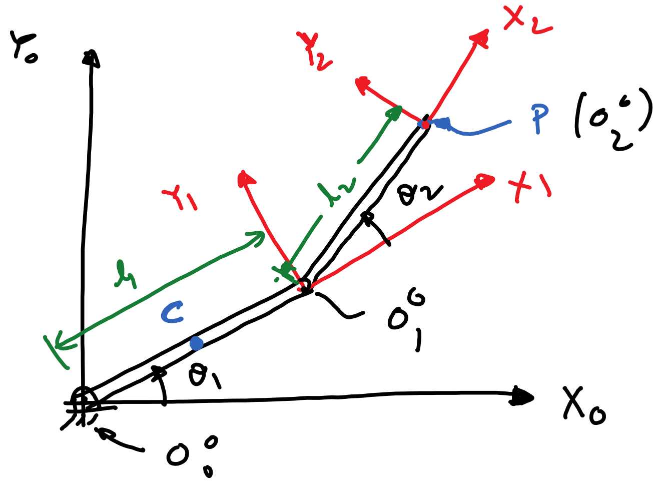
$$R^0_{i-1}\hat{k} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$R^0_{i-1}\hat{k} = \begin{bmatrix} r_{13} \\ r_{23} \\ r_{33} \end{bmatrix}$$ = First 3 rows of the third column of A^0_{i-1}

o^0_n = First 3 rows of the 4th column of A^0_n

o^0_{i-1} – First 3 rows of the 4th column of A^0_{i-1}

EXAMPLE



D-H Table

Link	a_i	α_i	d_i	θ_i
1	l_1	0	0	θ_1
2	l_2	0	0	θ_2

$R_{i-1}^0 k \times (o_n^0 - o_{i-1}^0)$

Consider the two link planar manipulator shown above. Point P is at the end of the second link and point C is mid-way on the first link. Find (a) Jacobian for P and (b) Jacobian for C

link

$$(a)\quad J_p(q) = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} J_{v_1} & J_{v_2} \\ J_{\omega_1} & J_{\omega_2} \end{bmatrix} = \begin{bmatrix} R_0^0 k \times (o_2^0 - o_0^0) & R_1^0 k \times (o_2^0 - o_1^0) \\ R_0^0 k & R_1^0 k \end{bmatrix}$$

$R_{i-1}^0 k$

$$A_0^0 = \begin{bmatrix} R_0^0 & d_0^0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_0^0 k$

o_0^0

$$A_1^0 = \begin{bmatrix} C_1 & -S_1 & 0 & l_1 C_1 \\ S_1 & C_1 & 0 & l_1 S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1^0 k$

o_1^0

$$A_2^1 = \begin{bmatrix} C_2 & -S_2 & 0 & l_2 C_2 \\ S_2 & C_2 & 0 & l_2 S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_1 = \cos\theta_1 \qquad S_1 = \sin\theta_1$

$C_2 = \cos\theta_2 \qquad S_2 = \sin\theta_2$

Use D-H formula

o_2^0

$$A_2^0 = A_1^0 A_2^1 = \begin{bmatrix} C_{12} & -S_{12} & 0 & l_1 C_1 + l_2 C_{12} \\ S_{12} & C_{12} & 0 & l_1 S_1 + l_2 S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_{12} = \cos(\theta_1 + \theta_2)$

$S_{12} = \sin(\theta_1 + \theta_2)$

$$J_p(q) = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} J_{v_1} & \underline{\underline{J_{v_2}}} \\ J_{\omega_1} & J_{\omega_2} \end{bmatrix} = \begin{bmatrix} \underline{\underline{R_0^0 k \times (o_2^0 - o_0^0)}} & R_1^0 k \times \underline{\underline{(o_2^0 - o_1^0)}} \\ \underline{\underline{R_0^0 k}} & \underline{\underline{R_1^0}}\, k \end{bmatrix}$$

From previous page

$R_0^0 k$ = 1st 3 rows of the 3rd column of A_0^0 $= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

R_1^0 = 1st 3 rows of the 3rd colum of A_1^0 $= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

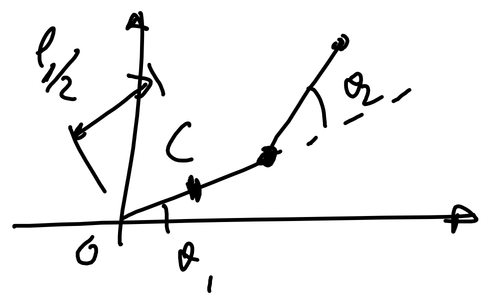
o_2^0 = 1st 3 rows of the 4th column of A_2^0 $= \begin{pmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ 0 \end{pmatrix}$

o_1^0 = " A_1^0 $= \begin{pmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{pmatrix}$

o_0^0 = " A_0^0 $= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$J_p(q) = \begin{array}{c} \begin{array}{cc} J_{v_1} \qquad\qquad & \qquad\qquad J_{v_2} \end{array} \\ \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \\ \begin{array}{cc} J_{\omega_1} \qquad\qquad & \qquad\qquad J_{\omega_2} \end{array} \end{array} \quad 6 \times 2$$

$$J_c(q) = \begin{bmatrix} J_{v_1} & J_{v_2} \\ J_{\omega_1} & J_{\omega_2} \end{bmatrix}$$



$$= \begin{bmatrix} R_0^0 k \times (o_c^0 - o_0^0) & 0 \\ R_0^0 k & 0 \end{bmatrix}$$

Because motion of θ_2 does not lead to motion of C

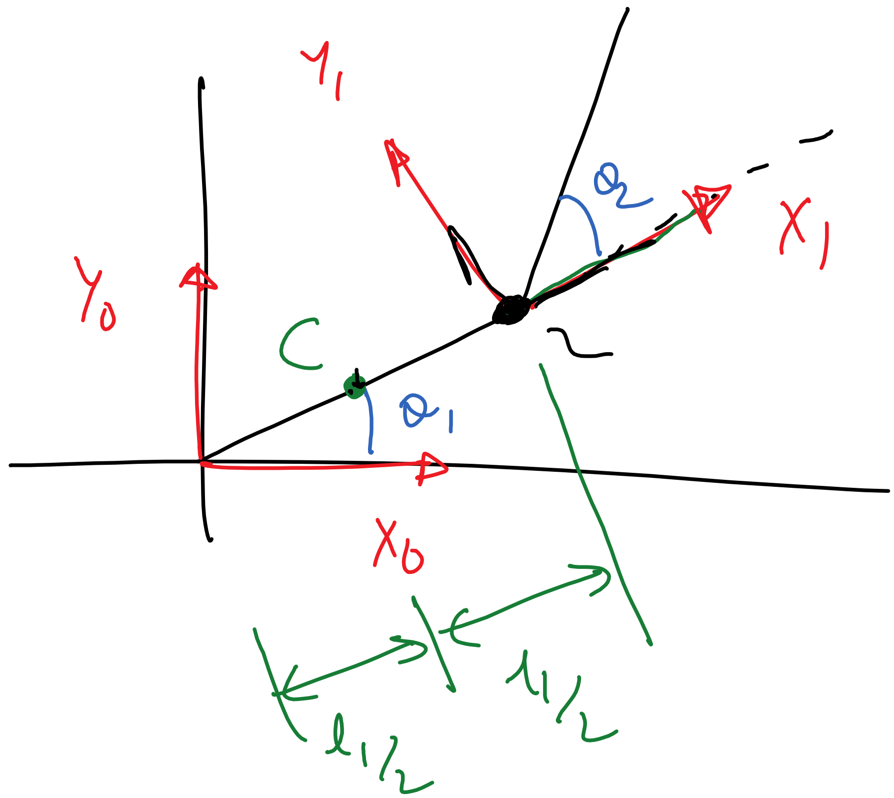
Revolute

$$J_{v_i} = R_{i-1}^0 k \times (o_n^0 - o_{i-1}^0)$$

end-effector

$$J_{\omega_i} = R_{i-1}^0 k$$

Lets evaluate o_c^0



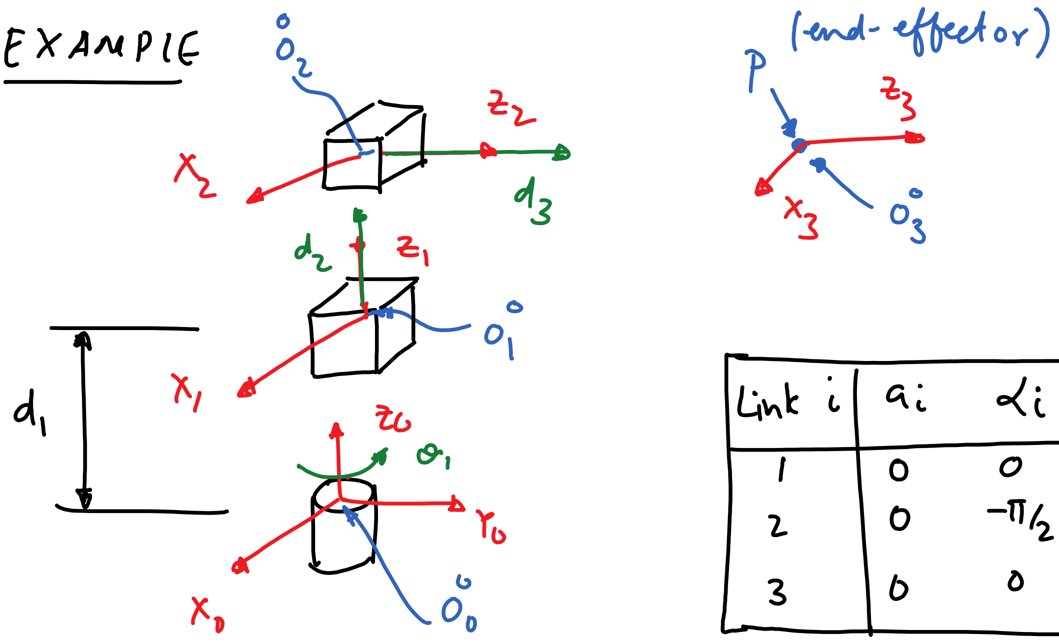
$$\underline{\underline{O_c^0}} = A_1^0 \, \underline{\underline{O_c^1}} = \begin{bmatrix} c_1 & -s_1 & 0 & l_1 c_1 \\ s_1 & c_1 & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -l_1/2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} l_1 c_1/2 \\ l_1 s_1/2 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{cross}\left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} l_1 c_1/2 \\ l_1 s_1/2 \\ 0 \end{pmatrix} \right)$$

$$J_c(q) = \begin{bmatrix} R_0^0 k \times (o_c^0 - o_0^0) & 0 \\ R_0^0 k & 0 \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} -l_1 s_1/2 \\ l_1 c_1/2 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \end{bmatrix}_{6\times 2}$$

EXAMPLE



Link i	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1
2	0	$-\pi/2$	d_2	0
3	0	0	d_3	0

Find the linear and angular velocity of the point P (the end-effector) for the three link manipulator shown above