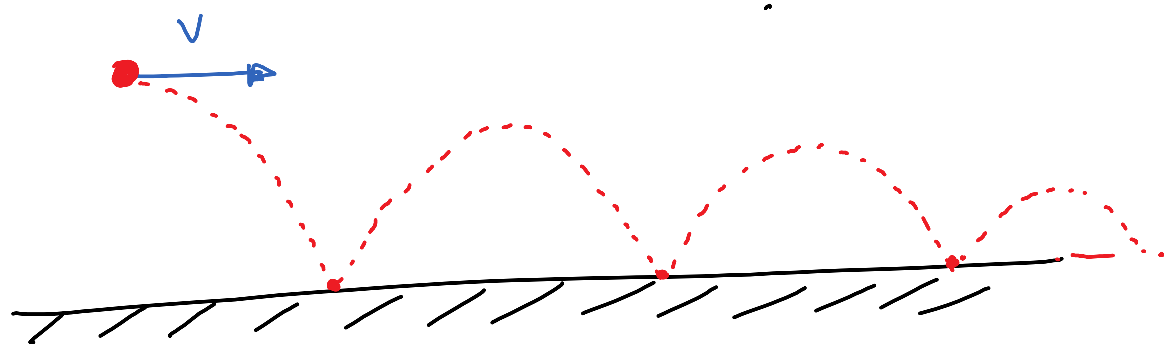
# Hybrid systems

A hybrid system has multiple phases in its motion. These phases are decribed by different equation. Two system (1) Bouncing ball (2) pogo stick or a spring-mass hopping robot.

## (1) Bouncing ball



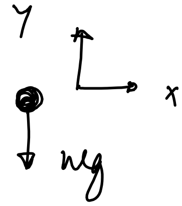

flight phase $\xrightarrow[\text{ground}]{\text{detect}}$ bounce phase $\longrightarrow$ flight phase $\xrightarrow[\text{contact}]{\text{detect}}$ bounce phase

Repeating element. / Building unit

Goal : model this system for one building unit and then loop

# Equations of motion

## Flight phase

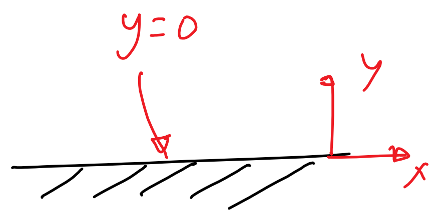



$$\ddot{y} = -g$$

$$\ddot{x} = 0$$

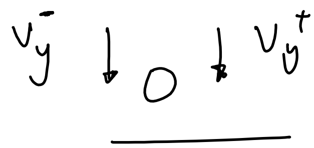
## Detect contact

$$y = 0$$




## Bounce phase

$v_y^+, v_y^-$ are the velocities in the y direction after and before collision respectively.




$$\boxed{v_y^+ = -e\, v_y^-}$$

e = co-efficient of restitution
constant for the particular ball-ground combination

$$0 < e \le 1$$

plastic        elastic

# Implementation in MATLAB

## Flight

[t, q] = ode45(@ball_rhs, [t0 tend], q0, options, parameters)

function: $\ddot{x} = 0$, $\ddot{y} = -g$

start and end time

initial conditions (q): $x_0, y_0$, $\dot{x}_0, \dot{y}_0$

gravity

options = odeset('AbsTol', 1e-8, 'Events', @ball_contact)

function [gstop, isterminal, direction] = ball_contact(t, q, params)

detect contact

y = q(3); % q(3) is y

gstop = y; % detect gstop = y = 0

isterminal = 1; % stop integration

direction = -1; % detect ground when gstop

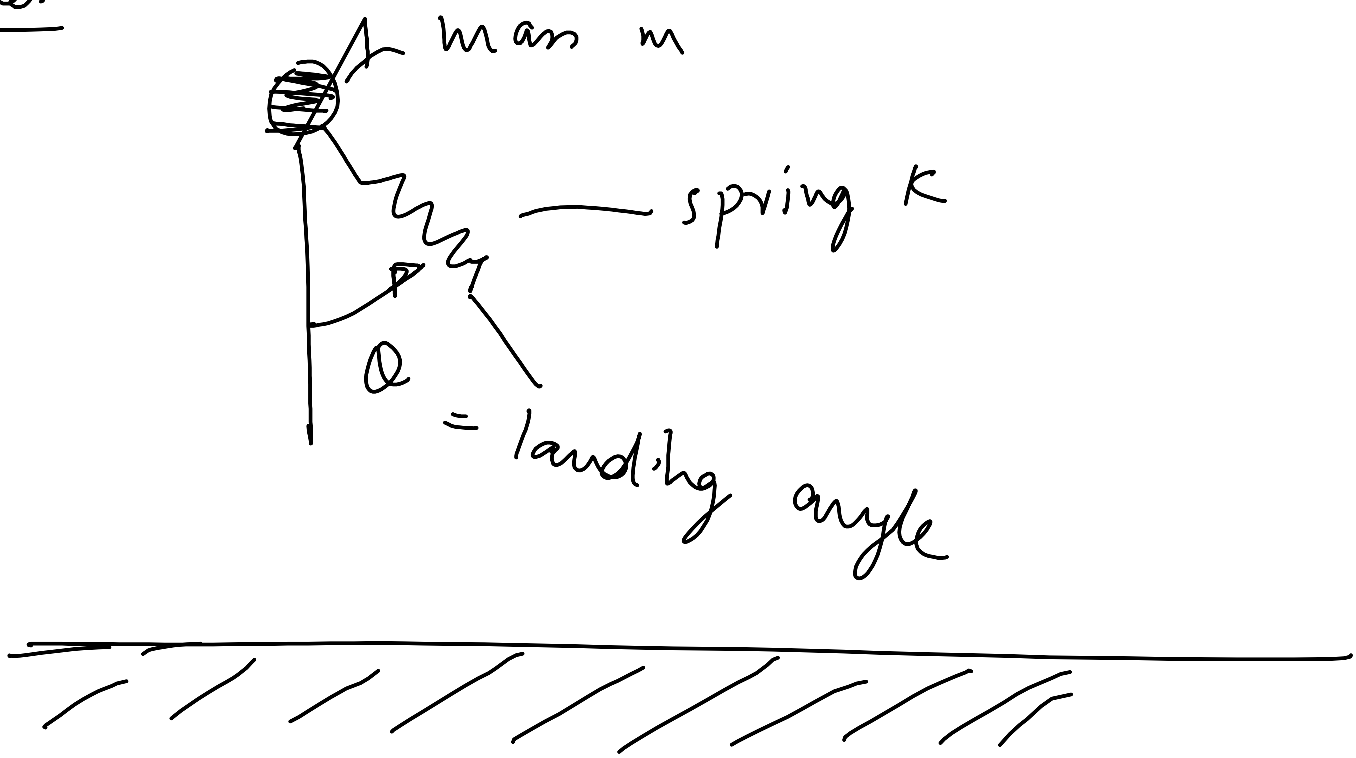
gstop = y - 1

direction = -1; % detect ... when gstop goes from + to -

gstop = y-1
if you want to detect y=1

## (2) Spring-mass hopping robot

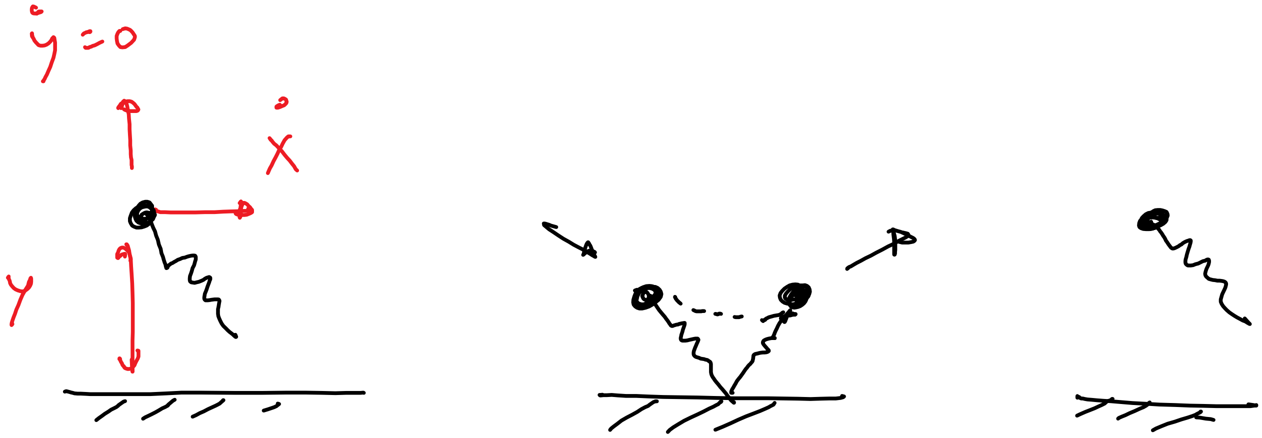
### Model




### Phases of motion

Flight $\xrightarrow[\text{contact}]{\text{detect}}$ Stance $\longrightarrow$ Flight

one step




apex  bounce/stance  apex

$\dot y = 0$  $y = 0$  $l = l_0$  $\dot y = 0$

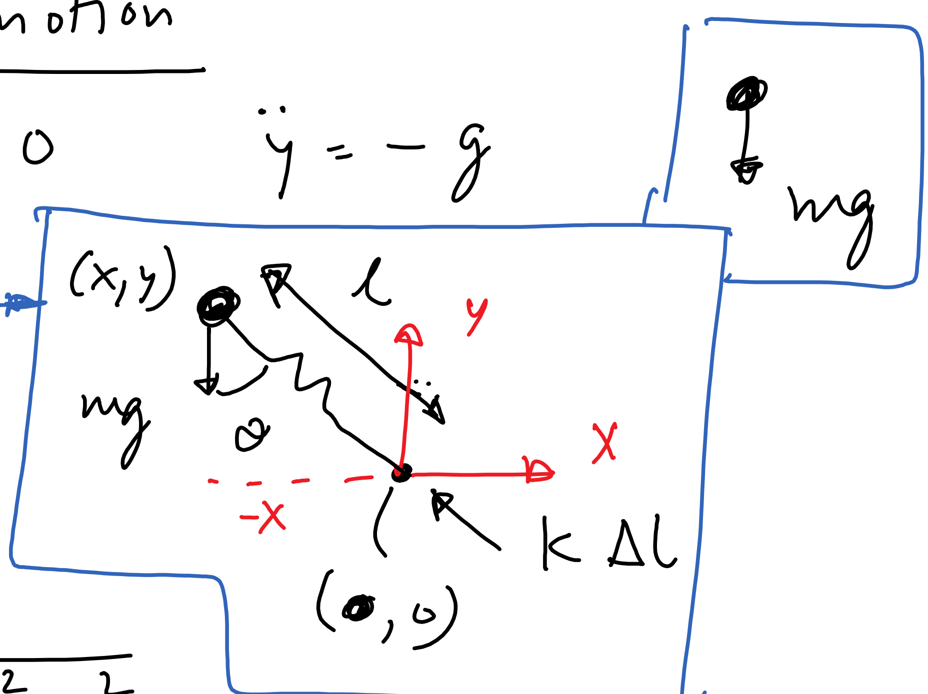
$\xrightarrow{\text{apex}}$ Flight $\xrightarrow{\text{ground}}$ Stance / Bounce $\xrightarrow{\text{takeoff}}$ Flight $\xrightarrow{\text{apex}}$

one step

## Equations of motion

**Flight:** $\ddot x = 0$  $\ddot y = -g$

**Bounce phase** →

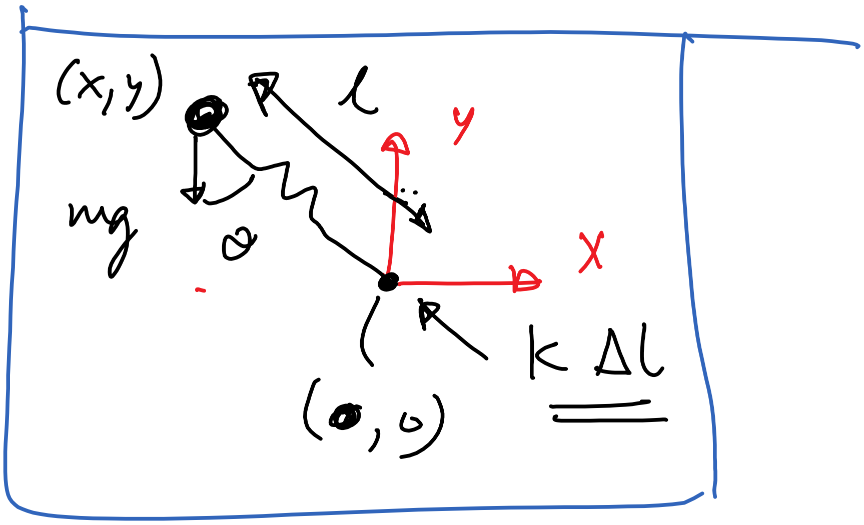



$$\Delta l = l_0 - l$$

$$= l_0 - \sqrt{x^2+y^2}$$

$\sin\theta = -x$  $\cos\theta = \frac{y}{}$

$$\sin\theta = \frac{-x}{\sqrt{x^2+y^2}} \qquad \cos\theta = \frac{y}{\sqrt{x^2+y^2}}$$




## Equations

$$m\ddot{x} = -k\Delta l \sin\theta$$

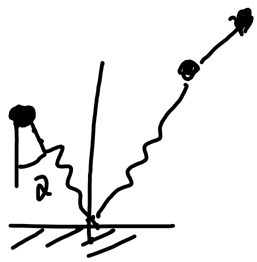
(1) — $$m\ddot{x} = -k\left(l_0 - \sqrt{x^2+y^2}\right)\left(\frac{-x}{\sqrt{x^2+y^2}}\right)$$

$$m\ddot{y} = -mg + k\Delta l \cos\theta$$

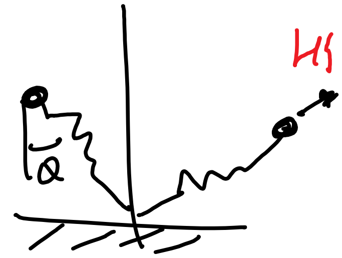
(2) — $$m\ddot{y} = -mg + k\left(l_0 - \sqrt{x^2+y^2}\right)\left(\frac{y}{\sqrt{x^2+y^2}}\right)$$

— Root finder = 0 & run the code

Simulation fails



Not enough momentum (fall backward)



High speed fall forward

For a given landing angle $\theta$ there is an optimum speed.

Root-finder_ON = 1 and simulate

Simulation: shows periodic motion

motion the repeats itself.