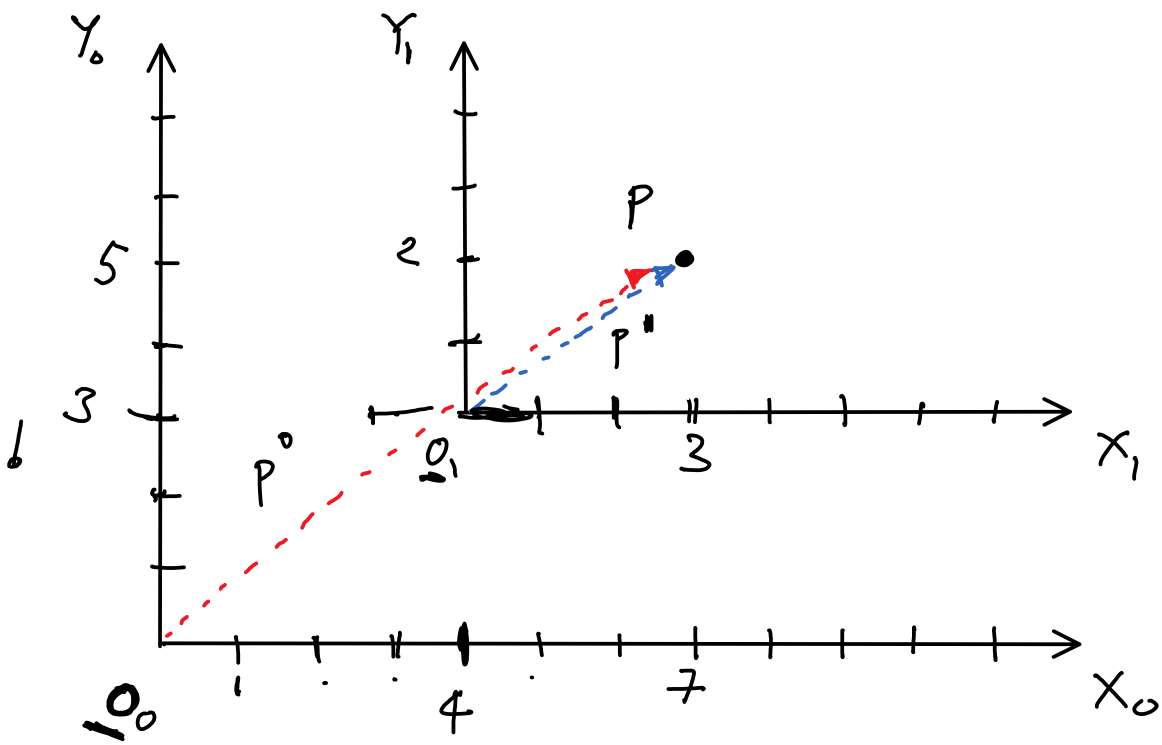
Representing position

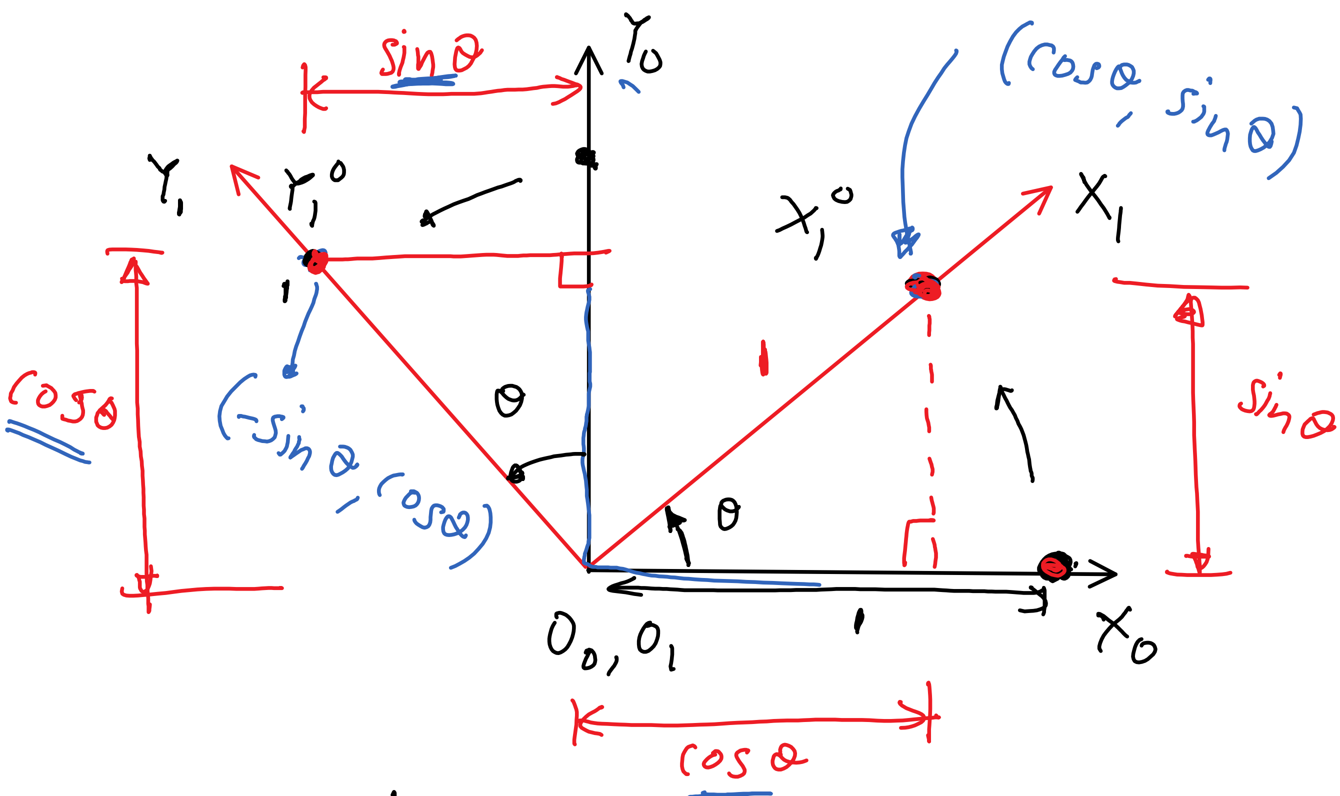


- P is a random point in the plane
- $O_0 X_0 Y_0$ and $O_1 X_1 Y_1$ are 2 co-ordinate frames.
- P can be represented in 2 ways

$$p^0 = \begin{bmatrix} 7 \\ 5 \end{bmatrix} \qquad p^1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$o_1^0 = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \qquad o_0^1 = \begin{bmatrix} -4 \\ -3 \end{bmatrix}$$

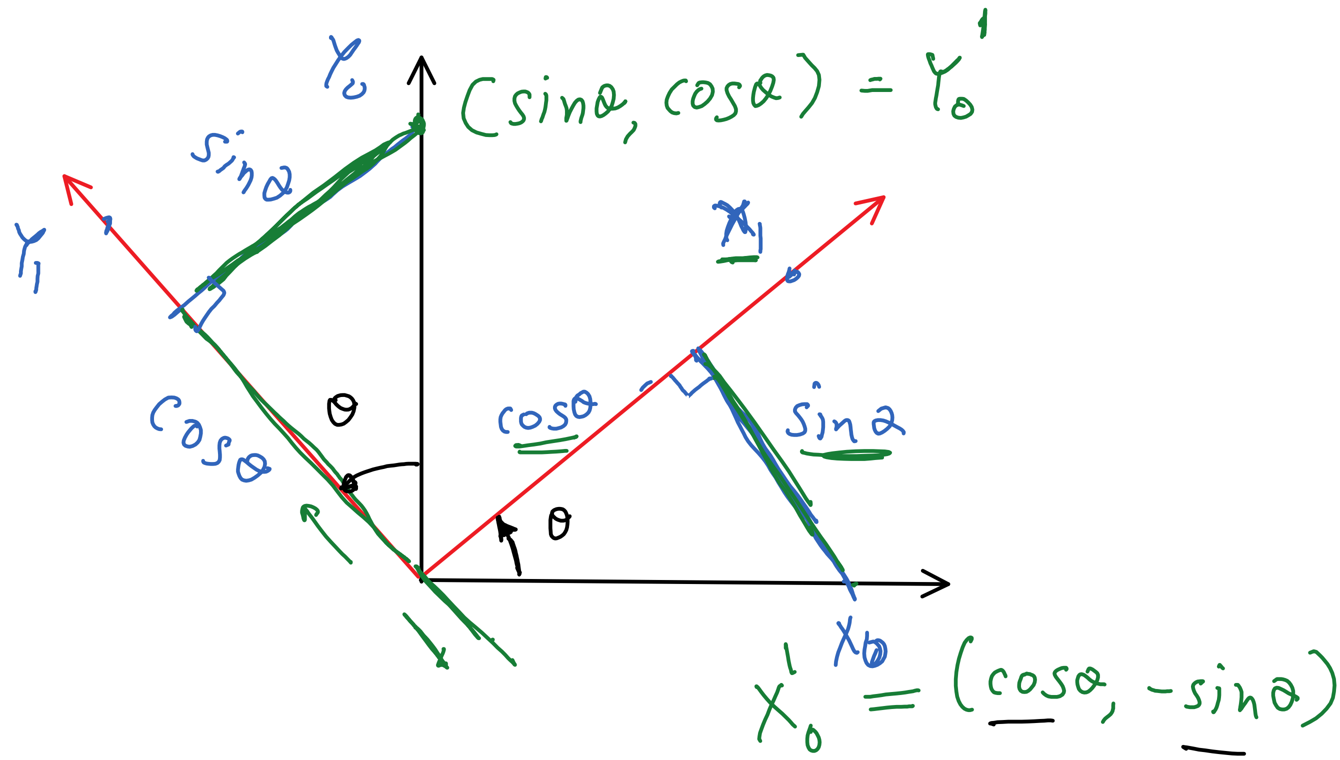
Representing Rotations



- original frame $O_0 X_0 Y_0$
- rotated frame $O_1 X_1 Y_1$
- θ counter clockwise and is positive

$$R_1^0 = \begin{bmatrix} \underset{2\times 1}{X_1^0} & \underset{2\times 1}{Y_1^0} \end{bmatrix}$$

$$\boxed{R_1^0 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}$$



$$R_0^1 = \begin{bmatrix} X_0^1 & Y_0^1 \end{bmatrix}$$

$$\boxed{R_0^1 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}}$$

$$\boxed{R_1^0 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}$$

From previous page

$$R_1^0 = (R_0^1)^T$$

$$\boxed{R_i^j = (R_j^i)^T}$$

Generalised

$$\left(R_1^0\right)^{-1} = \left(R_1^0\right)^T$$ from this

Property of rotation matrix $\underline{R^T R = I}$

$$\boxed{\left(R_i^j\right)^{-1} = \left(R_i^j\right)^T}$$ Generalized form

3-D rotations

$$R_1^0 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

rotation about z-axis

All 3-D rotations

$$R_{z,\theta} = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$s\theta = \sin\theta \qquad c\theta = \cos\theta$

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & -s\theta \\ 0 & s\theta & c\theta \end{bmatrix}$$

$$R_{y,\theta} = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}$$