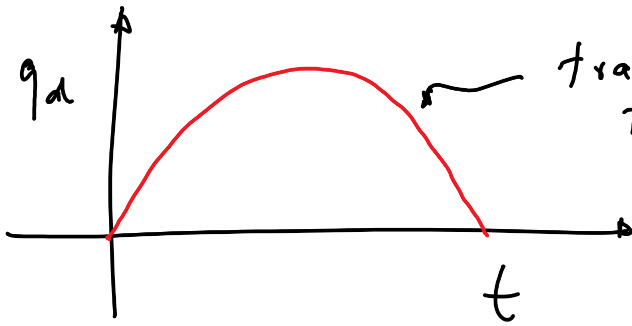
Control partitioning for trajectory tracking



$q_d(t)$ — given

$\dot{q}_d(t), \ddot{q}_d(t) \longrightarrow$ known.

Control

$$\tau = \hat{M}\left\{\ddot{q}_d - K_p(q - q_d) - K_d(\dot{q} - \dot{q}_d)\right\} + \hat{C}\dot{q} + \hat{G}q \qquad \text{I}$$

Analysis to shown that τ is able to track.

special case $\hat{M} = M \quad \hat{C} = C \quad \hat{G} = G$

Eq^n of manipulator

$$M\ddot{q} + C\dot{q} + Gq = \tau$$

q — actual motion

q_d — desired motion

Sub. I

$$\underline{\underline{M}}\ddot{q} + \cancel{C\dot{q}} + \cancel{Gq} = \underline{\underline{M}}\left\{\ddot{q}_d - K_p(q-q_d) - K_d(\dot{q}-\dot{q}_a)\right\} + \cancel{C\dot{q}} + \cancel{Gq}$$

$$M\left\{(\ddot{q} - \ddot{q}_d) + K_d(\dot{q} - \dot{q}_d) + K_p(q - q_d)\right\} = 0$$

$$M\left\{\ddot{e} + K_d\dot{e} + K_p e\right\} = 0 \qquad \{e = q - q_d\}$$

$$\bar{M}\left\{(\ddot{q} - \ddot{q}_d) + K_d(\dot{q} - \dot{q}_d) + K_p(q - q_d)\right\} = 0$$

$$\underset{\uparrow}{M}\left\{\ddot{e} + k_d\dot{e} + k_p e\right\} = 0 \qquad \left\{e = q - q_d\right\}$$

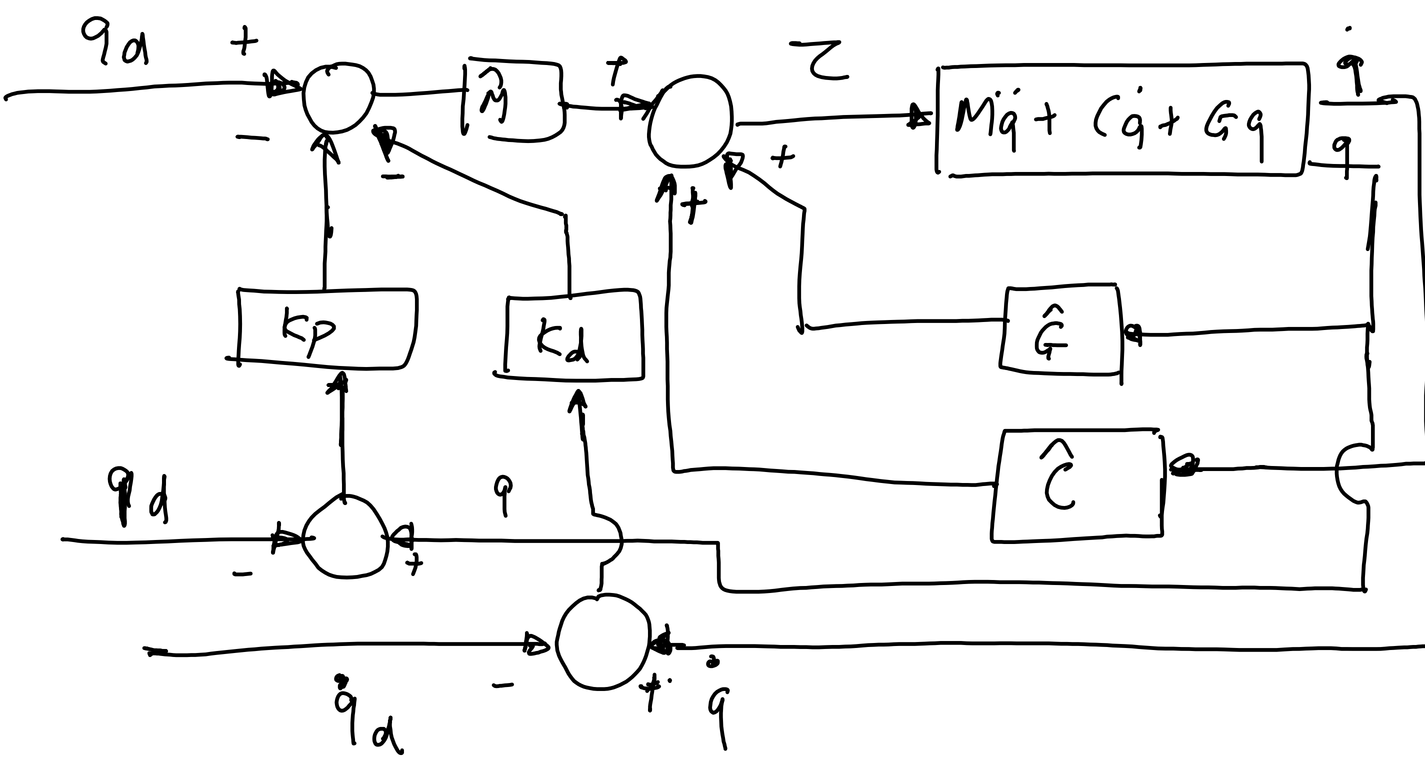
$$\left(\ddot{e} + K_d \dot{e} + K_p e\right) = 0$$

This looks like a spring-mass-damper system. By choosing K_p, K_d we can get the system to be critically damped.

$$\boxed{K_d = 2\sqrt{K_p}}$$

Block diagram

$$\tau = \hat{M}\left(\ddot{q}_d - K_p(q - q_d) - K_d(\dot{q} - \dot{q}_d)\right) + \hat{C}\dot{q} + \hat{G}q$$



$\Rightarrow$ add integral control

$$\tau = \hat{M}\left\{\ddot{q}_d - K_p(q - q_d) - K_d(\dot{q} - \dot{q}_d) - K_i\int(q - q_d)\,dt\right\}$$

$$+ \hat{C}\dot{q} + Gq$$

Cartesian - based control

The goal is to move from X_i to X_f where $X_i = [x_i, y_i, z_i]$ & $X_f = [x_f, y_f, z_f]$ in time t_f

① Find $X_d(t)$ using the polynomial of appropriate order

② Find $q_d, \dot{q}_d, \ddot{q}_d\,(t)$ using inverse kinematics

③ Use control partitioning with trajectory tracking

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① ✓

② $X_d = f(q_d)$ Forward kinematics

✓ $q_d = f^{-1}(X_d)$ inverse kinematics

$\dot{X}_d = J\,\dot{q}_d$ [first order approximation]

✓ $\dot{q}_d = J^{-1}\,\dot{X}_d$

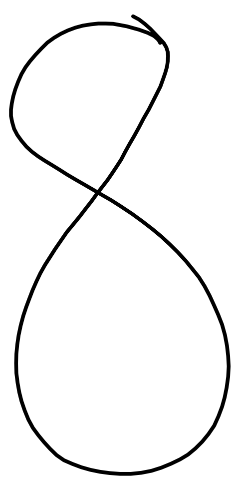
$$\ddot{x}_d = \dot{J}\dot{q}_d + J\underline{\ddot{q}_d}$$

$$\ddot{q}_d = J^{-1}\{\ddot{x}_d + \dot{J}\dot{q}_d\}$$

③ $$\tau = \hat{M}\{\ddot{q}_d - K_p(q - q_d) - K_d(\dot{q} - \dot{q}_d)\} + \hat{C}\dot{q} + \hat{G}q$$

MATLAB example

Figure 8



— Lemniscate

see MATLAB