Vectors — magnitude and direction

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$\hat{i}, \hat{j}, \hat{k}$ — unit vectors along axis

Magnitude

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Direction

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{a_x \hat{i} + a_y \hat{j} + a_z \hat{k}}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$$

$$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$$

Dot product

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z \quad \text{(Scalar)}$$

Cross product

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \hat{i}(a_y b_z - a_z b_y) - \hat{j}(a_x b_z - a_z b_x) + \hat{k}(a_x b_y - a_y b_x)$$

(Vector)

Another way to do cross-product (with matrix multiplication)

$$\vec{a} \times \vec{b} = \underset{3\times 3}{\begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}}$$

$$= \underset{3\times 1}{\begin{bmatrix} 0 - a_z b_y + a_y b_z \\ a_z b_x + 0 - a_x b_z \\ -a_y b_x + a_x b_y + 0 \end{bmatrix}}$$

$$\vec{a} \times \vec{b} = (-a_z b_y + a_y b_z)\,\hat{i} + (a_z b_x - a_x b_z)\,\hat{j} + (-a_y b_x + a_x b_y)\,\hat{k}$$

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$$\boxed{\begin{array}{c} \hat{i}\,(a_y b_z - a_z b_y) \\ -\hat{j}\,(a_x b_z - a_z b_x) \\ +\hat{k}\,(a_x b_y - a_y b_x) \\ \text{(vector)} \end{array}}$$