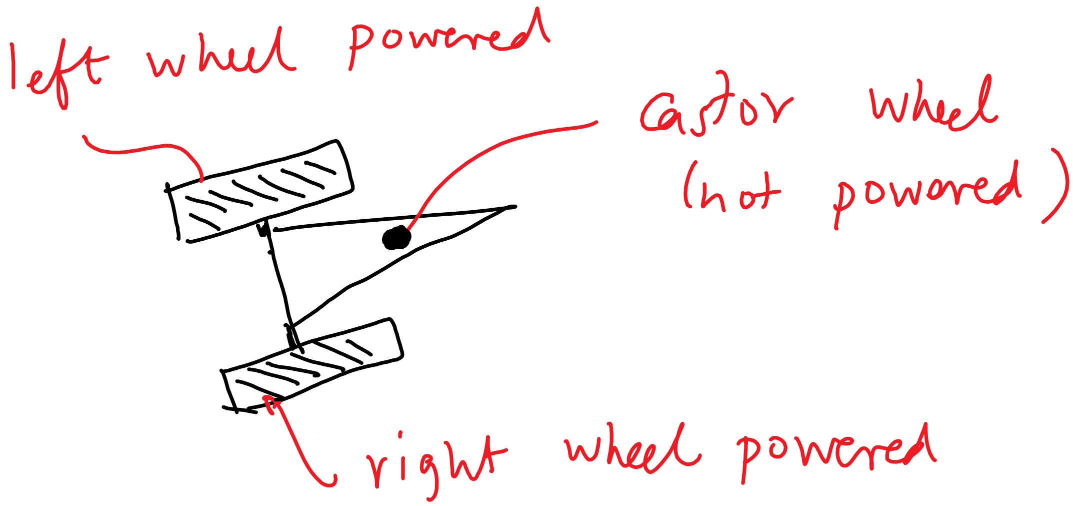
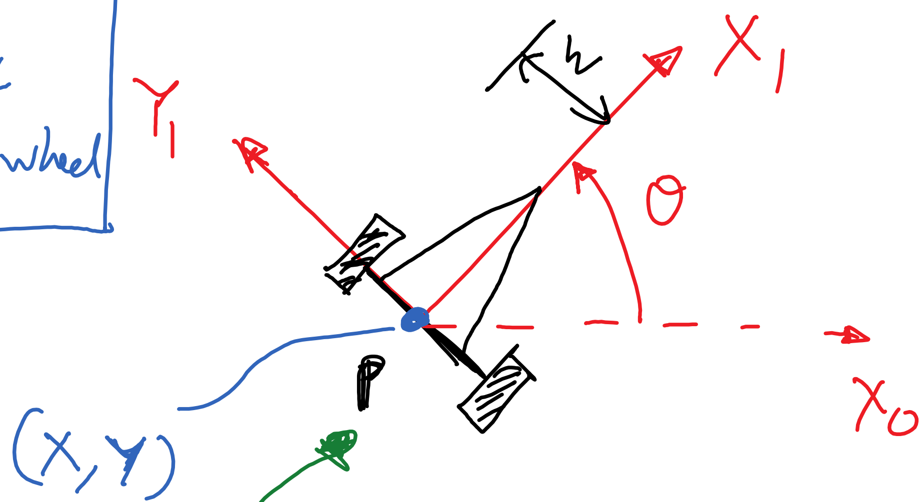
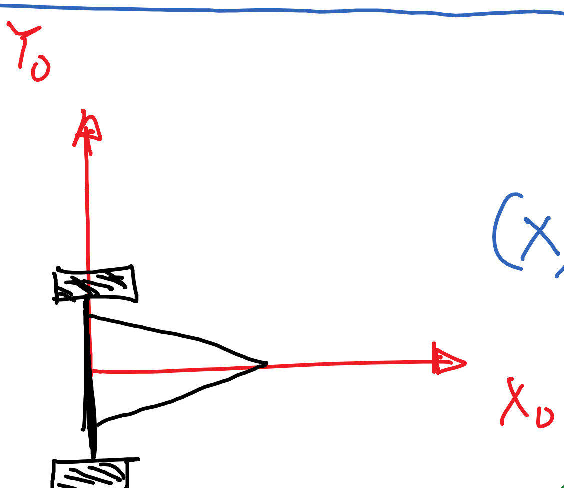
Kinematics of a differential drive car



Given: $\dot{\phi}_\ell$ & $\dot{\phi}_r$
angular velocity of
the left and right wheel

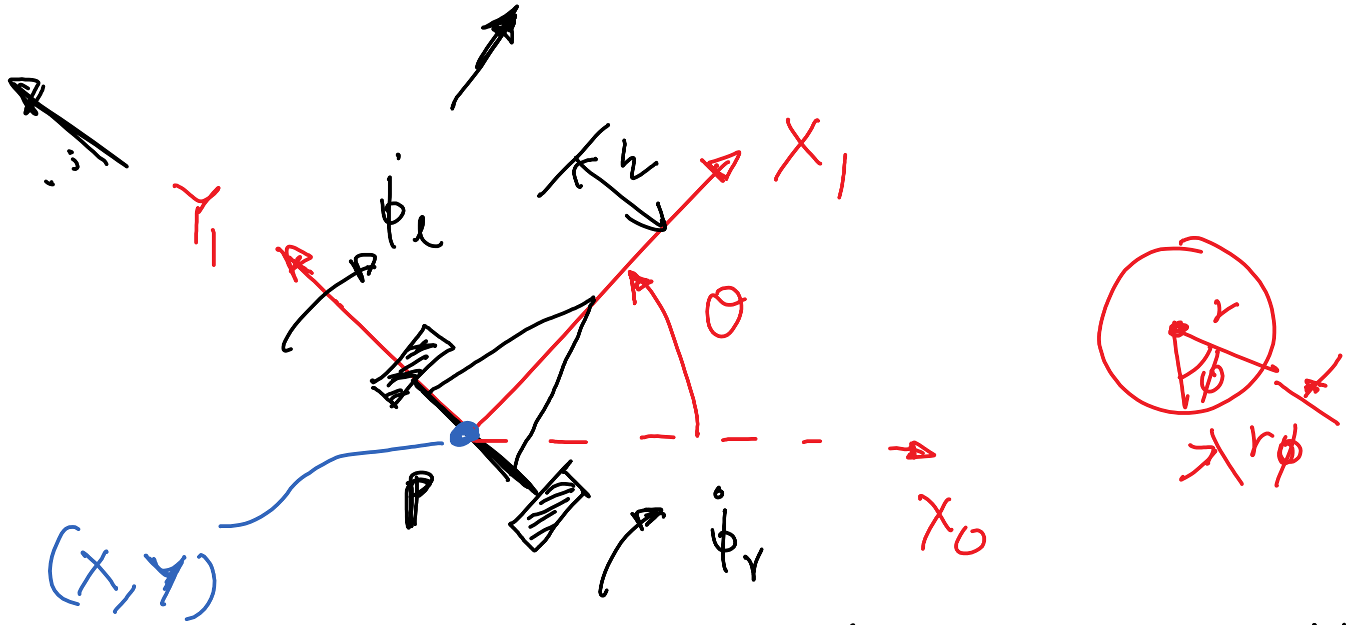




X_0, Y_0 – global frame
world frame

X_1, Y_1 – local frame
that moves
with car.

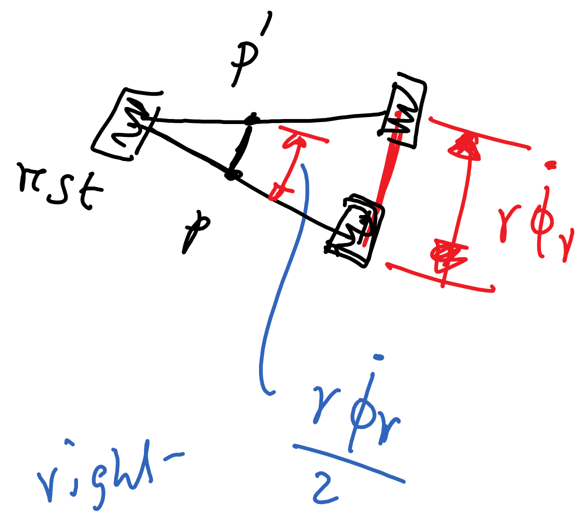
Goal:
We will keep track
of point P (x, y)
and θ



① Velocity of point P in frame $O_1X_1Y_1$

$$\dot{p}'_x = ?$$

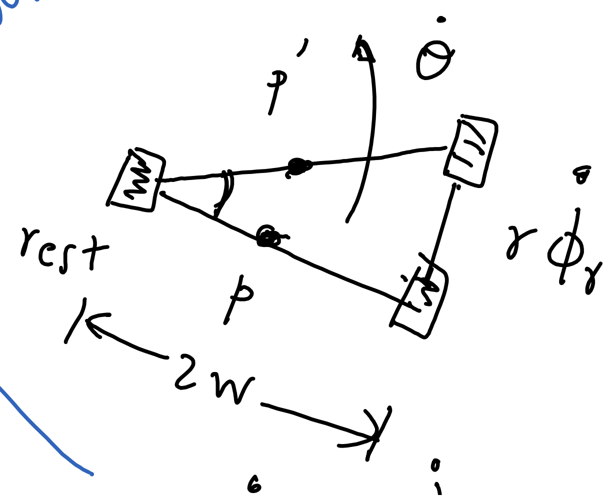
If $\dot\phi_r \neq 0$ $\quad \dot\phi_l = 0$



$$\dot{p}'_x = \frac{1}{2}\left(r\dot\phi_l + r\dot\phi_r\right)$$

$$\dot{p}'_y = 0$$

② $\dot\theta = \dfrac{r\dot\phi_r}{2w}$ // only for right



Both wheels turning

$$\dot\theta = \underline{r\dot\phi_r} - \underline{r\dot\phi_l}$$

$$\theta = \frac{r\phi_r}{2w} - \frac{r\phi_l}{2w}$$

$$2w\dot{\theta} = r\dot{\phi}_r$$

$$\dot{p}^1 = \begin{bmatrix} \dot{p}^1_x \\ \dot{p}^1_y \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r}{2}(\dot{\phi}_r + \dot{\phi}_l) \\ 0 \\ \frac{r}{2w}(\dot{\phi}_r - \dot{\phi}_l) \end{bmatrix}$$

World frame $\dot{p}^0$

$$\dot{p}^0 = R^0_1 \, \dot{p}^1$$

[see notes from last class]

$$\dot{p}^0 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{r}{2}(\dot{\phi}_r + \dot{\phi}_l) \\ 0 \\ \frac{r}{2w}(\dot{\phi}_r - \dot{\phi}_l) \end{bmatrix}$$

$$\boxed{\dot{p}^0 = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r}{2}(\dot{\phi}_r + \dot{\phi}_l)\cos\theta \\ \frac{r}{2}(\dot{\phi}_r + \dot{\phi}_l)\sin\theta \\ \frac{r}{2w}(\dot{\phi}_r - \dot{\phi}_l) \end{bmatrix}} \quad \text{— I}$$

Given $\dot{\phi}_r(t)$, $\dot{\phi}_l(t)$ and initial condition $x(0), y(0), \theta(0)$, we can use numerical integration on I to solve for $x(t), y(t), \theta(t)$

?

$$\dot{x} = \frac{r}{2}(\dot{\phi}_r + \dot{\phi}_l)\cos\theta = \frac{dx}{dt} = \frac{x(t_{i+1}) - x(t_i)}{dt}$$

$$\boxed{\begin{aligned} x(t_{i+1}) &= x(t_i) + dt\left(\frac{r}{2}\right)(\dot{\phi}_r + \dot{\phi}_l)\cos\theta \\ y(t_{i+1}) &= y(t_i) + dt\left(\frac{r}{2}\right)(\dot{\phi}_r + \dot{\phi}_l)\sin\theta \\ \theta(t_{i+1}) &= \theta(t_i) + dt\left(\frac{r}{2W}\right)(\dot{\phi}_r - \dot{\phi}_l) \end{aligned}}$$

II

In MATLAB.

define dt, $\dot{\phi}_r$, $\dot{\phi}_l$,

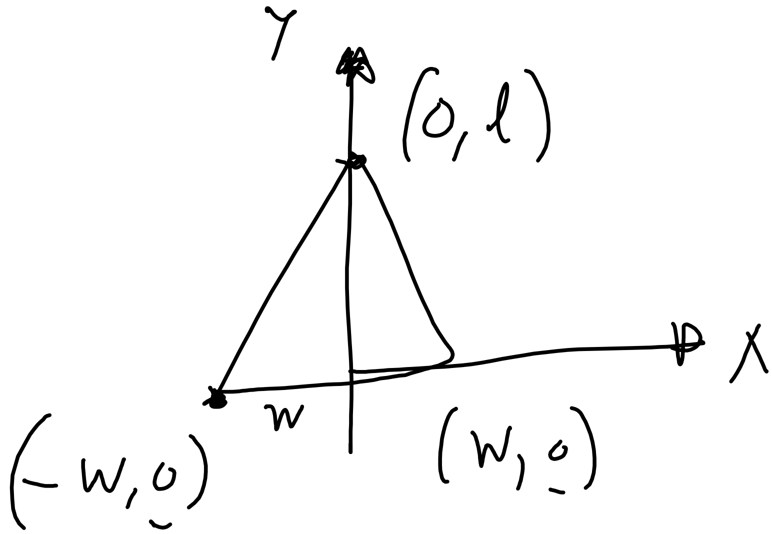
$x(0) = y(0) = \theta(0) = 0$

and use II to find

$x(dt)$, $x(2dt)$ . . . .

$y(dt)$, $y(2dt)$ . . . .

$\theta(dt)$, $\theta(2dt)$, . . .

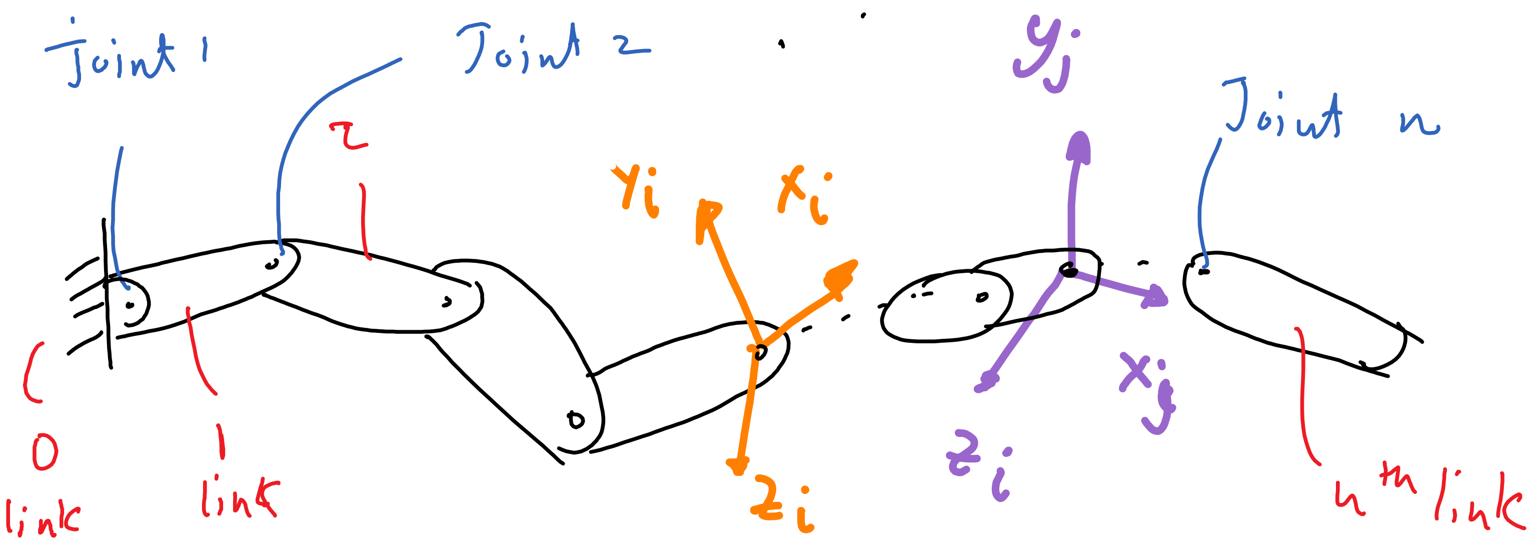


$$X_{vert} = [-W, W, 0]$$

$$Y_{vert} = [0, 0, l]$$

Kinematics of serial manipulator

- prismatic joints (d_i) {linear motion)
- revolute joints (θ_i) (rotational motion)



Links : 0, 1, 2, . . .

Joints : 1, 2, 3, . . .

$\Rightarrow$ Joint i is connected to links $i-1$ and i

$\Rightarrow$ $A_i^{i-1}(q_i)$ homogenous transformation linking i to $i-1$

q_i: degrees of freedom — d_i, θ_i

$$\boxed{T^i_j = A^i_{i+1} A^{i+1}_{i+2} \cdots A^{j-1}_j \qquad i < j \\ \quad = I \qquad\qquad\qquad\qquad i = j}$$

$$\boxed{T^j_i = \left(T^i_j\right)^{-1}}$$