Inverse kinematics

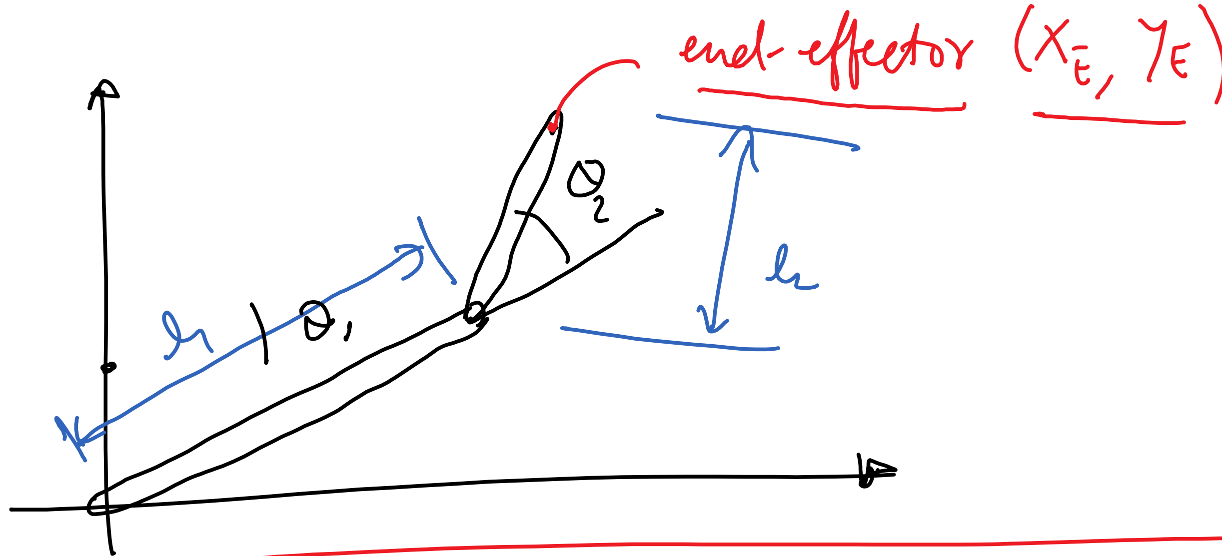
$$A_n^0 = A_1^0 A_2^1 \cdots A_n^{n-1} = \begin{bmatrix} \overset{\text{orientation}}{R_n^0} & \overset{\text{position}}{o_n^0} \\ 0 & 1 \end{bmatrix}$$

3 orientantions that describe R_n^0
3 positions that describle o_n^0

Use fsolve with 6 constraints

In this course we will mainly focus on meeting only the position constraints.

Inverse Kinematics



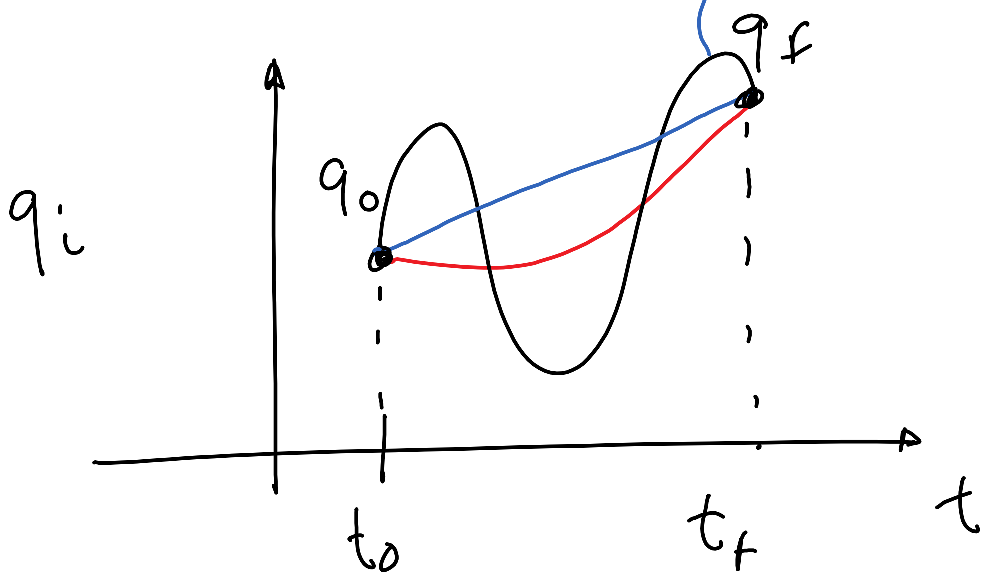
We have derived the formula for E

$$\left.\begin{aligned} x_E &= l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ y_E &= l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \end{aligned}\right\} - (I)$$

In MATLAB set 2 equations for x_E and y_E and then use fsolve

see MATLAB

Trajectory generation



which curve to use to connect the 2 points.

$q_0, q_f \rightarrow$ initial and final position

[e.g. 0.5 rad to 1 rad in time 5 sec ($t_0 = 0$, $t_f = 5$)

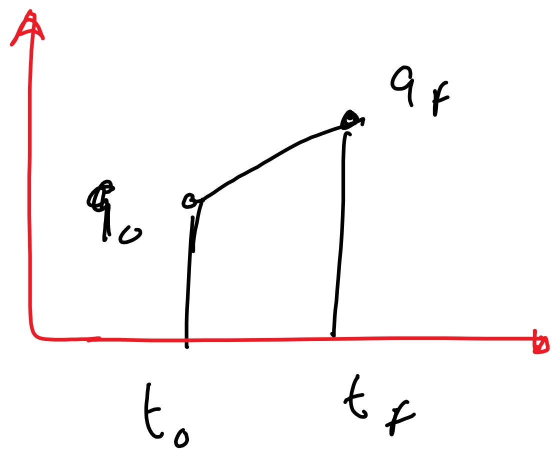
$q_0 = 0.5 \quad q_f = 1$]

Simplest $\longrightarrow$ straight line

$$q = a_0 + a_1 t \qquad a_0, a_1 \text{ constants}$$

$$q_0 = a_0 + a_1 t_0 \quad - (1)$$

$$q_f = a_0 + a_1 t_f \quad - (2)$$



Solve for a_0 and a_1

$$\begin{bmatrix} 1 & t_0 \\ 1 & t_f \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} q_0 \\ q_f \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 & t_0 \\ 1 & t_f \end{bmatrix}^{-1} \begin{bmatrix} q_0 \\ q_f \end{bmatrix}$$

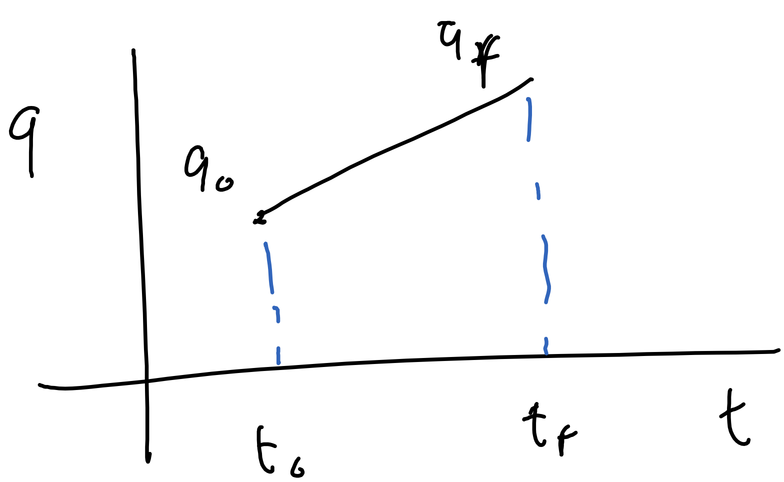
$$a_0 = \frac{q_0 t_f - q_f t_0}{t_f - t_0} \qquad a_1 = \frac{q_f - q_0}{t_f - t_0}$$

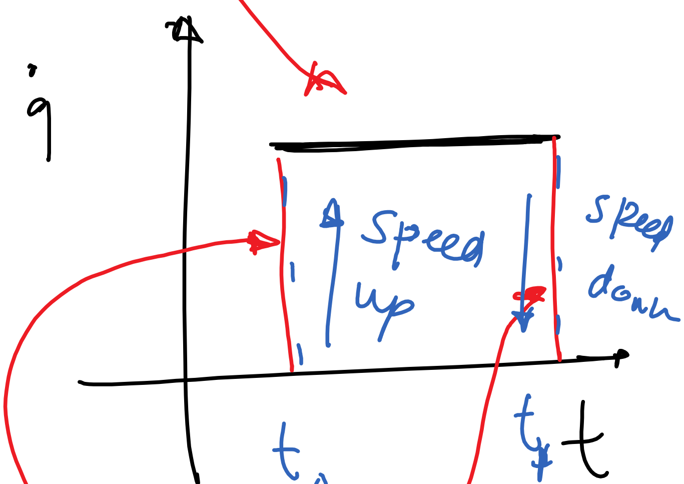
$$\boxed{q = \left(\frac{q_0 t_f - q_f t_0}{t_f - t_0}\right) + \left(\frac{q_f - q_0}{t_f - t_0}\right) t}$$

$$\boxed{q = \left(\frac{q_0 t_f - q_f t_0}{t_f - t_0}\right) + \left(\frac{q_f - q_0}{t_f - t_0}\right) t}$$

Differentiate with respect to time

$$\dot{q} = \underbrace{\frac{(q_f - q_0)}{(t_f - t_0)}}_{\text{constant}}$$





the velocity is discontinuous

We want to avoid this discontinuity.

To avoid this we impose the following conditions

$$\left.\begin{aligned} \dot{q}(t_0) &= 0 \\ \dot{q}(t_f) &= 0 \\ q(t_0) &= q_0 \\ q(t_f) &= q_f \end{aligned}\right\}$$

3rd order equation

$$q = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\Rightarrow \dot{q} = a_1 + 2a_2 t + 3a_3 t^2$$

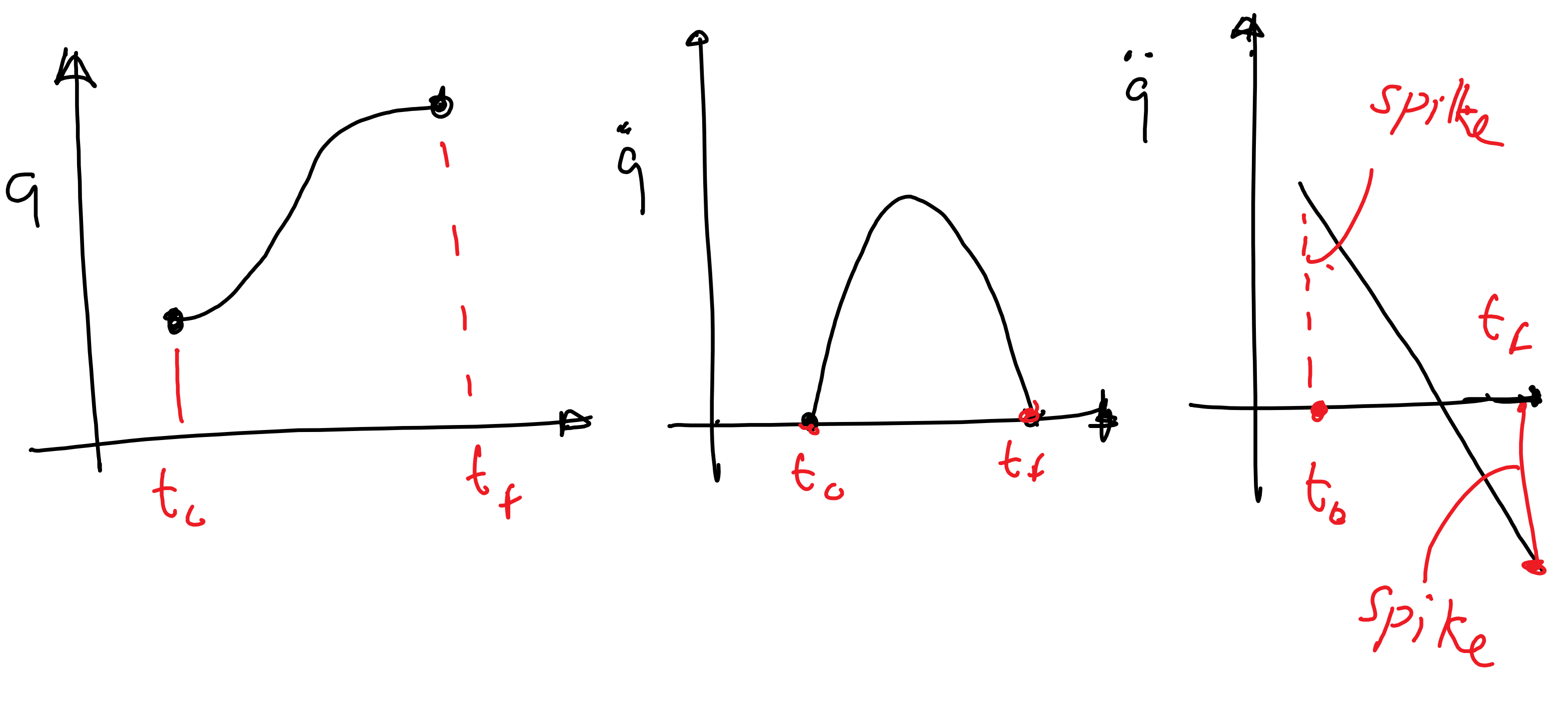
$$\left\{\begin{aligned} \dot{q}(t_0) = 0 &= a_1 + 2a_2 t_0 + 3a_3 t_0^2 \\ \dot{q}(t_f) = 0 &= a_1 + 2a_2 t_f + 3a_3 t_f^2 \\ q(t_0) = q_0 &= a_0 + a_1 t_0 + a_2 t_0^2 + a_3 t_0^3 \\ q(t_f) = q_f &= a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 \end{aligned}\right.$$

4 equations and 4 unknowns

a_0, a_1, a_2, a_3

$$\underbrace{\begin{bmatrix} 0 & 1 & 2t_0 & 3t_0^2 \\ 0 & 1 & 2t_f & 3t_f^2 \\ 1 & t_0 & t_0^2 & t_0^3 \\ 1 & t_f & t_f^2 & t_f^3 \end{bmatrix}}_{\text{invert using MATLAB}} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ q_0 \\ q_f \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \frac{1}{(t_0 - t_f)^3} \begin{bmatrix} q_f t_0^2 (t_0 - 3t_f) + q_0 t_f^2 (3t_0 - 3t_f) \\ 6 t_0 t_f (q_f - q_0) \\ 3(t_0 + t_f)(q_0 - q_f) \\ 2(q_f - q_0) \end{bmatrix}$$



Though the velocity starts and ends at zero speed. There is spike for acceleration at $t = t_0$ and $t = t_f$.

to avoid this, set the conditions $\ddot{q}(t_0) = 0$ and $\ddot{q}(t_f) = 0$

$\dddot{q}$ — jerk

$\ddddot{q}$ — crackle

$\overset{\cdot\cdot\cdot\cdot\cdot}{q}$ — pop

Keep adding more conditions.

Normally one goes upto jerk $\dddot{q}(t_0) = \dddot{q}(t_f) = 0$

Example 1: Find a time based parameterization for a revolute joint of a manipulator. The joint should move from 0 to 0.5 rad from time t= 0 to t=1 sec followed by movement from 0.5 rad to 1 rad in from t=1 to t=3 secs. Also, the velocity of the joint at the start of motion (t=0) and end of motion (t=3) should be 0 and the velocity of the joint at the intermediate point (t=1) should be 0.2 rad/s. Assume two minimal order polynomials of time, one for each movement.

$\Rightarrow q = 0 \quad t=0 \qquad \Rightarrow \dot{q} = 0 \quad t=0$

$\Rightarrow q = 0.5 \quad t=1 \qquad \dot{q} = 0 \quad t=3 \qquad \Leftarrow$ 4 cond

$q = 1 \quad t=3 \qquad \Rightarrow \dot{q} = 0.2 \quad t=1 \qquad \Rightarrow$ 4 cond

---

$$\Rightarrow q_1(t) = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3 \qquad 0 \le t \le 1$$

$$q_2(t) = a_{20} + a_{21}t + a_{22}t^2 + a_{23}t^3 \qquad 1 \le t \le 3$$

---

8 conditions.

$q_1(0) = 0 \qquad q_1(1) = 0.5 \quad | \quad q_2(1) = 0.5 \qquad \dot{q}_2(1) = 0.2$

$\dot{q}_1(0) = 0 \qquad \dot{q}_1(1) = 0.2 \quad | \quad q_2(3) = 1 \qquad \dot{q}_2(3) = 0$

Set 8 equations

$$q_1(0) = 0 = a_{10}$$

$$q_1(1) = 0.5 = a_{10} + a_{11} + a_{12} + a_{13}$$

$$\dot{q}_1(0) = 0 = a_{11}$$

$$\dot{q}_1(0.2) = 0.2 = a_{11} + 2a_{12} + 3a_{13}$$

$$q_2(1) = 0.5 = a_{20} + a_{21} + a_{22} + a_{23}$$

$$q_2(3) = 1 = a_{20} + 3a_{21} + 9a_{22} + 27a_{23}$$

$$\dot{q}_2(1) = 0.2 = a_{21} + 2a_{22} + 3a_{23}$$

$$\dot{q}_2(3) = 0 = a_{21} + 6a_{22} + 27a_{23}$$

Set up a 8 equations in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 & 9 & 27 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 6 & 27 \end{bmatrix} \begin{bmatrix} a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \\ 1 \\ 0 \\ 0.2 \\ 0.2 \\ 0 \end{bmatrix}$$

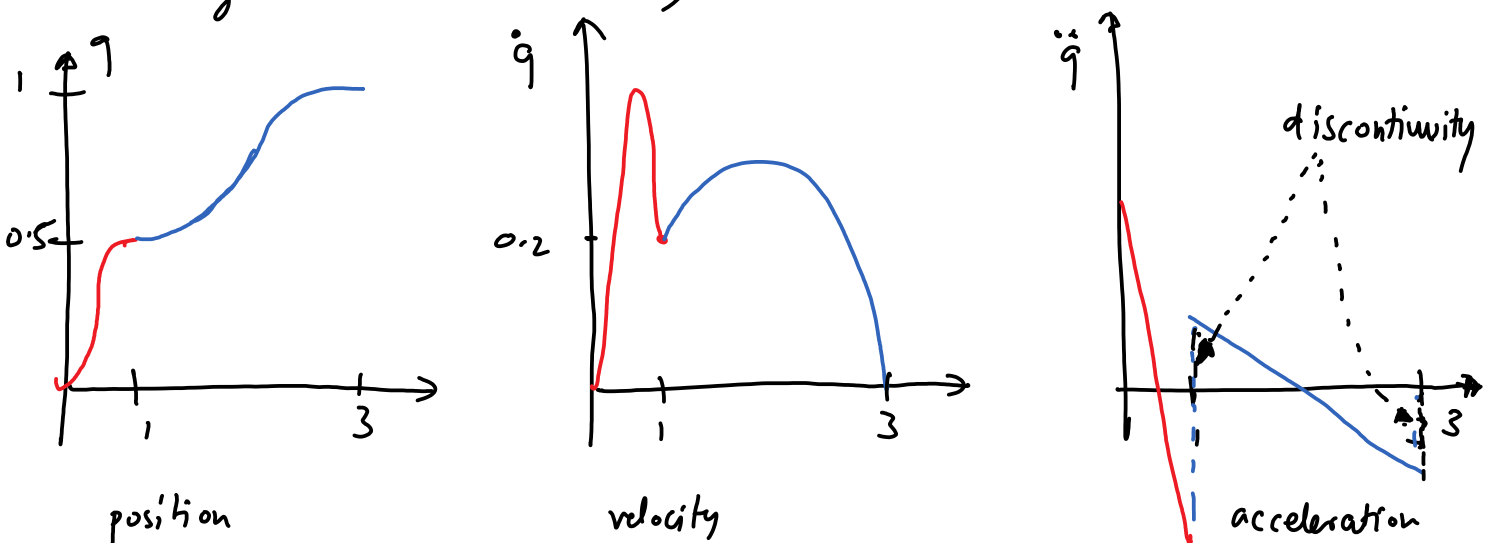
$$A \qquad\qquad X \qquad = b$$

In MATLAB do $X = inv(A)b$ to get

$a_{10} = 0 \quad a_{11} = 0 \quad a_{12} = 1.3 \quad a_{13} = -0.8$

$a_{20} = 0.55 \quad a_{21} = -0.375 \quad a_{22} = 0.4 \quad a_{23} = -0.075$

Plotting (also see MATLAB)



position

velocity

acceleration