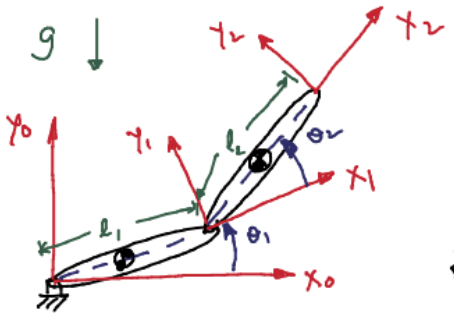
```
clc
clear all
```

Equations of motion of a two-link manipulator



Find the equation of motion of the two link manipulator using Euler-Lagrange Method.

{Center of mass is mid-way of each link}

SOLUTION

Link i	a_i	α_i	d_i	θ_i
1	l_1	0	0	θ_1
2	l_2	0	0	θ_2

```
%define symbolic quantities
syms l1 l2 m1 m2 I1 I2 g real %parameters
syms T1 T2 real %external torques on joints
syms theta1 theta2 theta1dot theta2dot theta1ddot theta2ddot real

%compute jacoboians
s1 = sin(theta1); s12 = sin(theta1+theta2);
c1 = cos(theta1); c12 = cos(theta1+theta2);

Jvc1 = [-l1*s1/2 0; l1*c1/2 0; 0 0];
Jvc2 = [-l1*s1-l2*s12/2 -l2*s12; l1*c1+l2*c12/2 l2*c12; 0 0];

Jwc1 = [0 0; 0 0; 1 0];
Jwc2 = [0 0; 0 0; 1 1];

X = [theta1 theta2]';
Xdot = [theta1dot theta2dot]';
K = 0.5*Xdot'*(m1*(Jvc1'*Jvc1) + m2*(Jvc2'*Jvc2) + I1*(Jwc1'*Jwc1)+I2*(Jwc2'*Jwc2)
P = m1*g*l1*s1/2 + m2*g*(l1*s1+l2*s12/2);
L = K - P;
```

Since there are two generalized co-ordinates, theta1 & theta2, we will have two equations.

```
dLdXdot(1) = diff(L,theta1dot);
ddt_dLdXdot(1) = diff(dLdXdot(1),theta1)*theta1dot+...
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                diff(dLdXdot(1),theta2)*theta2dot+...
                diff(dLdXdot(1),theta1dot)*theta1ddot+...
                diff(dLdXdot(1),theta2dot)*theta2ddot;
dLdX(1) = diff(L,theta1);

dLdXdot(2) = diff(L,theta2dot);
ddt_dLdXdot(2) = diff(dLdXdot(2),theta1)*theta1dot+...
                diff(dLdXdot(2),theta2)*theta2dot+...
                diff(dLdXdot(2),theta1dot)*theta1ddot+...
                diff(dLdXdot(2),theta2dot)*theta2ddot;
dLdX(2) = diff(L,theta2);

EOM1 = ddt_dLdXdot(1) - dLdX(1) - T1;
EOM2 = ddt_dLdXdot(2) - dLdX(2) - T2;
%EOM1 looks like this
%              M11(X)*theta1ddot+M12(X)*theta2ddot
%              + C11(X,Xdot)*theta1dot+C12(X,Xdot)*theta2dot+ G1(X) = 0
%Similarly EOM2 looks like this
%              M21(X)*theta1ddot+M22(X)*theta2ddot
%              + C21(X,Xdot)*theta1dot+C22(X,Xdot)*theta2dot+ G2(X) = 0


%Finally, we will simplify the expression.

%To get G1 and G2, we put Xdot, Xddot to zero.
G1 = subs(EOM1,[theta1ddot theta2ddot theta1dot theta2dot theta1 theta2],[0 0 0 0
G1 = simplify(G1);
G2 = subs(EOM2,[theta1ddot theta2ddot theta1dot theta2dot theta1 theta2],[0 0 0 0
G2 = simplify(G2);

%Display for easy copy-paste
disp(['G1 = ', char(G1),';']);
disp(['G2 = ', char(G2),';']);

%To get C terms, we put theta1ddot theta2ddot each equal to zero and subtract from
C1 = subs(EOM1,[theta1ddot theta2ddot],[0 0])-G1; %C1 = C11(X,Xdot)*theta1dot+C12(
C1 = simplify(C1);

C2 = subs(EOM2,[theta1ddot theta2ddot],[0 0])-G2; %C2 = C21(X,Xdot)*theta1dot+C22(
C2 = simplify(C2);

%Display for easy copy-paste
disp(['C1 = ', char(C1),';']);
disp(['C2 = ', char(C2),';']);

%Finally to get M, we subtract C and G from EOM
M1 = EOM1 - C1 - G1; %M1; = M11(X)*theta1ddot+M12(X)*theta2ddot
M2 = EOM2 - C2 - G2; %M2; = M21(X)*theta1ddot+M22(X)*theta2ddot

%Now we will extract M11, M12, M21, M22 by putting various values for theta1ddot a
M11 = subs(M1,[theta1ddot theta2ddot],[1 0]); M11 = simplify(M11);
M12 = subs(M1,[theta1ddot theta2ddot],[0 1]); M12 = simplify(M12);
M21 = subs(M2,[theta1ddot theta2ddot],[1 0]); M21 = simplify(M21);
M22 = subs(M2,[theta1ddot theta2ddot],[0 1]); M22 = simplify(M22);
```

```
disp(['M11 = ', char(M11),';']);
disp(['M12 = ', char(M12),';']);
disp(['M21 = ', char(M21),';']);
disp(['M22 = ', char(M22),';']);

disp('Equations solved');
disp('Copy paste Ms, Cs, and Gs');
```

```
G1 = (g*l1*m1)/2 - T1 + g*m2*(l1 + l2/2);
G2 = (g*l2*m2)/2 - T2;
C1 = (g*l2*m2*cos(theta1 + theta2))/2 - g*l1*m2 - (g*l2*m2)/2 - (g*l1*m1)/
C2 = (l2*m2*(g*cos(theta1 + theta2) - g + l1*theta1dot^2*sin(theta2)))/2;
M11 = I1 + I2 + (l1^2*m1)/4 + l1^2*m2 + (l2^2*m2)/4 + l1*l2*m2*cos(theta2)
M12 = I2 + (l2^2*m2)/2 + l1*l2*m2*cos(theta2);
M21 = I2 + (l2^2*m2)/2 + l1*l2*m2*cos(theta2);
M22 = I2 + l2^2*m2;
Equations solved
Copy paste Ms, Cs, and Gs
```

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