Manipulator system

Equations of motion

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B(q)u$$

$M(q)$: $n \times n$ — mass; $\ddot{q}$: $n \times 1$; $C(q,\dot{q})\dot{q}$: $n \times 1$ — coriolis; $G(q)$: $n \times 1$ — gravity; $B(q)$: $n \times m$ — selection matrix; u: $m \times 1$

n – states

m – controls

$B(q)$ – control selection matrix

We want $\dot{x} = f(x,u)$

$$\underline{x} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

$$\dot{\underline{x}} = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \underbrace{\begin{bmatrix} \dot{q} \\ M(q)^{-1}\left(B(q)u - C(q,\dot{q})\dot{q} - G(q)\right) \end{bmatrix}}_{f(x,u)}$$

Linearization: $f(x, u) = \begin{bmatrix} \dot{q} \\ M^{-1}(q)[Bu - C(q,\dot{q})\dot{q} - G(q)] \end{bmatrix}$

$$\underline{A} = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial \dot{q}}{\partial q} \overset{0}{} & \frac{\partial}{\partial \dot{q}}\dot{q} = I \\ \underbrace{\frac{\partial}{\partial q}\left[M^{-1}(Bu - C\dot{q} - G)\right]}_{1} & \underbrace{\frac{\partial}{\partial \dot{q}}\left[M^{-1}(Bu - C - G)\right]}_{2} \end{bmatrix}$$

(1) $\frac{\partial}{\partial q}\left[M(q)^{-1}\left[B(q)u - C(q,\dot{q})\dot{q} - G(q)\right]\right]$

$$= \frac{\partial M(q)^{-1}}{\partial q}\underbrace{\left[B(q)u - C(q,\dot{q})\dot{q} - G(q)\right]}_{0} +$$

$M(q)\ddot{q}$ $\overset{0}{}$ steady state

$$M^{-1}(q)\left[\frac{\partial B}{\partial q}u - \frac{\partial C}{\partial q}\dot{q} - \frac{\partial G(q)}{\partial q}\right]$$

$$= M^{-1}(q)\left[\frac{\partial B}{\partial q}u - \frac{\partial C}{\partial q}\dot{q} - \frac{\partial G}{\partial q}\right]$$

② $$\frac{\partial}{\partial \dot{q}} \left\{ M(q)^{-1} \left(\underline{B(q)u - C(q,\dot{q})\dot{q} - G(q)} \right) \right\}$$

$$= \cancel{\frac{\partial M(q)^{-1}}{\partial \dot{q}}} \left[B(q)u - C(q,\dot{q})\dot{q} - G(q) \right] +$$

$$M(q)^{-1} \left[\cancel{\frac{\partial B(q)}{\partial \dot{q}}} u - \frac{\partial C}{\partial \dot{q}} \dot{q} - \cancel{\frac{\partial G(q)}{\partial \dot{q}}} \right]$$

$$= M(q)^{-1} \left(- \frac{\partial C}{\partial \dot{q}} \dot{q} \right)$$

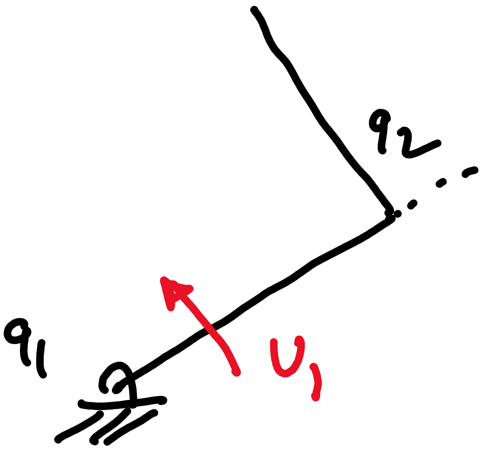
$$B = \frac{\partial f}{\partial u} = \begin{bmatrix} \cancel{\frac{\partial}{\partial u} \dot{q}} \\ \frac{\partial}{\partial u} \underbrace{M^{-1} \left(\underline{B(q)}u - \underline{C\dot{q} - G} \right)}_{M(q)^{-1} B(q)} \end{bmatrix}$$

Summary

$$\dot{x} = Ax + Bu$$

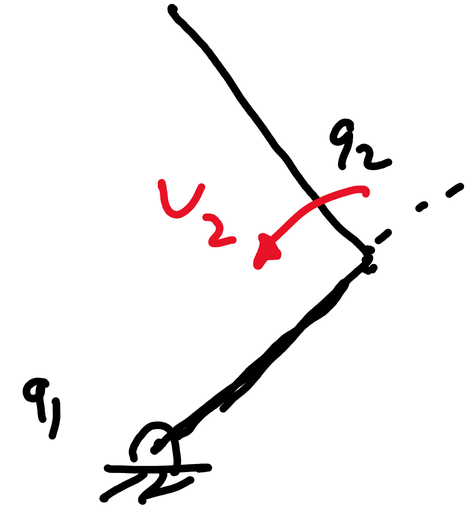
$$A = \begin{bmatrix} 0 & I \\ M^{-1}\left(-\frac{\partial C}{\partial q}\dot{q} - \frac{\partial G}{\partial q} + \frac{\partial B}{\partial q}u\right) & M^{-1}\left(-\frac{\partial C}{\partial \dot{q}}\dot{q}\right) \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} 0 \\ M^{-1}\,\underline{B}(q) \end{bmatrix}$$



1 actuator at q_1

Pendubot



1 actuator at q_2

Acrobot

Goal: Stabilize about the vertical position

Solution: ① EOM : $M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu$

② Linearize : $\dot{X} = AX + Bu$

$$X = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

③ Use LQR : $u = -KX$

$$u_i = -\underbrace{(K_1 \;\; K_2 \;\; K_3 \;\; K_4)}_{\text{LQR}} \overset{\text{Sensor}}{\begin{bmatrix} q_1 \\ q_2 \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}}$$

$i = 1, 2$