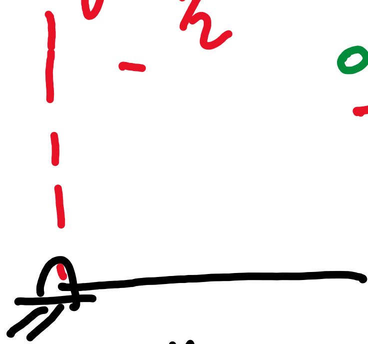
EXAMPLE: Trajectory Tracking for a 1-link pendulum

$\theta = \pi/2$

$\underline{0} \le t \le \underline{t_f}$



$\underline{\theta = 0}$

Trajectory generation

$$\theta_{des} = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \quad \text{— 4 constants}$$

4 constants

$$\left.\begin{aligned} \theta_{ref}(t=0) &= 0 \\ \theta_{ref}(t=t_f) &= \pi/2 \\ \dot{\theta}_{ref}(t=0) &= 0 \\ \dot{\theta}_{ref}(t=t_f) &= 0 \end{aligned}\right\} \text{4 equation}$$

Solve for 4 constants using 4 equations.

Trajectory Tracking

$$\tau = M\left[\ddot{q}_{des} - K_p\left(q - q_{des}\right) - K_d\left(\dot{q} - \dot{q}_{des}\right)\right] + C(q,\dot{q})\,\dot{q} + G(q) \qquad \text{— ①}$$

General control partitioning / feedback linearization controller.

Why does this form of τ work?

Substitute ① in

$$M(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + G(q) = \tau$$

$$M(q)\,\ddot{q} + \cancel{C(q,\dot{q})\dot{q}} + \cancel{G(q)} = M(q)\left[\ddot{q}_{des} - K_p\left(q - q_{des}\right) - K_d\left(\dot{q} - \dot{q}_{des}\right)\right] + \cancel{C(q,\dot{q})\,\dot{q}} + \cancel{G(q)}$$

$$M(q)\left[\left(\ddot{q} - \ddot{q}_{des}\right) - K_p\left(q - q_{des}\right) - K_d\left(\dot{q} - \dot{q}_{des}\right)\right] = 0$$

$$\underline{M(q)}\left[(\ddot{q} - \underline{\underline{\ddot{q}_{des}}}) - K_p(q - \underline{q_{des}}) - K_d(\dot{q} - \underline{\dot{q}_{des}})\right] = 0$$

$$e = q - q_{des} \,;$$

$$\dot{e} = \dot{q} - \dot{q}_{des}$$

$$\ddot{e} = \ddot{q} - \ddot{q}_{des}$$

$$M(q)\left[\ddot{e} - K_p e - K_d \dot{e}\right] = 0$$

$$\boxed{K_d = 2\sqrt{K_p}}$$

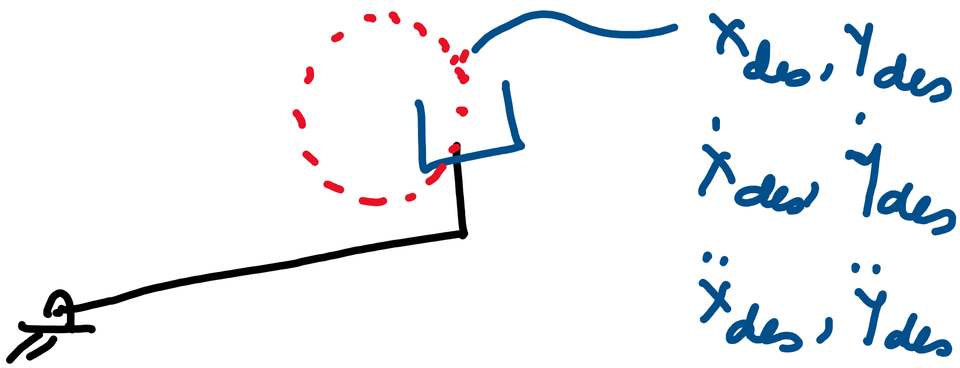
Tracking

$$\tau = (I + ml^2)\left[\ddot{\theta}_{des} - K_p(\theta - \theta_{des}) - K_d(\dot{\theta} - \dot{\theta}_{des})\right]$$

<4a_pendulum_trajectory_tracking>

<4b_double pendulum_trajectory_tracking>

Feedback linearization in the task space



Trajectory generation

8 unknowns

$$\left.\begin{aligned} x_{des} &= a_{x0} + a_{x1} t + a_{x2} t^2 + a_{x3} t^3 \\ y_{des} &= a_{y0} + a_{y1} t + a_{y2} t^2 + a_{y3} t^3 \end{aligned}\right\}$$

Given

$$\left.\begin{aligned} x_{des}(0) &= \checkmark \qquad y_{des}(0) = \checkmark \\ x_{des}(t_f) &= \checkmark \qquad y_{des}(t_f) = \checkmark \end{aligned}\right\} \text{4 cond}$$

$$0 = \underbrace{\dot{x}_{des}(0) = \dot{y}_{des}(0) = \dot{x}_{des}(t_f) = \dot{y}_{des}(t_f)}_{\text{4 cond}}$$

8 constants / 8 conditions.

Mapping x_{des}, y_{des} to q_{des}

$$\begin{bmatrix} x_{des} \\ y_{des} \end{bmatrix} = f(q_{des})$$ Forward kinematics

① – $q_{des} = f^{-1} \begin{bmatrix} x_{des} \\ y_{des} \end{bmatrix}$ Inverse kinematics

Root finding

$$\begin{bmatrix} \dot{x}_{des} \\ \dot{y}_{des} \end{bmatrix} = J \, \dot{q}_{des}$$

Jacobian

$J = \frac{\partial f}{\partial q}$

② – $\dot{q}_{des} = J^{-1} \begin{bmatrix} \dot{x}_{des} \\ \dot{y}_{des} \end{bmatrix}$ just J^{-1}

$$\begin{bmatrix} \ddot{x}_{des} \\ \ddot{y}_{des} \end{bmatrix} = J \, \underline{\underline{\ddot{q}_{des}}} + \dot{J} \, \dot{q}_{des}$$

③ – $\ddot{q}_{des} = J^{-1} \left(\begin{bmatrix} \ddot{x}_{des} \\ \ddot{y}_{des} \end{bmatrix} - \dot{J} \, \dot{q}_{des} \right)$

Now use

$$\tau = M\left(\ddot{q}_{des} - K_p(q - q_{des}) - K_d(\dot{q} - \dot{q}_{des})\right) + G(q,\dot{q})\dot{q} + C(q)$$

Subs. $\ddot{q}_{des}$, $\dot{q}_{des}$, q_{des} from 1, 2, 3

<5_doublependulum_cartesian_control>