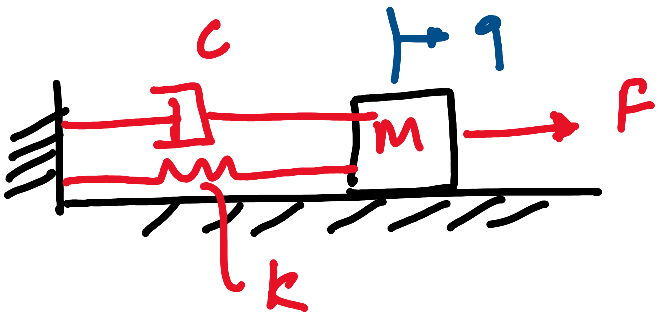
Simple system with equations similar to a manipulator

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$$

$$\boxed{m\ddot{q} + c\dot{q} + kq = F}$$

(spring mass damper)



lets assume $F = 0$ (free vibration)

$$\ddot{q} + \frac{c}{m}\dot{q} + \frac{k}{m}q = 0$$

$$\boxed{\omega_n = \sqrt{\frac{k}{m}}}$$

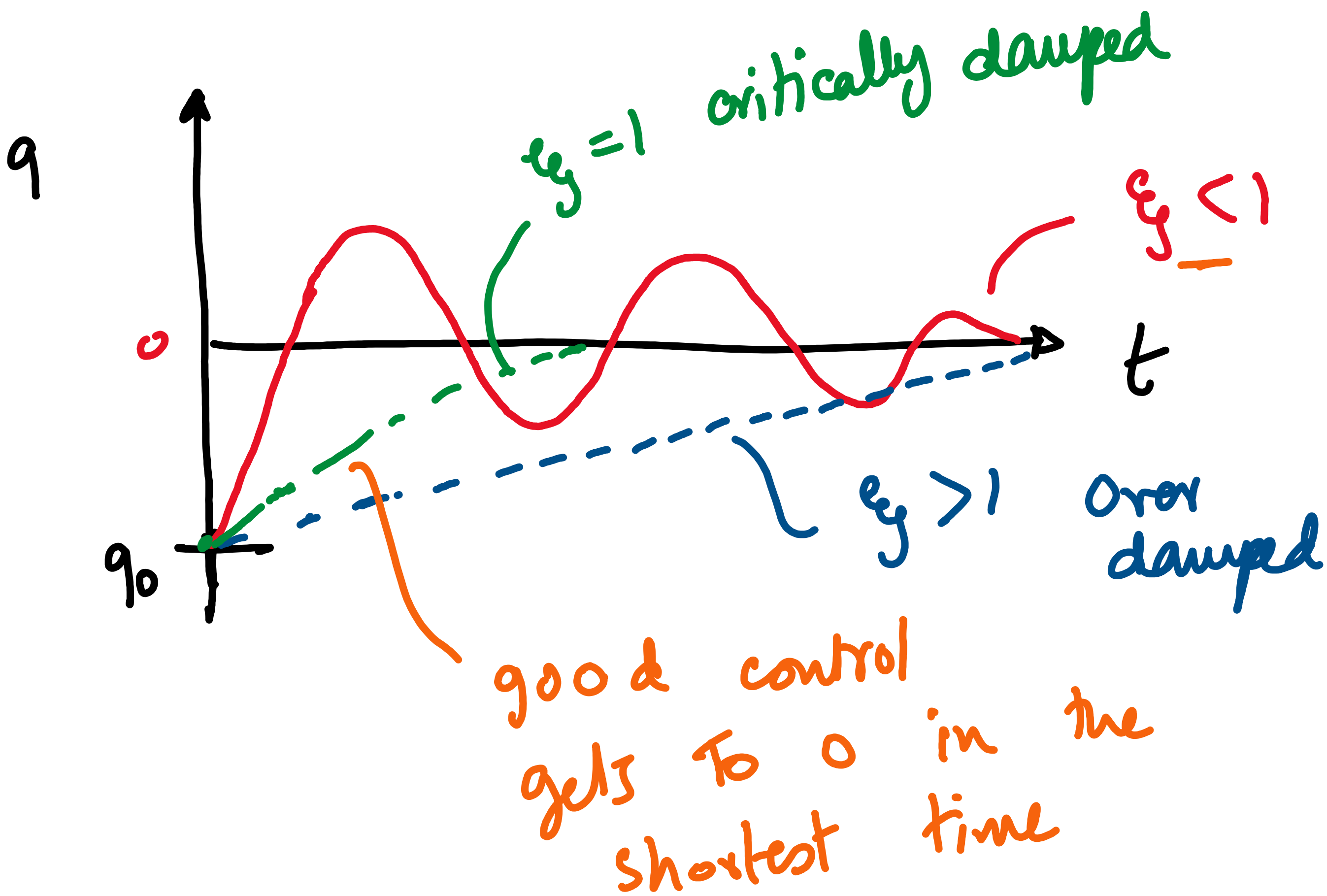
$$2\xi\omega_n = \frac{c}{m}$$

Xi

$$\boxed{\xi = \frac{c}{2\sqrt{km}}}$$

3 cases

1	$\boxed{\xi > 1}$	$c > 2\sqrt{km}$	Overdamped
2	$\boxed{\xi = 1}$	$c = 2\sqrt{km}$	Critical damped
3	$\boxed{\xi < 1}$	$c < 2\sqrt{km}$	Under-damped



$$m\ddot{q} + c\dot{q} + kq = F \qquad - (1)$$

Design F such that the system is critically damped.

Assume $F = -k_p q - k_d \dot{q} \qquad - (2)$

proportional - derivative control

Substitute (2) in (1)

$$m\ddot{q} + c\dot{q} + kq = -k_p q - k_d \dot{q}$$

$$m\ddot{q} + (c + k_d)\dot{q} + (k + k_p) q = 0$$

free vibrations

Choose k_p, k_d such that the system is critically damped

$$(c + k_d) = 2\sqrt{(k + k_p)\, m}$$

2 constants and 1 equation.
Fix one & use the equation to compute the second one

Fix k_p, solve for k_d

$$\boxed{k_d = -c + 2\sqrt{(k+k_p)m}}$$

system will be critically damped.

See python file in the folder
1_simple_control_partitioning