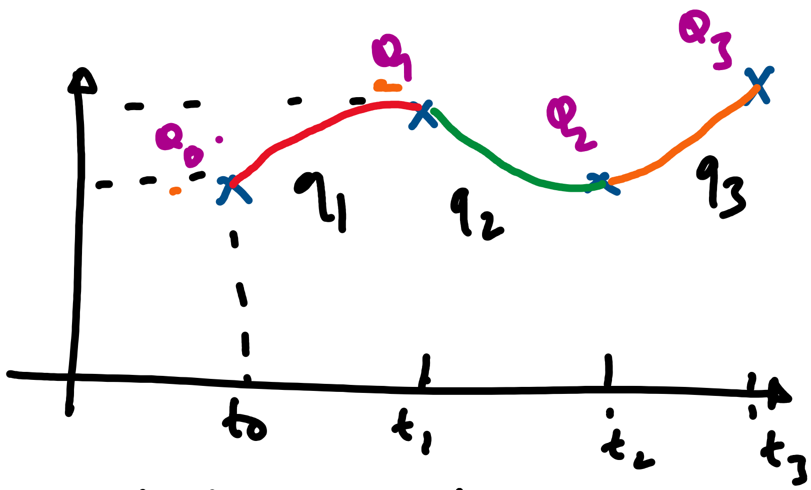
Piecewise splines



q_1, q_2, q_3

3rd order polynomials

Given data points

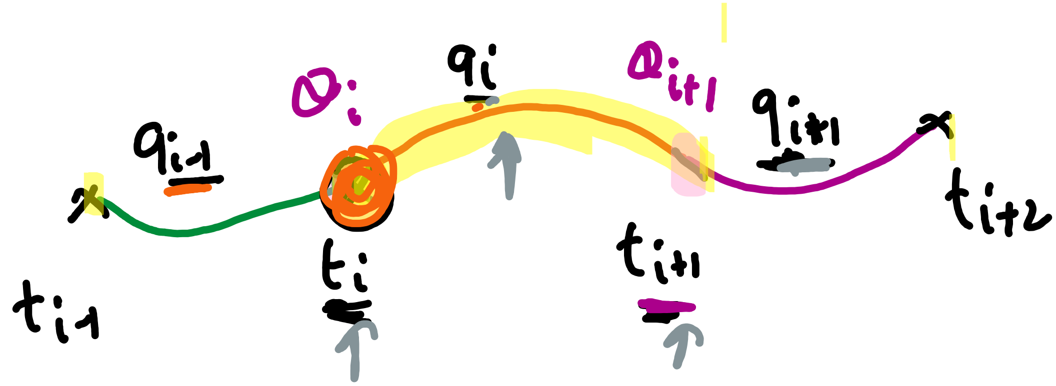
$[t_0, Q_0], [t_1, Q_1], [t_2, Q_2], [t_3, Q_3] \ldots$

$[t_n, Q_n]$

$(n+1)$ data points.

Assume a 3rd order polynomial

$$q_i = a_{i0} + a_{i1}(t - t_i) + a_{i2}(t - t_i)^2 + a_{i3}(t - t_i)^3$$



$$q_i = a_{i0} + a_{i1}(t-t_i) + a_{i2}(t-t_i)^2 + a_{i3}(t-t_i)^3$$

$$q_i(t_i) = a_{i0} = Q_i$$

$$q_i(t_{i+1}) = a_{i0} + a_{i1}(t_{i+1}-t_i) + \dots$$
$$a_{i2}(t_{i+1}-t_i)^2 + a_{i3}(t_{i+1}-t_i)^3 = Q_{i+1}$$

$$q_i'(t_i) = q_{i-1}'(t_i) \checkmark$$

$$q_i''(t_i) = q_{i-1}''(t_i) \checkmark$$

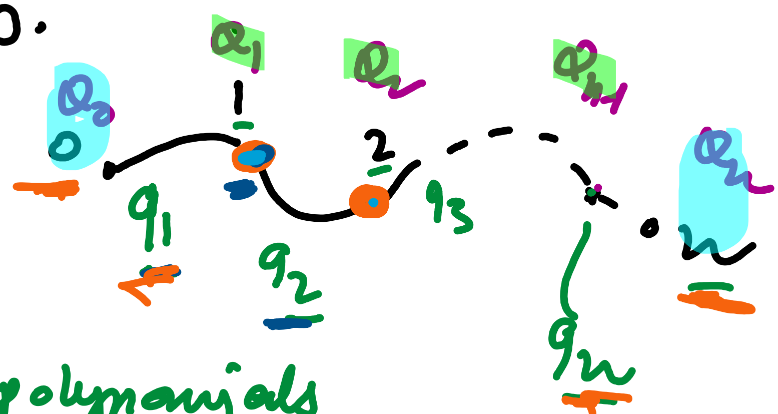
$$q_i'(t_{i+1}) = q_{i+1}'(t_{i+1}) \checkmark$$

$$q_i''(t_{i+1}) = q_{i+1}''(t_{i+1}) \checkmark$$

vel

acc

Compute the # of constants and # of conditions.



$n+1$ points

n 3rd order polynomials

4 constant for every 3rd order poly.

constants : $4n$

$2(n-1)$ position conditions

$(n-1)$ velocity conditions

$(n-1)$ acceleration conditions

2 θ_0 and θ_n

$$= 2(n-1) + n-1 + n-1 + 2$$

$$= 2n - \cancel{2} + n-1 + n-1 \cancel{+2}$$

$$= 4n-2$$

$4n$ constants $>$ $4n-2$ conditions.
(equations)

We need to choose 2 more conditions/equations to compute all $4n$ constants.

Here are few ways of imposing the 2 conditions

1. Natural spline

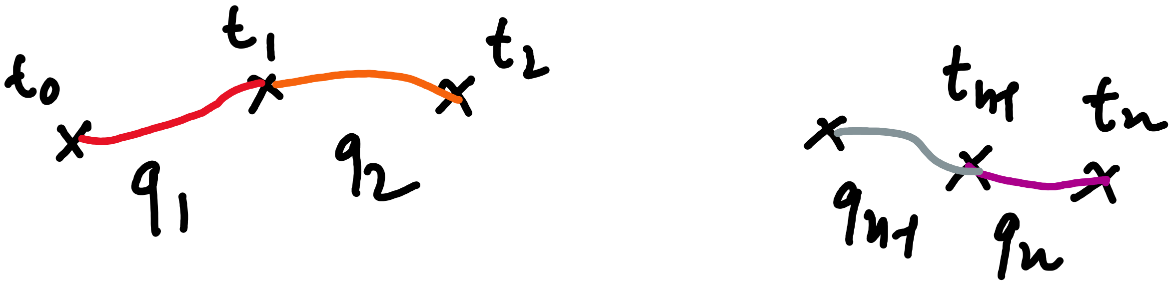
$$F''(t_0)=0 \quad \& \quad F''(t_n)=0$$

2. Clamped condition

$$f'(t_0)=0 \quad \& \quad f'(t_n)=0$$

3. Not-a-Knot condition

③ Not-a-Knot condition



$$q_1'''(t_1) = q_2'''(t_1)$$

$$q_i = a_{i0} + a_{i1}(t-t_i) + a_{i2}(t-t_i)^2 + a_{i3}(t-t_i)^3$$

$$q_i''' = 6 \, a_{i3}$$

$$6 a_{13} = 6 a_{23}$$

$$q_n'''(t_{n-1}) = q_{n-1}'''(t_{n-1})$$