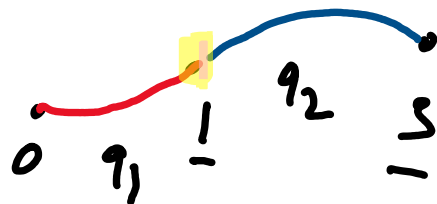


Example 1: Find a time based parameterization for a revolute joint of a manipulator. The joint should move from 0 to 0.5 rad from time t=0 to t=1 sec followed by movement from 0.5 rad to 1 rad in from t=1 to t=3 secs. Also, the velocity of the joint at the start of motion (t=0) and end of motion (t=3) should be 0 and the velocity of the joint at the intermediate point (t=1) should be 0.2 rad/s. Assume two minimal order polynomials of time, one for each movement.

0 - 0.5 rad
0.5 - 1

$t \in [0, 1]$
 $t \in [1, 3]$
 $v = 0.2$



$$t=0 \quad q_1 = 0$$

$$t=1 \quad q_1 = 0.5$$

$$t=0 \quad \dot{q}_1 = 0$$

$$t=1 \quad \dot{q}_1 = 0.2$$

$$q_1 = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3$$

$$\dot{q}_1 = a_{11} + 2a_{12}t + 3a_{13}t^2$$

4 conditions
on q_1

$$t=1 \quad q_2 = 0.5$$

$$t=3 \quad q_2 = 1$$

$$t=3 \quad \dot{q}_2 = 0$$

$$t=1 \quad \dot{q}_2 = 0.2$$

$$q_2 = a_{20} + a_{21}t + a_{22}t^2 + a_{23}t^3$$

$$\dot{q}_2 = a_{21} + 2a_{22}t + 3a_{23}t^2$$

4 conditions
on q_2

$$q_1(0) = 0$$

$$0 = q_{10}$$

$$q_1(1) = 0.5$$

$$0.5 = q_{10} + q_{11} + q_{12} + q_{13}$$

$$\dot{q}_1(0) = 0$$

$$0 = q_{11}$$

$$\dot{q}_1(1) = 0.2$$

$$0.2 = q_{11} + 2q_{12} + 3q_{13}$$

$$q_2(1) = 0.5$$

$$0.5 = q_{20} + q_{21} + q_{22} + q_{23}$$

$$q_2(3) = 1$$

$$1 = q_{20} + 3q_{21} + 9q_{22} + 27q_{23}$$

$$\dot{q}_2(3) = 0$$

$$0 = q_{21} + 6q_{22} + 27q_{23}$$

$$\dot{q}_2(1) = 0.2$$

$$0.2 = q_{21} + 2q_{22} + 3q_{23}$$

8 eqn
8 unknowns

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 & 9 & 27 \\ 0 & 0 & 0 & 0 & 0 & 1 & 6 & 27 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} q_{10} \\ q_{11} \\ q_{12} \\ q_{13} \\ q_{20} \\ q_{21} \\ q_{22} \\ q_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 0 \\ 0.2 \\ 0.5 \\ 1 \\ 0 \\ 0.2 \end{bmatrix}$$

$$\begin{bmatrix}
 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1 & 2 & 3 & 0 & 0 & 0 & 0 \\
 \hline
 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
 0 & 0 & 0 & 0 & 1 & 3 & 9 & 27 \\
 0 & 0 & 0 & 0 & 0 & 1 & 6 & 27 \\
 0 & 0 & 0 & 6 & 0 & 1 & 2 & 3
 \end{bmatrix}
 \begin{bmatrix}
 a_{10} \\
 a_{11} \\
 a_{12} \\
 a_{13} \\
 a_{20} \\
 a_{21} \\
 a_{22} \\
 a_{23}
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0.5 \\
 0 \\
 0.2 \\
 0.5 \\
 1 \\
 0 \\
 0.2
 \end{bmatrix}$$

A
 X
 b

$$A X = b$$

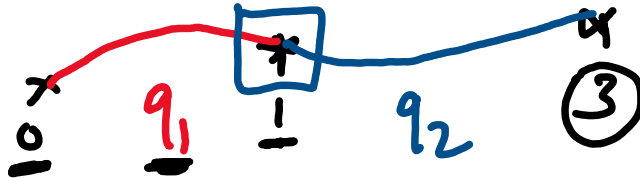
$$X = A^{-1} b$$

Using python

$$X = [0 \quad 0 \quad 1.3 \quad -0.8 \quad 0.55 \quad -0.375 \quad -0.4 \quad -0.075]$$

$[a_{10}$
 a_{11}
 a_{12}
 a_{13}
 a_{20}
 a_{21}
 a_{22}
 a_{23}

Example 2: Find a time based parameterization for a revolute joint of a manipulator. The joint should move from 0 to 0.5 rad from time $t=0$ to $t=1$ sec followed by movement from 0.5 rad to 1 rad in from $t=1$ to $t=3$ secs. Also, the velocity of the joint at the start of motion ($t=0$) and end of motion ($t=3$) should be 0 and the acceleration of the joint at the intermediate point ($t=1$) should be continuous. Assume two minimal order polynomials of time, one for each movement.



$$q_1(t=0) = 0 \quad ; \quad q_1(t=1) = 0.5$$

$$q_2(t=1) = 0.5 \quad ; \quad q_2(t=3) = 1$$

$$\dot{q}_1(t=0) = 0 \quad ; \quad \dot{q}_2(t=3) = 0$$

$$\ddot{q}_1(t=1) = \ddot{q}_2(t=1)$$

$$\dot{q}_1(t=1) = \dot{q}_2(t=1)$$

— 1 cond

— 1 cond.

} 6 cond

$$q_1 = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3$$

$$q_2 = a_{20} + a_{21}t + a_{22}t^2 + a_{23}t^3$$

} 8 unknowns

8 equations / 8 unknowns