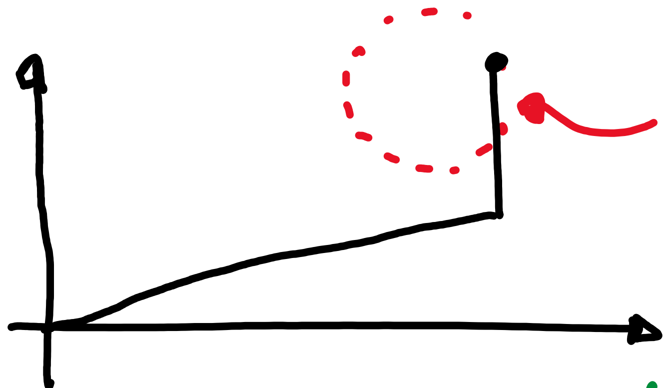
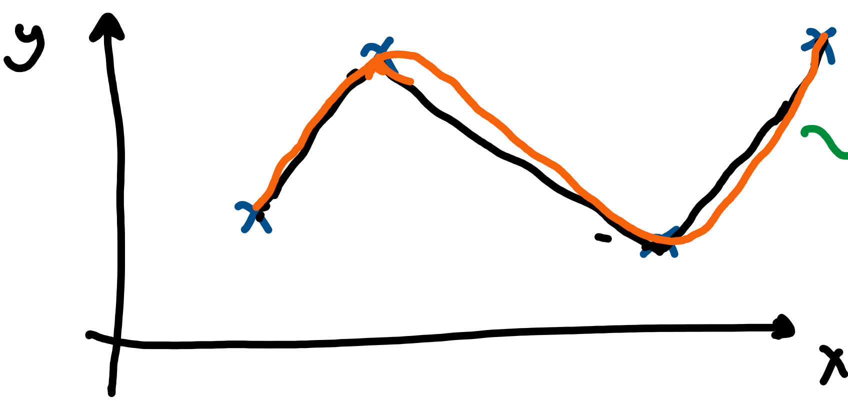
Trajectory Generation



$x = r\cos t + x_0$
$y = r\sin t + y_0$

t = time



How to generate the trajectory?

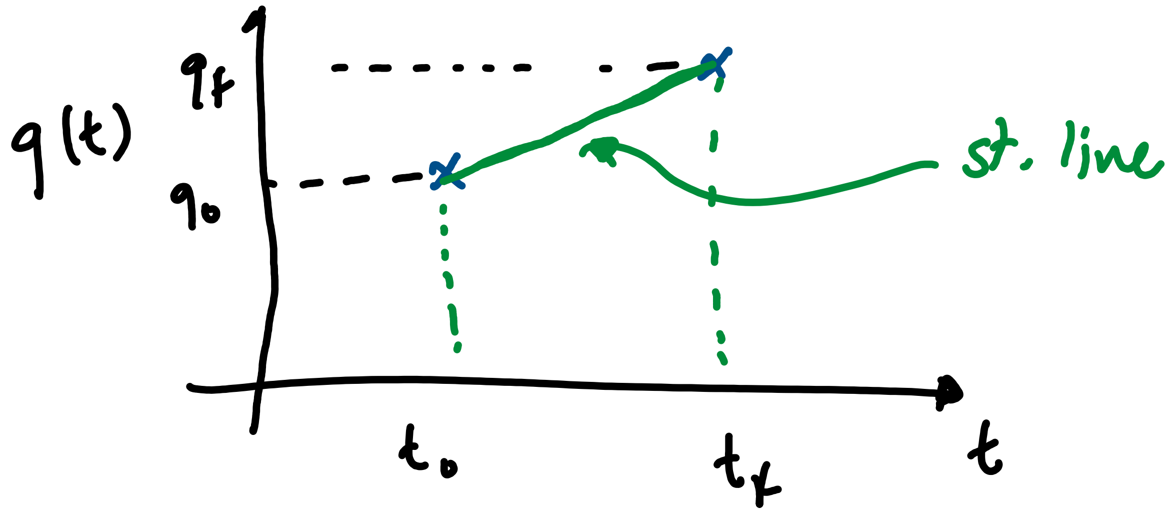
Conditions:

i) Pass through given points

ii) Smooth

⋮

① Linear profile



$$q(t) = a_0 + a_1 t$$ a_0, a_1 are unknowns.

$$q_0 = a_0 + a_1 t_0$$
$$q_f = a_0 + a_1 t_f$$

$$\underbrace{\begin{bmatrix} q_0 \\ q_f \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} 1 & t_0 \\ 1 & t_f \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}}_{X}$$

$$b = AX$$

$$\Rightarrow X = A^{-1} b$$

$$X = A^{-1} b$$

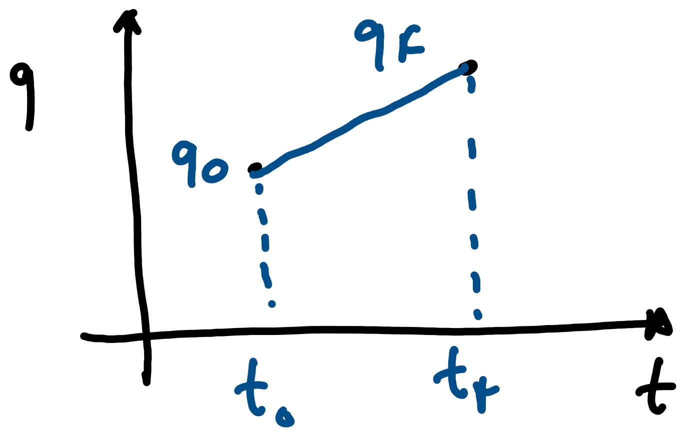
$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 & t_0 \\ 1 & t_f \end{bmatrix}^{-1} \begin{bmatrix} q_0 \\ q_f \end{bmatrix}$$

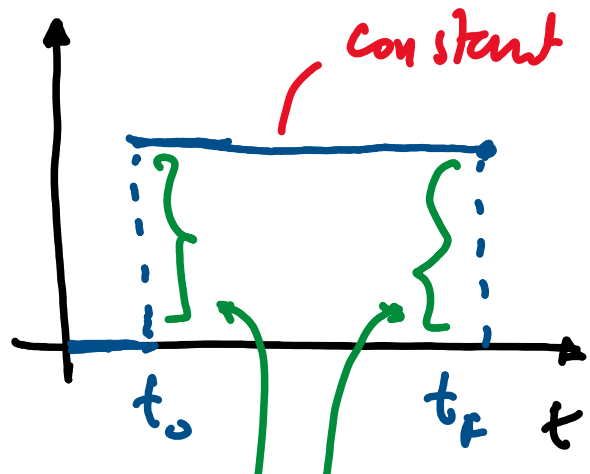
$$= \frac{1}{(t_f - t_0)} \begin{bmatrix} t_f & -t_0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} q_0 \\ q_f \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \left\{ \dfrac{q_0 t_f - q_f t_0}{t_f - t_0} \right\} \\ \left\{ \dfrac{q_f - q_0}{t_f - t_0} \right\} \end{bmatrix}$$

$$\boxed{q(t) = \left(\frac{q_0 t_f - q_f t_0}{t_f - t_0} \right) + \left(\frac{q_f - q_0}{t_f - t_0} \right) t}$$

$$\boxed{\dot{q} = \left(\frac{q_f - q_0}{t_f - t_0} \right) = \text{constant}}$$





jump in the velocity.

✓

Bad for motors

To avoid this, we set 4 conditions.

$t = t_0 \quad q = q_0$

$t = t_f \quad q = q_f$

$t = t_0 \quad \dot{q} = 0$

$t = t_f \quad \dot{q} = 0$

4 conditions

$\rightarrow q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$

4 constants, 4 conditions.

$\rightarrow \dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2$

$\ddot{q}(t) = 2a_2 + 6a_3 t$ — st. line

$t = t_0 \qquad q_0 = a_0 + a_1 t_0 + a_2 t_0^2 + a_3 t_0^3$

$t = t_f \qquad q_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3$

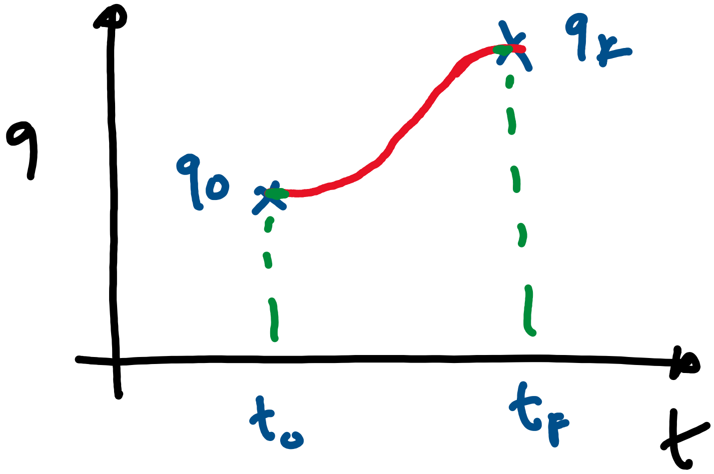
$t = t_0 \qquad 0 = a_1 + 2a_2 t_0 + 3a_3 t_0^2$

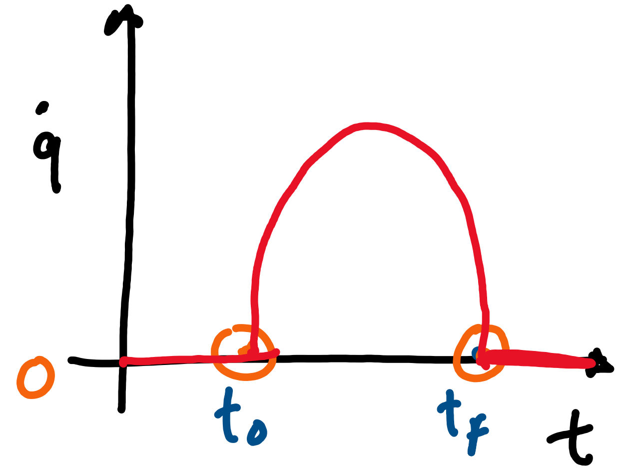
$t = t_f \qquad 0 = a_1 + 2a_2 t_f + 3a_3 t_f^2$

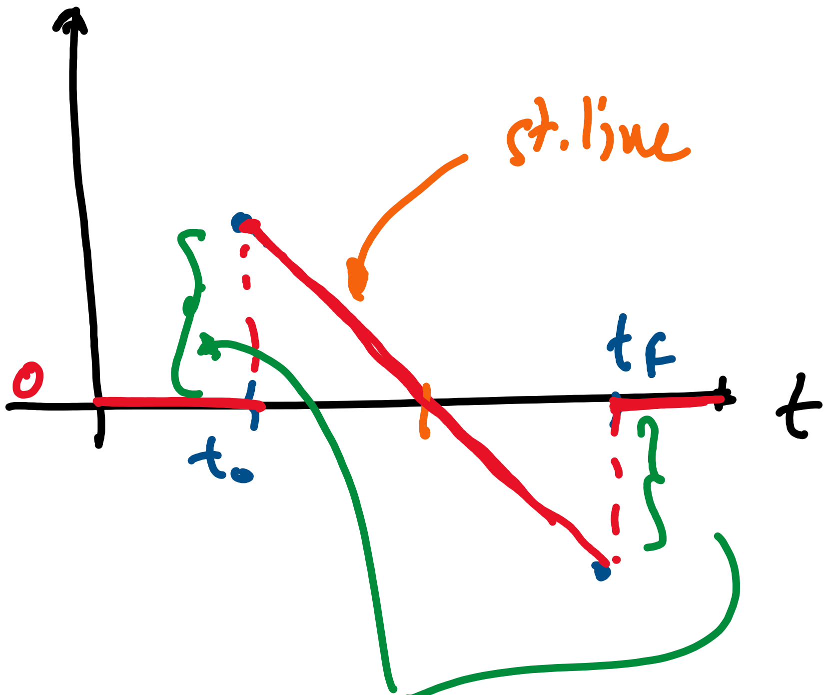
$$\underbrace{\begin{bmatrix} q_0 \\ q_f \\ 0 \\ 0 \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_0 & 3t_0^2 \\ 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}}_{X}$$

$b = AX$

$X = A^{-1} b$







acceleration is discontinuous.

bad for motors

To avoid this, we add 2 more $\ddot{}$ conditions

$t = t_0 \qquad q = q_0$

$t = t_f \qquad q = q_f$

$t = t_0 \qquad \dot{q} = 0$

$t = t_f \qquad \dot{q} = 0$

$t = t_0 \qquad \ddot{q} = 0$

$t = t_f \qquad \ddot{q} = 0$

6 conditions

Assume $q = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$

6 constant

$\dddot{q}$ will be discontinuous.

jerk.

add 2 more conditions. 7^{th} order poly.

$\ddddot{q}$ — snap

$\overset{\cdot\cdot\cdot\cdot\cdot}{q}$ — crackle

$\overset{\cdot\cdot\cdot\cdot\cdot\cdot}{q}$ — pop

Stop at 6 conditions $\ddot{q}$ is continuous

8 conditions $\dddot{q}$ is continuous.