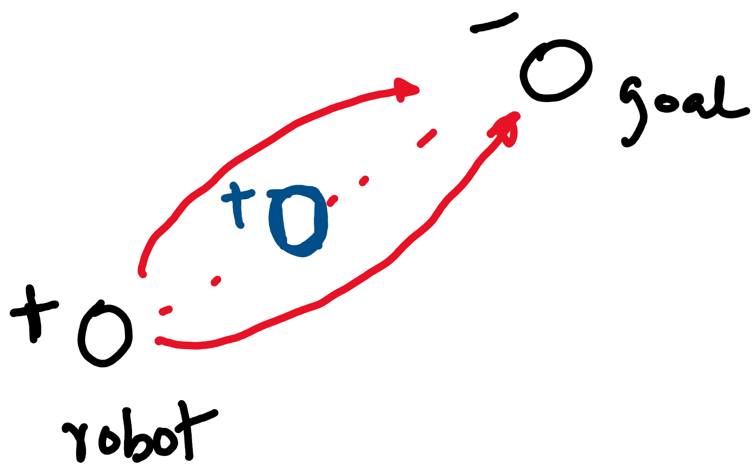
② Potential fields



U_{att} - attractive field

U_{rep} - repulsive field

$U = U_{att} + U_{rep}$.

$F = -\nabla U(q)$ ⇝ jacobian

(a) Attractive potential field U_{att}

(i) Conic potential field

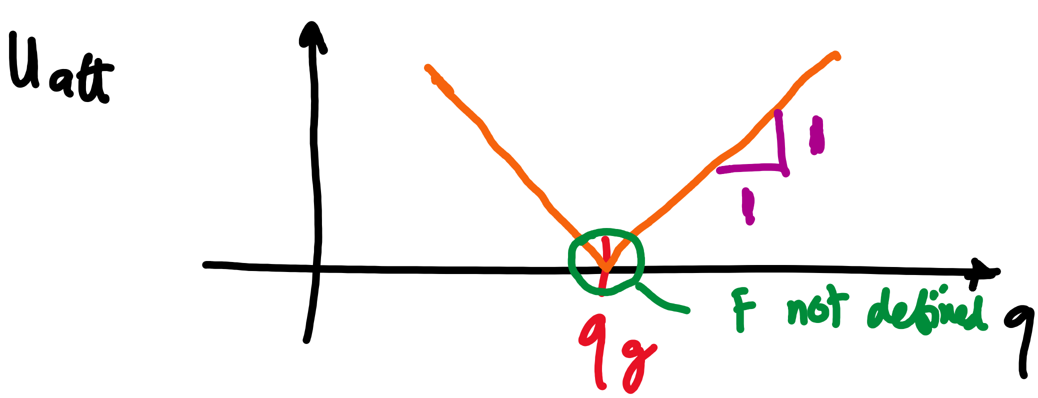
$\cdot\, p(q_g)$ goal

$\cdot\, p(q)$ degrees of freedom

position

$$U_{att} = \|p(q) - p(q_g)\| \quad \text{norm}$$

$$= \sqrt{(p(q) - p(q_g)) \cdot (p(q) - p(q_g))}$$



$$F = -\nabla U_{att} = \frac{(p(q) - p(q_g))}{\|p(q) - p(q_g)\|} = \frac{\text{vector}}{\|\text{vector}\|}$$

$$U_{att} = \sqrt{x(q)\cdot x(q)} = \|x(q)\|$$

$$F = -\frac{\partial U_{att}}{\partial q} = \underline{\qquad}$$

$$\frac{d}{dx} x^n = n\, x^{n-1}$$

$$U = \left((x - x_g)^2 + (y - y_G)^2 \right)^{1/2}$$

$$\frac{\partial U}{\partial (x,y)} = \frac{1}{2}\left[(x - x_g)^2 + (y - y_G)^2 \right]^{\frac{1}{2}-1} \cdots$$

$$\left[2(x - x_G),\ 2(y - y_G) \right]$$

$$= \left[\frac{\cancel{2}(x - x_G)}{\cancel{2}\sqrt{(x - x_g)^2 + (y - y_G)^2}},\ \frac{\cancel{2}(y - y_G)}{\cancel{2}\sqrt{(x - x_G)^2 + (y - y_G)^2}} \right]$$

$$F = -\frac{\partial U}{\partial q}$$

$$= -\left[\frac{(x - x_G)}{\sqrt{(x - x_g)^2 + (y - y_G)^2}},\ \frac{(y - y_G)}{\sqrt{(x - x_G)^2 + (y - y_G)^2}} \right]$$

$$F = -\frac{\text{vector}}{\|\text{vector}\|}$$

$$= \frac{p(q) - p(q_g)}{\| p(q) - p(q_g) \|}$$

NOTE: $\|F\| = 1$

At $q = q_g$ F is not defined

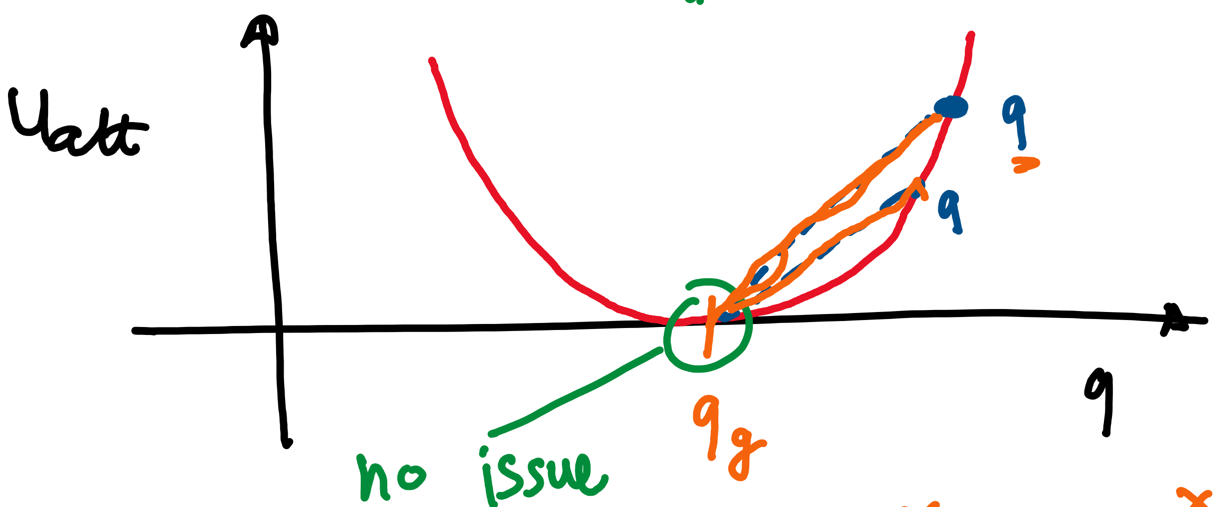
$$F = -\nabla U_{att} \qquad q \neq q_g$$

$$= 0 \qquad q = q_g$$

(ii) Parabolic potential

$$U_{att} = \frac{1}{2}\,\xi\,\| p(q) - p(q_g) \|^2$$

constant



$$F = -\frac{\partial U}{\partial q} = -\frac{\partial}{\partial q}\,\frac{\xi}{2}\left[\underbrace{\left(p(q) - p(q_g)\right)}_{x}\cdot\underbrace{\left(p(q) - p(q_g)\right)}_{x}\right]$$

$$= -\frac{\xi}{\cancel{2}}\,\cancel{2}\,\underbrace{\left[p(q) - p(q_g)\right]}_{2x}$$

$$F = -\xi\left[p(q) - p(q_g)\right]$$

Conic:

Constant force away from q_g Adv

F is not defined at q_g

Parabolic

F is defined at q_g Adv.

F is proportional to distance from q_g

Combine Conic & Parabolic

Choose a distance d from q_g

If robot is at a distance $\geq d$ use conic potential

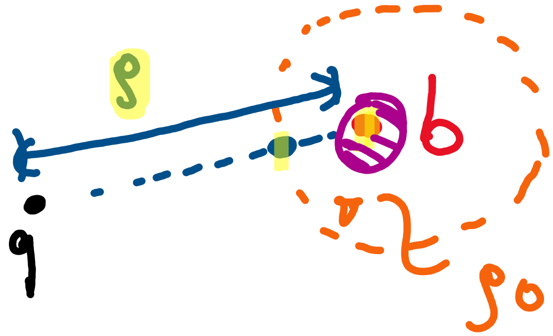
If robot is at a distance $< d$ use parabolic potential.

(iii) Combined conic / parabolic field

$$U_{att} = \begin{cases} \frac{1}{2}\xi \|p(q)-p(q_g)\|^2 & \|p(q)-p(q_g)\| \leq d \\ d\xi\|p(q)-p(q_g)\| - \underbrace{\frac{1}{2}\xi d^2}_{\text{constant}} & \|p(q)-p(q_g)\| > d \end{cases}$$

$$\boxed{F_{att} = \begin{cases} -\xi \|p(q)-p(q_g)\| & \|p(q)-p(q_g)\| \leq d \\ -d\xi \dfrac{p(q)-p(q_g)}{\|p(q)-p(q_g)\|} & \|p(q)-p(q_g)\| > d \end{cases}}$$

ⓑ Repulsive field



$$\rho(q) = \| p(q) - b^{0} \|$$

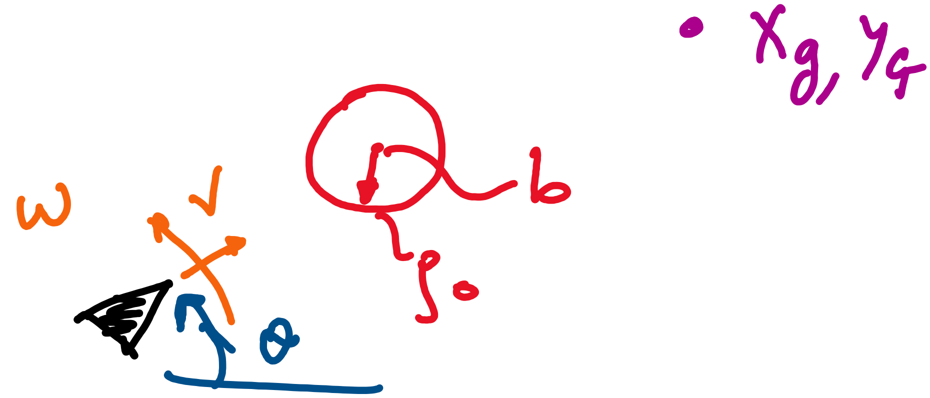
$$U_{rep} \begin{cases} 0 & \rho(q) > \rho_0 \\ \dfrac{\eta}{2} \left\{ \dfrac{1}{\rho(q)} - \dfrac{1}{\rho_0} \right\}^2 & \rho(q) \leq \rho_0 \end{cases}$$

$$F_{rep} = -\partial U_{ref}$$

$$F_{rep} \begin{cases} 0 & \rho(q) > \rho_0 \\ \eta \left\{ \dfrac{1}{\rho(q)} - \dfrac{1}{\rho_0} \right\} \dfrac{1}{\rho^2(q)} \nabla \rho & \rho(q) \leq \rho_0 \end{cases}$$

$$\nabla \rho = \frac{p(q) - b}{\| p(q) - b \|}$$ {Unit vector}

(i) Motion planning of a car



$$U = U_{att} + U_{rep}$$

$$F = -\partial U = -\partial U_{att} - \partial U_{rep}$$

$$\underline{F} = F_{att} + F_{rep}$$

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2 controls : v, ω

$v = v_0$ (nominal speed)

$$\theta_{des} = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$\omega = K(\theta_{des} - \theta)$$

↑ user chose