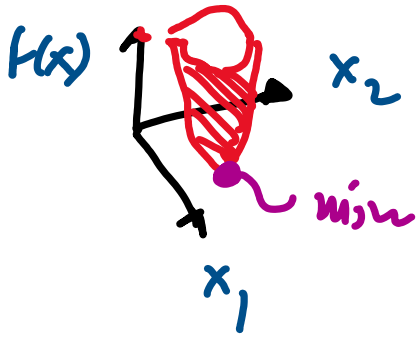
Optimization

Unconstrained optimization

$$\min_{x_1 x_2} f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

Ways to solve

① Graph $f(x)$ vs x



② Guess

$f(x) \geq 0$ (sum of squares)

$$\begin{cases} x_2 - x_1^2 = 0 \\ x_2 = x_1^2 = 1 \end{cases} \quad \& \quad \begin{cases} 1 - x_1 = 0 \\ \Rightarrow x_1 = 1 \end{cases}$$

$$\boxed{(x_1, x_2) = (1, 1)}$$

③ $\frac{df}{dx} = 0$ (extremum) — root-finding, fsolve

1×2

$\frac{d^2 f}{dx^2} > 0$ min < 0 max

We will use scipy.minimize

(I) Unconstrained optimization

$$\min_{x_1, x_2} f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

(II) Constrained optimization

$$\min_{x_1, x_2, x_3, x_4, x_5} F(x) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2$$

subject to:

$x_1 + x_2 + x_3 = 5$ ← Linear equality constraint

$x_3^2 + x_4 = 5$ ← non-linear equality constraint

Bound

$0.3 \leq x_1 \leq \infty$

$x_1 \geq 0.3$

$-\infty \leq x_3 \leq 5$

$x_3 \leq 5$

$-\infty \leq x_2, x_4, x_5 \leq \infty$

$x_4^2 + x_5^2 \leq 5$ ← non-linear inequality constraint.