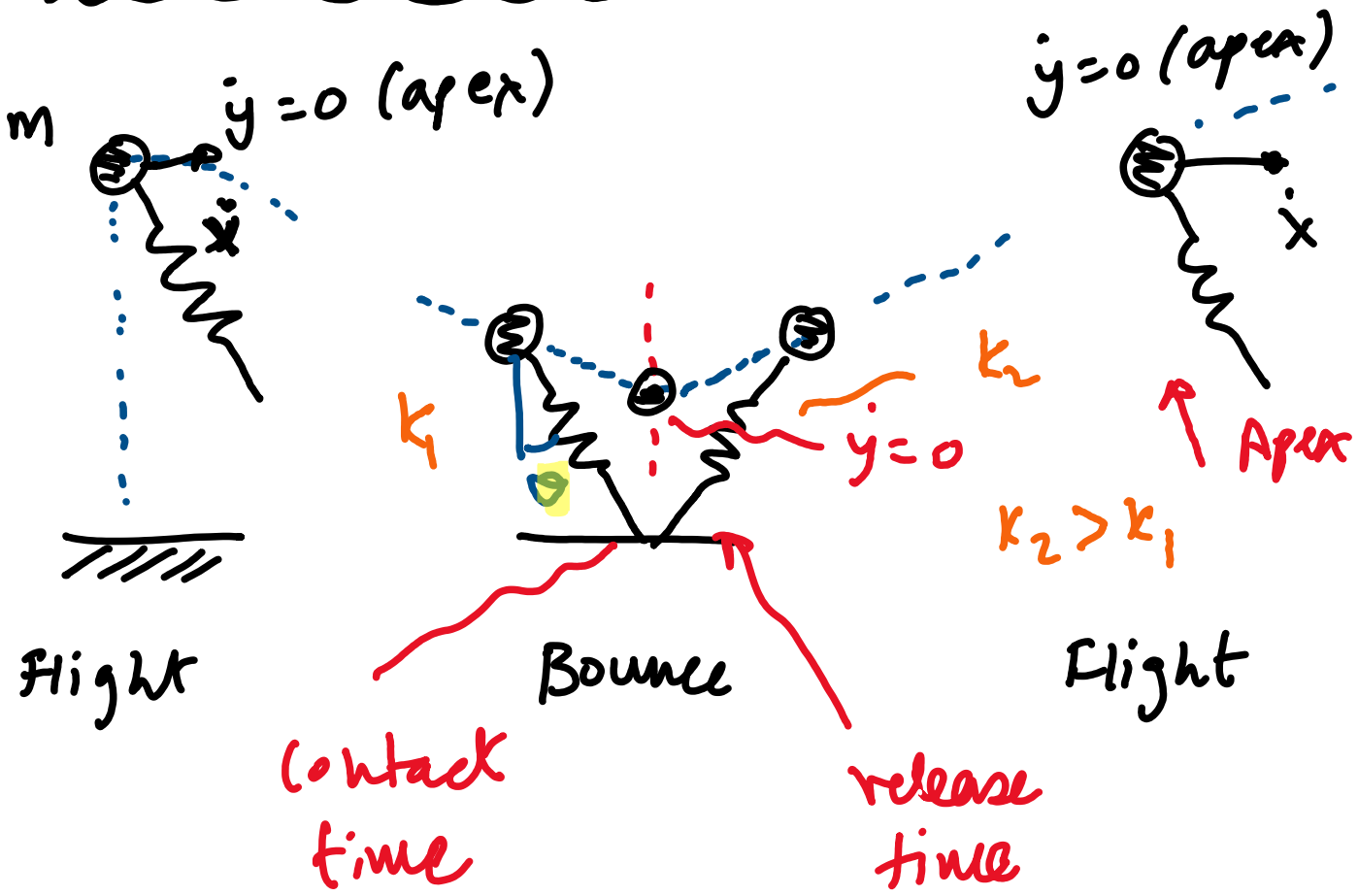
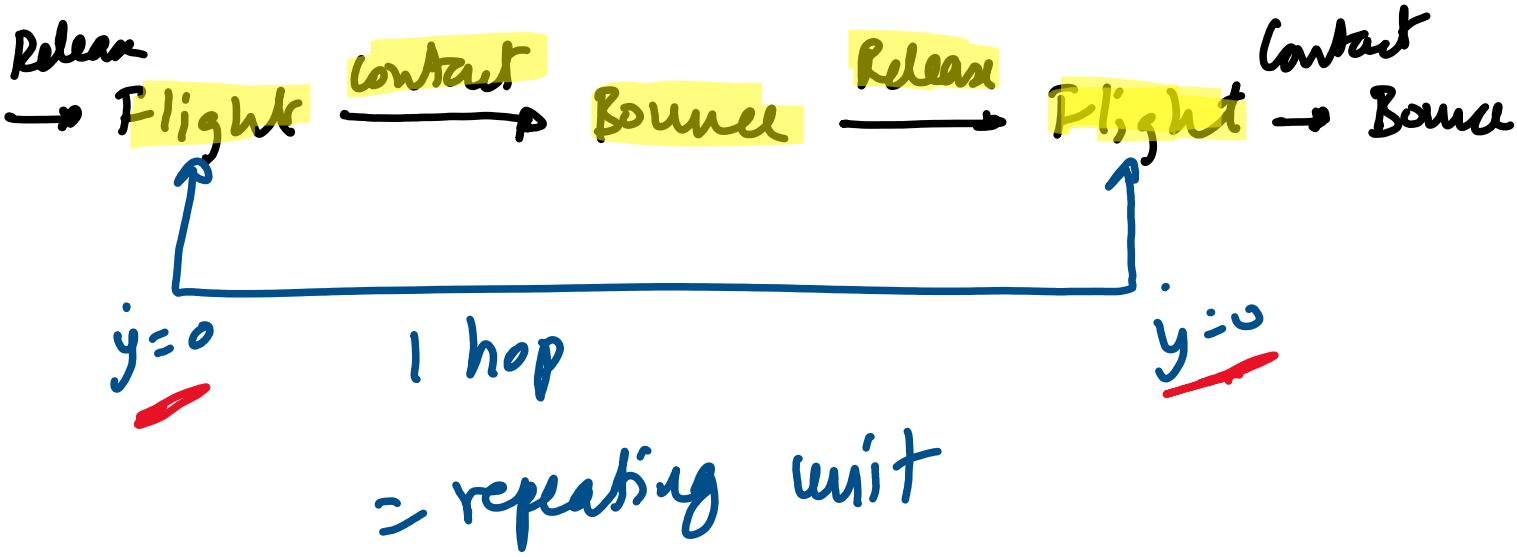
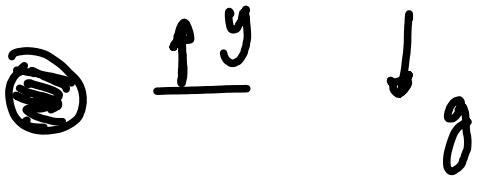
Pogo-stick hopper.





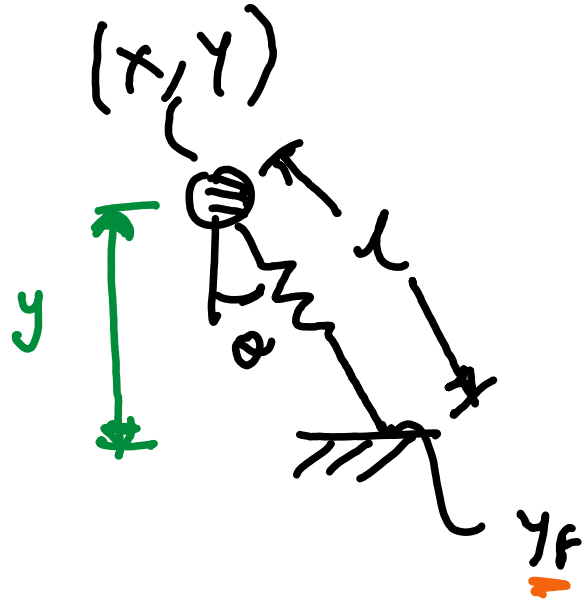
= repeating unit

① Flight



$$\ddot{x} = 0$$
$$\ddot{y} = -g$$

② Contact



$$y - l\cos\theta = y_f = 0$$

③ Bounce phase

(i) x, y
$\dot{x}, \dot{y}$

(ii) $\mathcal{L} = T - V$

$$= \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) - \left\{ mgy + \frac{1}{2} k (l - l_0)^2 \right\}$$

$$l = \sqrt{(x - x_f)^2 + (y - y_f)^2}$$

0

$$\mathcal{L} = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2) - mgy - 0.5k\left[\sqrt{x - x_f^2 + y^2} - l_0\right]^2$$

(iii) $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = Q_j$

$q_j = x, y$

EOMs:

$$m\ddot{x} = k(l_0 - l)\left(\frac{x - x_f}{l}\right)$$

$$m\ddot{y} = k(l_0 - l)\left(\frac{y}{l}\right) - mg$$

where $l = \sqrt{(x - x_f)^2 + y^2}$

(4) Release: $F_s = k(l_0 - l) = 0$

$\Rightarrow l_0 = l = \sqrt{(x - x_f)^2 + y^2}$

$$l_0 - \sqrt{(x - x_f)^2 + y^2} = 0$$

(5) Apex: $\dot{y} = 0$

Simulate in Python

① Apex → Flight → (Contact) → Bounce → (Release) → Flight → Apex →

Repeating unit
(1 hop / 1 bounce)

② solve_ivp('EOM', z_0, $[t_0\ t_f]$, parm, events = '...')