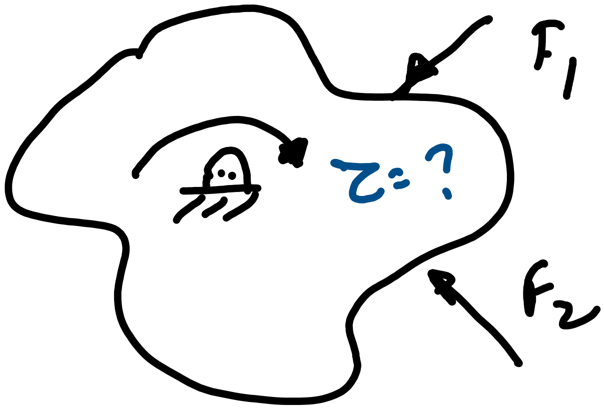
② Computing Static Forces



Given $F_1, F_2, \ldots F_n$, compute the motor torque needed to keep the body from rotating.

Theory

Virtual work

$$\text{Work} = \sum \underset{\sim}{F}^T \delta r$$

virtual displacement

$$\underbrace{1\times2 \quad 2\times1}_{1\times1}$$

$$\text{Work} = \tau^T \delta\theta$$

virtual rotation

$1\times1 \quad 1\times1$

$$\tau^T \delta\theta = F^T \delta r$$

Divide by $\delta\theta$

$$\tau^T = F^T \frac{\delta r}{\delta\theta}$$

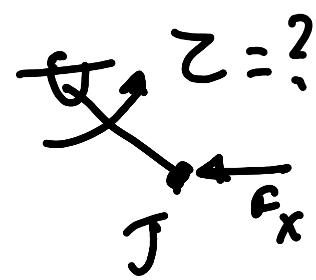
$$\tau^T = F^T J$$

Take transpose of both sides

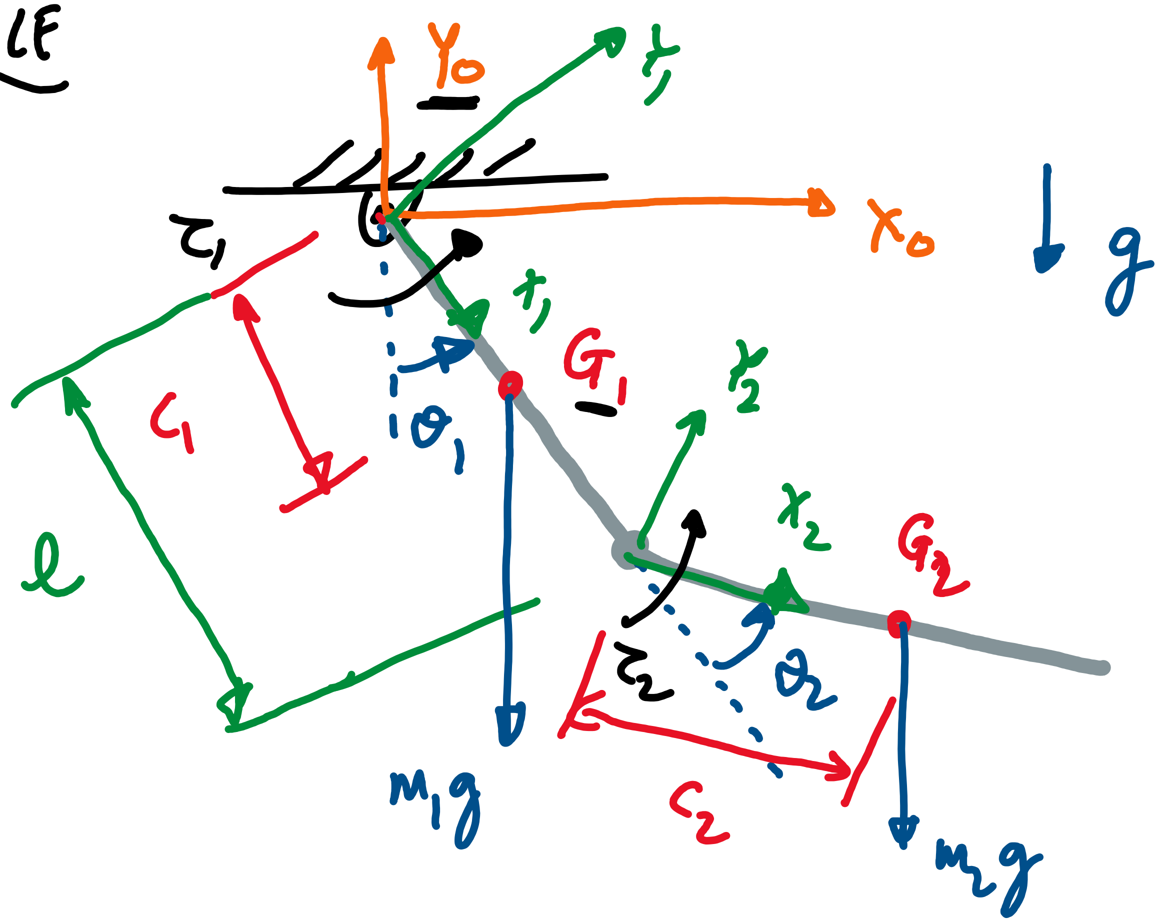
$$\tau = (F^T J)^T$$

$$= J^T F \qquad \{(AB)^T = B^T A^T\}$$

$$\boxed{\tau = J^T F}$$



EXAMPLE



Compute motor torque τ_1 and τ_2 such that the double pendulum is in static equilibrium $\theta_1 \neq \theta_2 \neq 0$

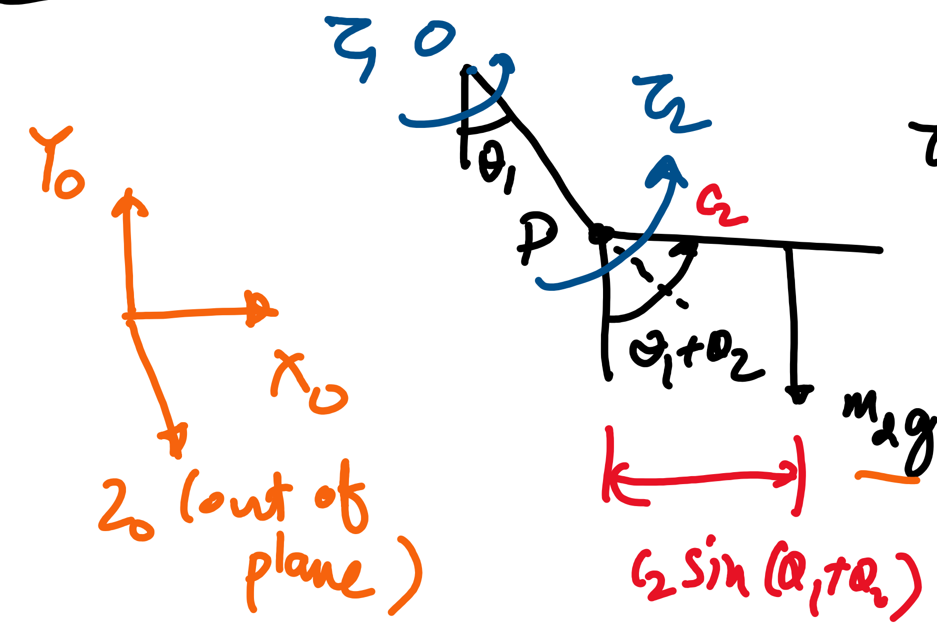
$$\tau = \sum J^T F$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = J_{G_1}^T \begin{bmatrix} 0 \\ -m_1 g \end{bmatrix} + J_{G_2}^T \begin{bmatrix} 0 \\ -m_2 g \end{bmatrix}$$

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \underbrace{\begin{bmatrix} c_1\cos\theta_1 & c_1\sin\theta_1 \\ 0 & 0\end{bmatrix}}_{J_{G_1}^T}\begin{bmatrix}0\\ -m_1 g\end{bmatrix} + \cdots$$

$$\underbrace{\begin{bmatrix} c_2\cos(\theta_1+\theta_2) + l\cos\theta_1 & c_2\sin(\theta_1+\theta_2)+l\sin\theta_1 \\ c_2\cos(\theta_1+\theta_2) & c_2\sin(\theta_1+\theta_2)\end{bmatrix}}_{J_{G_2}^T}\begin{bmatrix}0\\ -m_2 g\end{bmatrix}$$

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix} -m_1 g\, c_1\sin\theta_1 - m_2 g\sin(\theta_1+\theta_2) - m_2 g\, l\sin\theta_1 \\ -m_2 g\, c_2\sin(\theta_1+\theta_2)\end{bmatrix}$$



$\tau_2 = -m_2 g\; c_2\sin(\theta_1+\theta_2)$

CW −ive

CCW +ive