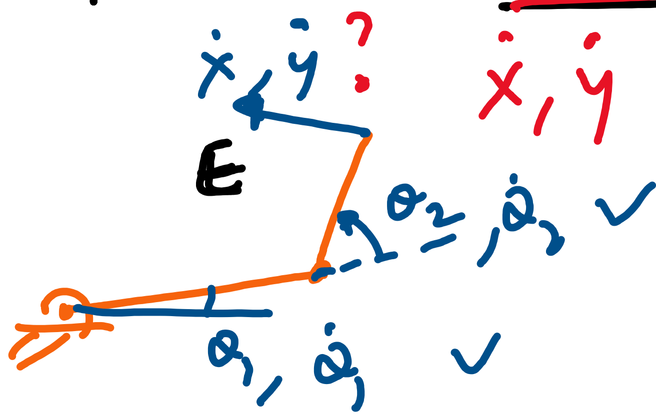
Applications of J (3)

Application 1: compute the cartesian velocity



Theory

$$\underline{e}^0 = F(q) \quad \text{Forward kinematics}$$

$$\underline{q} = [\underline{\theta_1}, \underline{\theta_2}]$$

$$J = \frac{\partial F}{\partial q} \quad (\text{definition})$$

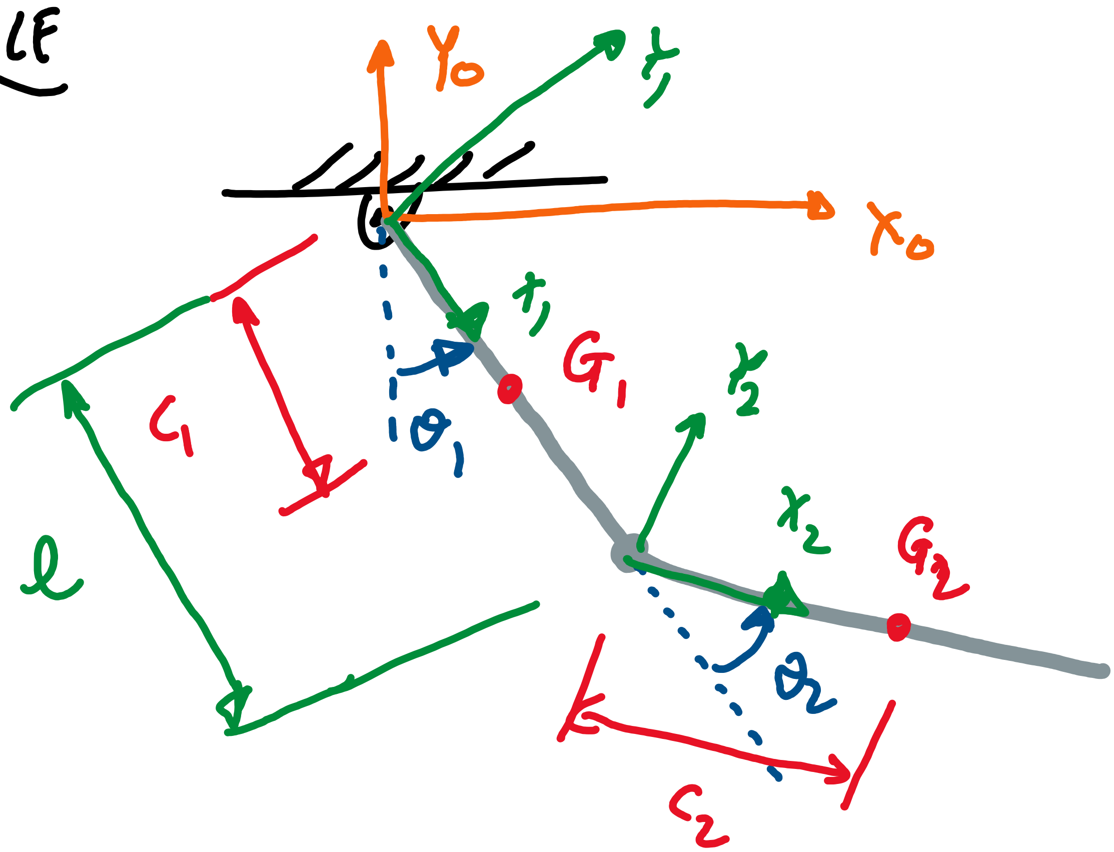
$$J \partial q = \partial F \qquad \frac{de^0}{dt}$$

$$J \frac{dq}{dt} = \frac{dF}{dt} \qquad t \to \text{time}$$

$$J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \dot{x}_e^0 \\ \dot{y}_e^0 \end{bmatrix}$$

$$\boxed{J \dot{q} = v_e^0}$$

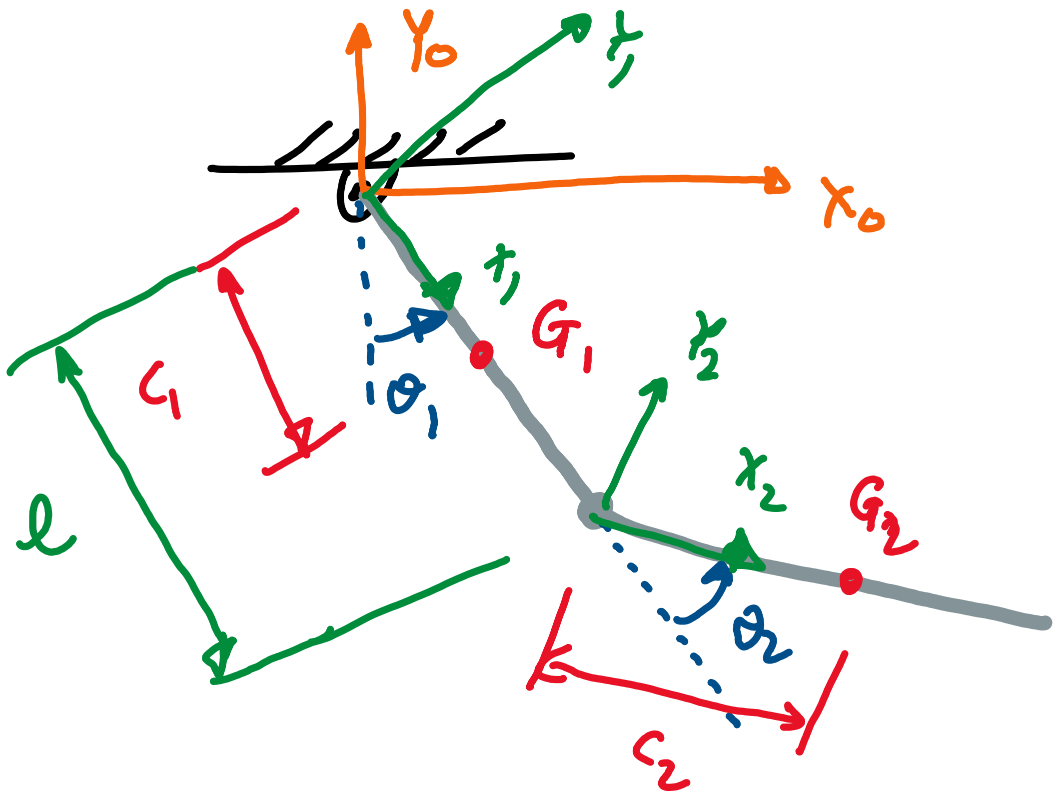
EXAMPLE



Compute the cartesian velocity of the center of mass, G_1 and G_2

Given; c_1, c_2, l, θ_1, θ_2, $\dot{\theta}_1$, $\dot{\theta}_2$

$\therefore$ $V^0_{G_1} = ?$ $\quad V^0_{G_2} = ?$

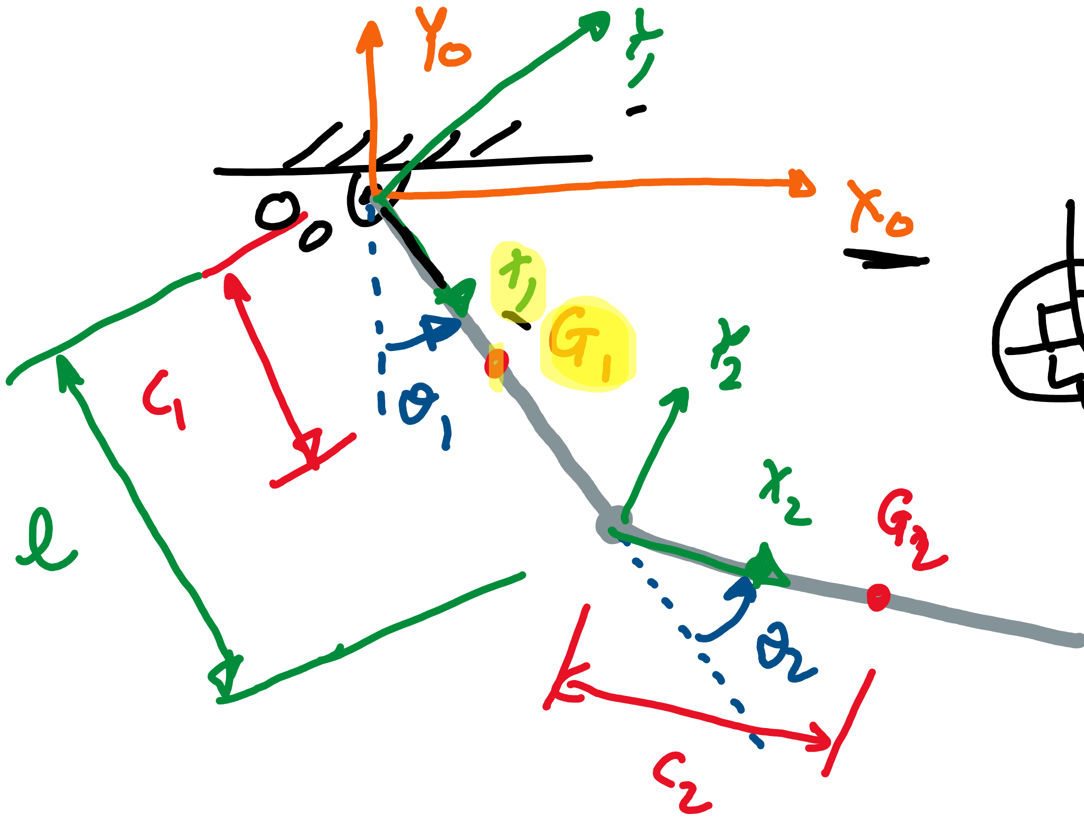


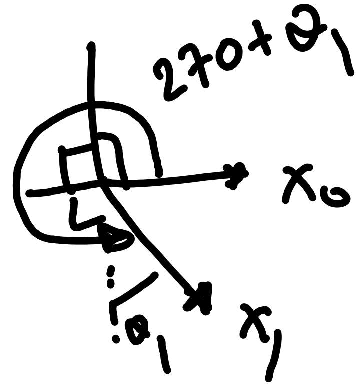
$$v^0_{G_1} = J_{G_1}\,\dot{q} = J_{G_1}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}$$

$$v^0_{G_2} = J_{G_2}\,\dot{q} = J_{G_2}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}$$

$$J_{G_1} = \frac{\partial r_{G_1}}{\partial q} \quad ; \quad J_{G_2} = \frac{\partial r_{G_2}}{\partial q}$$

$$J_{G_1} = \frac{\partial g^0_1}{\partial q} \quad ; \quad J_{G_2} = \frac{\partial g^0_2}{\partial q}$$





$$g_1^0 = H_1^0 \, g_1^1$$

$$= \begin{bmatrix} \cos(270+\theta_1) & -\sin(270+\theta_1) & 0 \\ \sin(270+\theta_1) & \cos(270+\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \left.\begin{bmatrix} c_1 \sin\theta_1 \\ -c_1 \cos\theta_1 \\ 1 \end{bmatrix}\right\} g_1^0$$

$$J_{G_1} = \frac{\partial g_1^0}{\partial q} \qquad g_1^0 = [\overbrace{\underline{c_1 \sin\theta_1}}^{x^0_{G_1}} \quad \overbrace{\underline{-c_1 \cos\theta_1}}^{y^0_{G_1}}]$$

$$q = [\theta_1 \quad \theta_2]$$

$$= \begin{bmatrix} \frac{\partial x^0_{G_1}}{\partial \theta_1} & \frac{\partial x^0_{G_1}}{\partial \theta_2} \\ \frac{\partial y^0_{G_1}}{\partial \theta_1} & \frac{\partial y^0_{G_1}}{\partial \theta_2} \end{bmatrix}$$

$$J_{G_1} = \begin{bmatrix} +c_1 \cos\theta_1 & 0 \\ +c_1 \sin\theta_1 & 0 \end{bmatrix} \quad 2\times 2$$

$$\boxed{V_{G_1} = J_{G_1} \dot{q} = \begin{bmatrix} c_1 \cos\theta_1 \dot{\theta}_1 \\ c_1 \sin\theta_1 \dot{\theta}_1 \end{bmatrix}} \quad 2\times 1$$

$$v_{G_2} = \frac{\partial g_2^0}{\partial q} \dot{q}$$

$$G_2^0 = H_1^0 H_2^1 G_2^2$$

Check this calculation at home

$$v_{G_2} = \underbrace{\begin{bmatrix} c_2 \cos(\theta_1+\theta_2) + l\cos\theta_1 & c_2\cos(\theta_1+\theta_2) \\ c_2 \sin(\theta_1+\theta_2) + l\sin\theta_1 & c_2 \sin(\theta_1+\theta_2) \end{bmatrix}}_{2\times 2 \quad J_{G_2}} \underbrace{\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}}_{2\times 1 \quad \dot{q}}$$