Jacobian

$$f(q) = \underbrace{\begin{bmatrix} f_1(q) & f_2(q) & f_3(q) \dots & f_m(q) \end{bmatrix}}_{\text{vector of functions}}$$

$$q = \begin{bmatrix} x_1, x_2, x_3, \dots & x_n \end{bmatrix}$$

$$J = \frac{\partial f}{\partial q}_{m \times n} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

$m \times n$

$$F = [x_2^2 + y^2,\ 2x + 3y + 5]$$

Compute $J = \frac{\partial F}{\partial q}$

$$F = [F_1, F_2] = [\underbrace{x^2 + y^2}_{f_1},\ \underbrace{2x + 3y + 5}_{f_2}] \quad 1 \times 2$$

$$q = [x, y] \quad 1 \times 2$$

$$J = \frac{\partial F}{\partial q} = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} \quad 2 \times 2$$

$$= \begin{bmatrix} 2x & 2y \\ 2 & 3 \end{bmatrix}$$

$$J\Big|_{\substack{x=1 \\ y=2}} = \begin{bmatrix} 2(1) & 2(2) \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix}$$