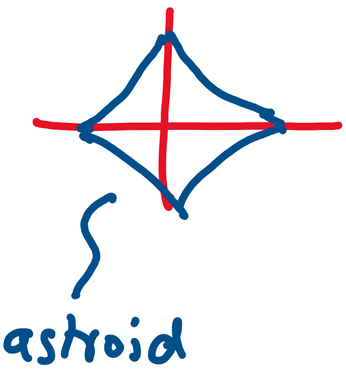
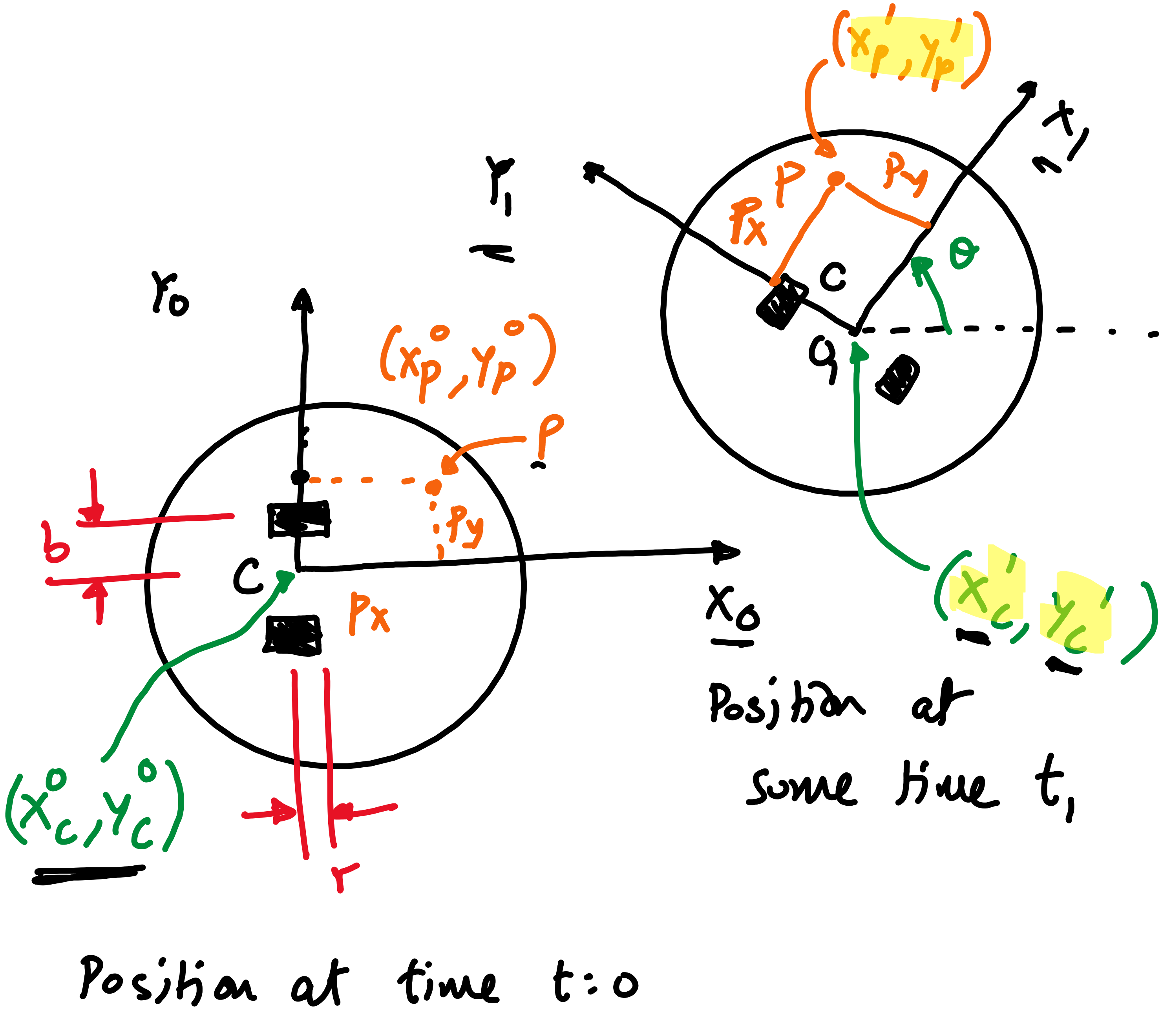
Inverse kinematics of a differential drive car

Compute $v(t)$ and $\omega(t)$ [controls]

such that $x_c(t) = x_{ref}(t)$

$y_c(t) = y_{ref}(t)$





Goal: Get the point P to track $(x_{ref}(t), y_{ref}(t))$

$$c^0 = R_1^0 c^1 + d_1^0$$

$$p^0 = R_1^0 p^1 + d_1^0$$

$$p^0 - c^0 = R_1^0 (p^1 - c^1)$$

$$\begin{bmatrix} x_p^0 - x_c^0 \\ y_p^0 - y_c^0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_p^1 - x_c^1 \\ y_p^1 - y_c^1 \end{bmatrix}$$

$$\begin{bmatrix} x_p^0 - x_c^0 \\ y_p^0 - y_c^0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

Given p^0, compute c^0

Needed for IK code

$$\begin{bmatrix} x_c^0 \\ y_c^0 \end{bmatrix} = \begin{bmatrix} x_p^0 \\ y_p^0 \end{bmatrix} - \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

Given c^0, compute p^0

$$\begin{bmatrix} x_p^0 \\ y_p^0 \end{bmatrix} = \begin{bmatrix} x_c^0 \\ y_c^0 \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

Differential with respect to time

$$\begin{bmatrix} \dot{x}_p^o \\ \dot{y}_p^o \end{bmatrix} = \begin{bmatrix} \dot{x}_c^o \\ \dot{y}_c^o \end{bmatrix} + \begin{bmatrix} -\sin\theta\,\dot{\theta} & -\cos\theta\,\dot{\theta} \\ \cos\theta\,\dot{\theta} & -\sin\theta\,\dot{\theta} \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_p \\ \dot{y}_p \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \end{bmatrix} + \begin{bmatrix} (-\sin\theta)\omega\, p_x - (\cos\theta)\omega\, p_y \\ (\cos\theta)(\omega)\, p_x - (\sin\theta)\omega\, p_y \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \dot{x}_p^o \\ \dot{y}_p^o \end{bmatrix}}_{\text{Track ref}} = \underbrace{\underbrace{\begin{bmatrix} \cos\theta & (-p_x\sin\theta - p_y\cos\theta) \\ \sin\theta & (p_x\cos\theta - p_y\sin\theta) \end{bmatrix}}_{p_x,\, p_y,\, \theta}}_{2\times 2} \underbrace{\begin{bmatrix} v \\ \omega \end{bmatrix}}_{\text{controls}}$$

Here is how to do IK

$$\left.\begin{aligned} \dot{x}_p^o &= K_{px}\,(x_{ref} - x_p^o) \\ \dot{y}_p^o &= K_{py}\,(y_{ref} - y_p^o) \end{aligned}\right\}\ \text{Feedback}$$

measured (sensor)

$$\underbrace{\begin{bmatrix} k_{px}(x_{ref}-x_p^o) \\ k_{py}(y_{ref}-y_p^o)\end{bmatrix}}_{\substack{b \\ \text{known}}} = \underbrace{\begin{bmatrix}\cos\theta & (-p_x\sin\theta - p_y\cos\theta) \\ \sin\theta & (p_x\cos\theta - p_y\sin\theta)\end{bmatrix}}_{\substack{A \\ \text{known}}} \underbrace{\begin{bmatrix} v \\ \omega\end{bmatrix}}_{\substack{X \\ \text{unknown}}}$$

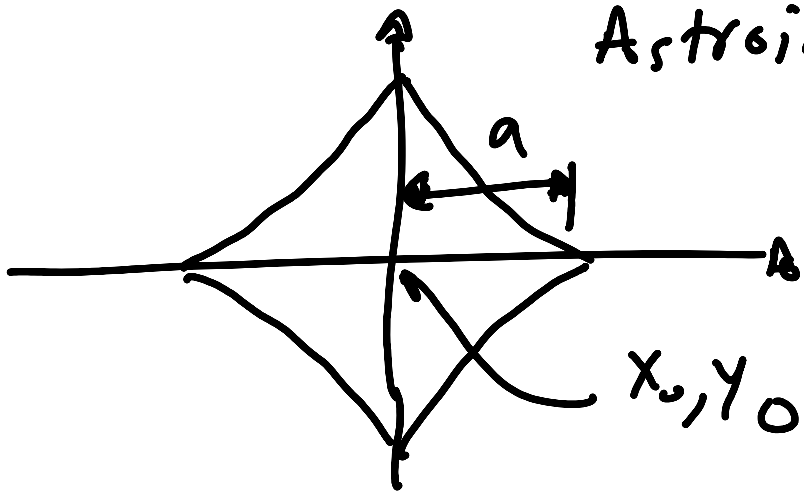
$$b = AX$$

$$\Rightarrow X = \begin{bmatrix} v \\ \omega \end{bmatrix} = A^{-1} b$$

$$A^{-1} = \begin{bmatrix} \cos\theta - \left(\frac{p_y}{p_x}\right)\sin\theta & \sin\theta + \left(\frac{p_y}{p_x}\right)\cos\theta \\ -\frac{1}{p_x}\sin\theta & \frac{1}{p_x}\cos\theta \end{bmatrix}$$

$p_x \neq 0$

Astroid



$$x = x_0 + a\cos^3(2\pi t)$$

$$y = y_0 + a\sin^3(2\pi t)$$