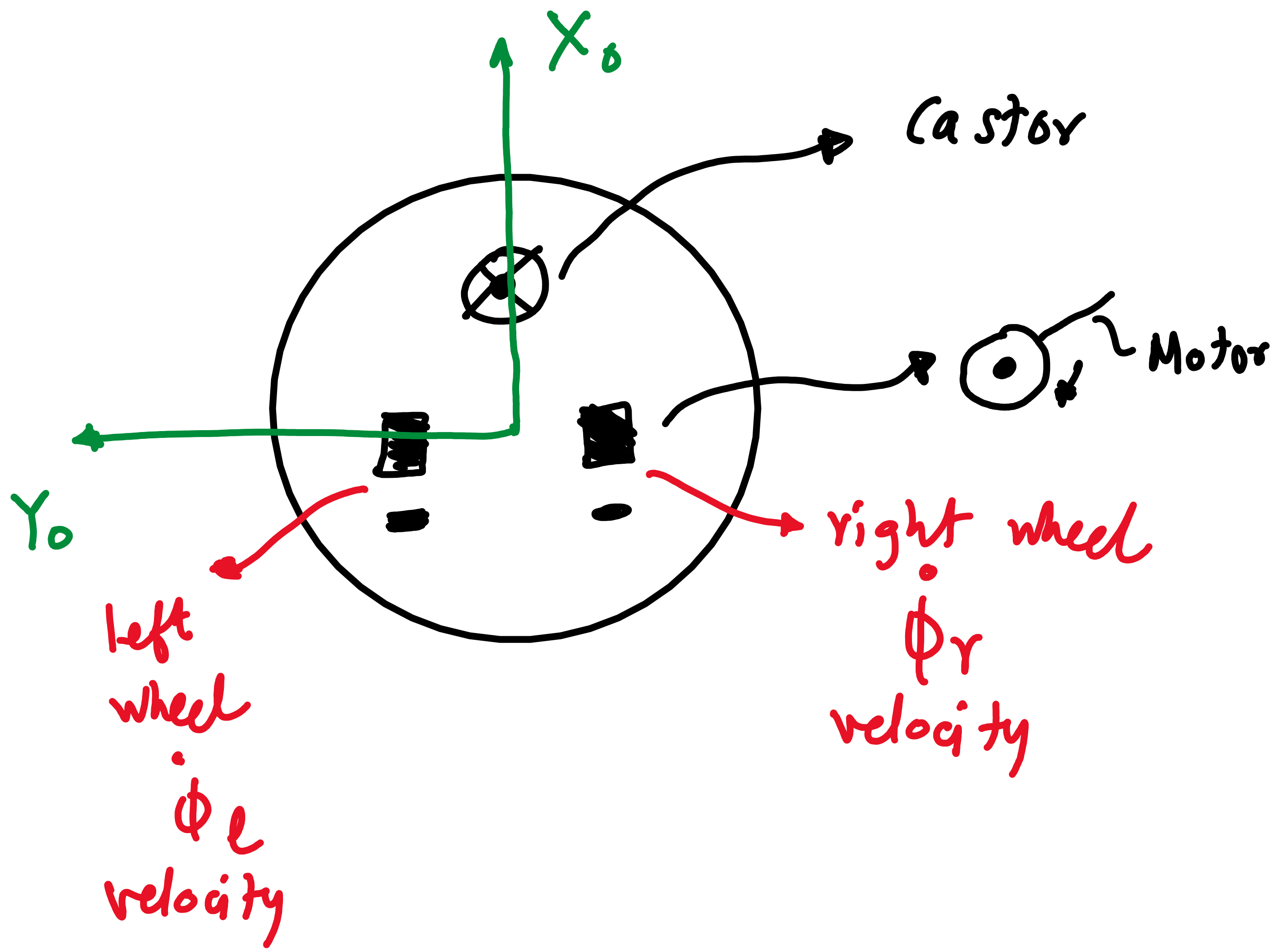
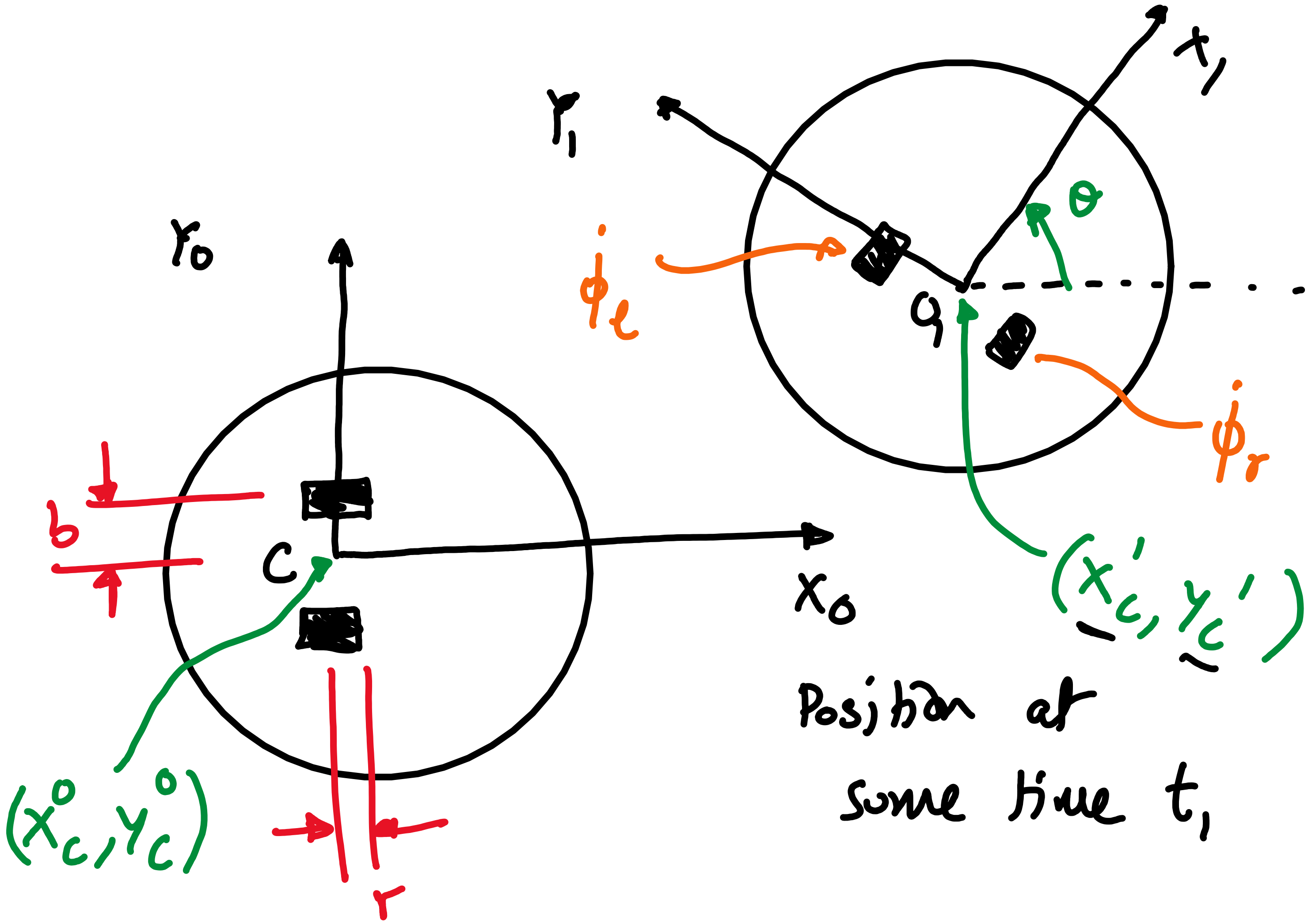
Differential Drive car



1. Move straight $\dot{\phi}_r = \dot{\phi}_l$
2. Turn right : $\dot{\phi}_l > \dot{\phi}_r$
3. Turn left : $\dot{\phi}_r > \dot{\phi}_l$



Position at some time t_1

Position at time $t=0$

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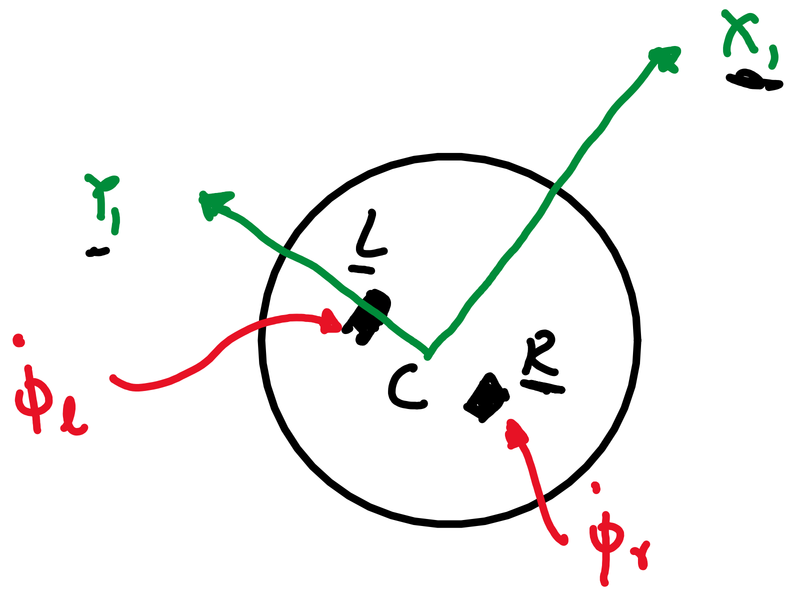
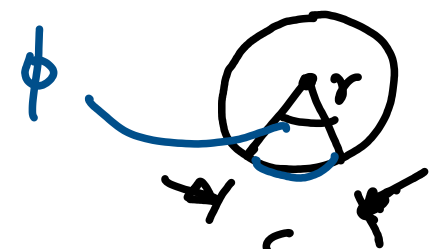
compute $\underline{\dot{x}_c^0, \dot{y}_c^0, \dot{\theta}} = ?$

x_c, y_c, θ can be found by integration.

Derivation

① Compute $\dot{x}_c^{\,0}, \dot{y}_c^{\,0}$

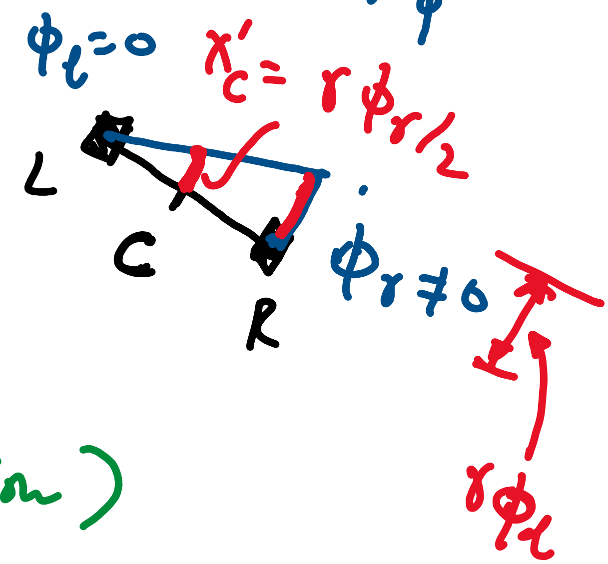




$S = r\phi$

$\dot{S} = r\dot{\phi}$

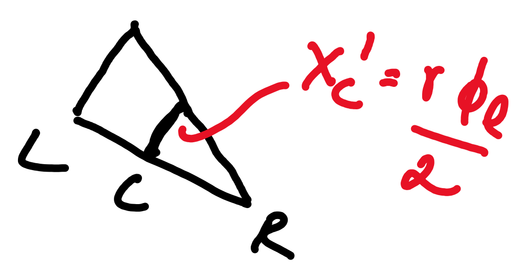
$\dot{x}_c^{\,1}, \dot{y}_c^{\,1} = ?$



Assume $\dot{\phi}_l = 0 \quad \dot{\phi}_r \neq 0$

$$\begin{cases} \dot{x}_c^{\,1} = \dfrac{r\dot{\phi}_r}{2} & \text{(Triangle assumption)} \\ \dot{y}_c^{\,1} = 0 \end{cases}$$

Assume $\dot{\phi}_r = 0 \quad \dot{\phi}_l \neq 0$



$$\begin{cases} \dot{x}_c^{\,1} = \dfrac{r\dot{\phi}_l}{2} \qquad \dot{y}_c^{\,1} = 0 \end{cases}$$

$$\begin{cases} \dot{x}_c{}' = \dfrac{r\dot{\phi}_l}{2} & \dot{y}_c{}' = 0 \end{cases}$$

If $\dot{\phi}_l \neq 0 \quad \dot{\phi}_r \neq 0$

$$\dot{x}_c^1 = \frac{r}{2}(\dot{\phi}_r + \dot{\phi}_l)$$

$$\dot{y}_c^1 = 0$$

$$c^0 = R_1^0 c^1$$

$$\begin{bmatrix} x_c^0 \\ y_c^0 \end{bmatrix} = R_1^0 \begin{bmatrix} x_c^1 \\ y_c^1 \end{bmatrix}$$

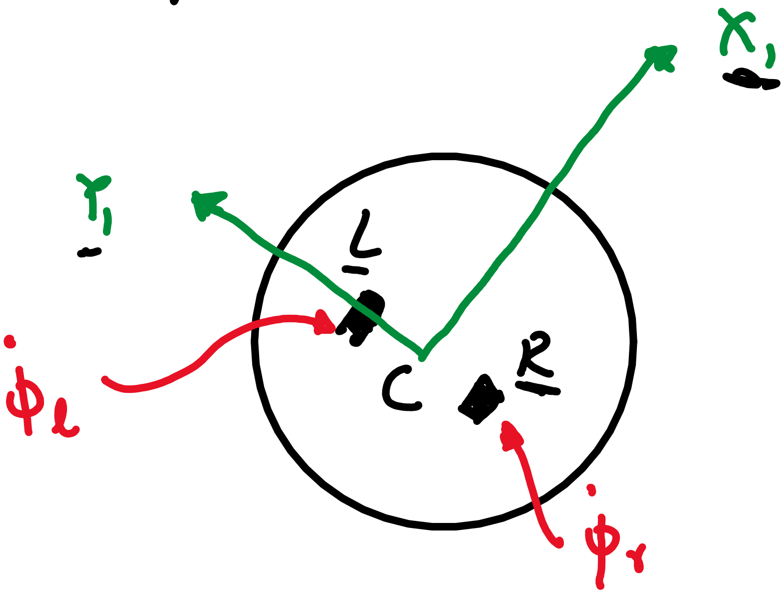
No rotation here

$$\begin{bmatrix} \dot{x}_c^0 \\ \dot{y}_c^0 \end{bmatrix} = R_1^0 \begin{bmatrix} \dot{x}_c^1 \\ \dot{y}_c^1 \end{bmatrix} + \dot{R}_1^0 \begin{bmatrix} x_c^1 \\ y_c^1 \end{bmatrix}$$

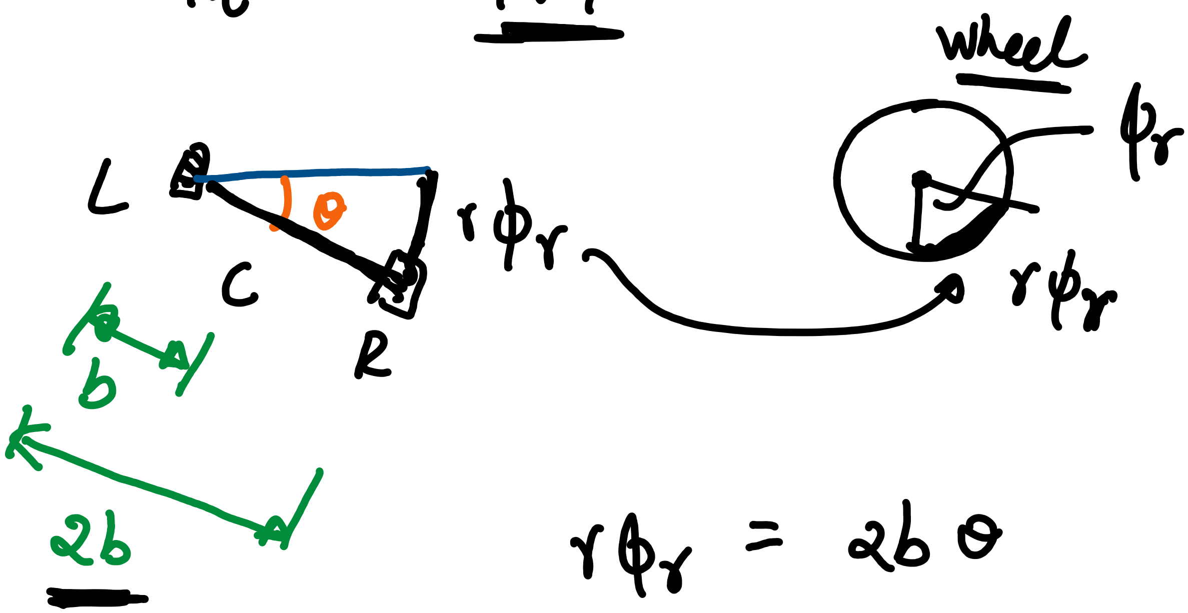
$$\begin{bmatrix} \dot{x}_c^0 \\ \dot{y}_c^0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \frac{r}{2}(\dot{\phi}_l + \dot{\phi}_r) \\ 0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \dot{x}_c^0 \\ \dot{y}_c^0 \end{bmatrix} = \begin{bmatrix} \frac{r}{2}(\dot{\phi}_l + \dot{\phi}_r)\cos\theta \\ \frac{r}{2}(\dot{\phi}_l + \dot{\phi}_r)\sin\theta \end{bmatrix}} \quad \text{I}$$

② Compute $\dot{\theta}$



Assume $\dot{\phi}_l = 0$ & $\underline{\dot{\phi}_r \neq 0}$

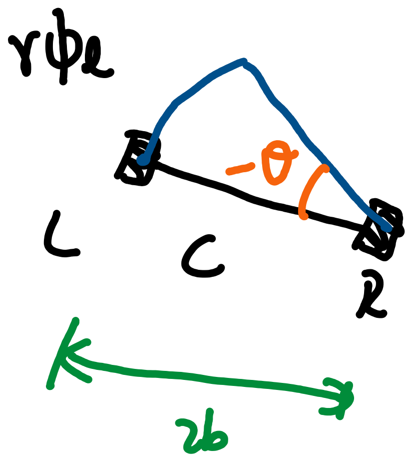


$$r\phi_r = 2b\,\theta$$

$$\theta = \frac{r}{2b}\,\phi_r$$

$$\underline{\dot{\theta}} = \underline{\frac{r}{2b}\,\dot{\phi}_r}$$

Assume $\dot{\phi}_r = 0$ & $\dot{\phi}_l \neq 0$



$$r\phi_l = 2b(-\theta)$$

$$\theta = \frac{-r}{2b}\phi_l$$

$$\dot{\theta} = \frac{-r}{2b}\dot{\phi}_l$$

If $\dot{\phi}_r \neq 0$ & $\dot{\phi}_l \neq 0$

$$\boxed{\dot{\theta} = \frac{r}{2b}(\dot{\phi}_r - \dot{\phi}_l)} \quad \text{— II}$$

$$v = 0.5r\,(\dot{\phi}_r + \dot{\phi}_l)$$

$$\omega = \frac{0.5r}{b}\,(\dot{\phi}_r - \dot{\phi}_l)$$

$$\dot{x}_c^{\circ} = v\cos\theta$$

$$\dot{y}_c^{\circ} = v\sin\theta$$

$$\dot{\theta} = \omega$$

compute $\dot{x}_c, \dot{y}_c, \dot{\theta}$ given the history of $v(t)$, $\omega(t)$

Eulers Method

$$\boxed{\begin{aligned} x_c(t+h) &= x_c(t) + h\, v(t)\cos(\theta(t)) \\ y_c(t+h) &= y_c(t) + h\, v(t)\sin(\theta(t)) \\ \theta(t+h) &= \theta(t) + h\,\omega(t) \end{aligned}}$$

Given

h - step size e.g. $h \approx 0.001$

$x_c(\underline{t=0})$, $y_c(\underline{t=0})$, $\underline{\theta(t=0)}$ are known

Compute $x(0), x(h), x(2h) \ldots$

$y(0), y(h), y(2h) \ldots$

$\theta(0), \theta(h), \theta(2h) \ldots$

$\theta(0), \quad \theta(h), \quad \theta(2h) \ldots$