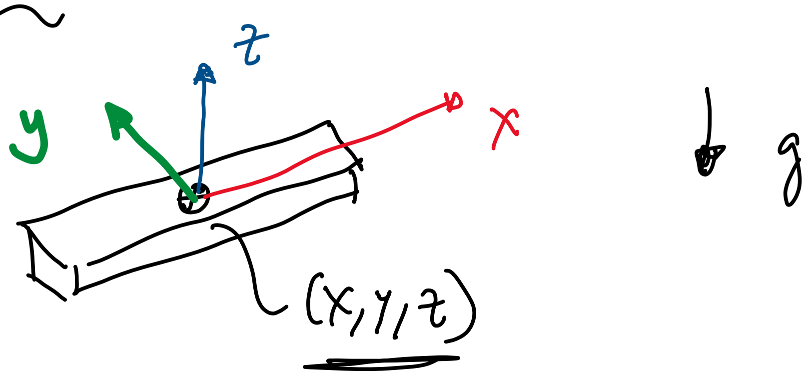
3D Dynamics



Given an initial position/orientation and linear/angular velocity, describe the motion of the object.

- Equations ✓
- Simulation (ode) ✓
- Animation ✓

(A) Euler-Lagrange Equations

(1) Position / Orientation

x, y, z Euler angles 3-2-1

$z - y - x$

$\psi - \theta - \phi$

Linear / Angular

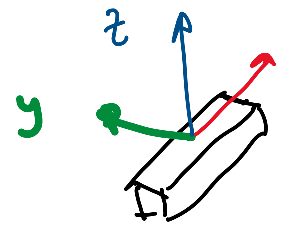
$\dot{x}, \dot{y}, \dot{z}$ ω world frame $\omega_x, \omega_y, \omega_z$

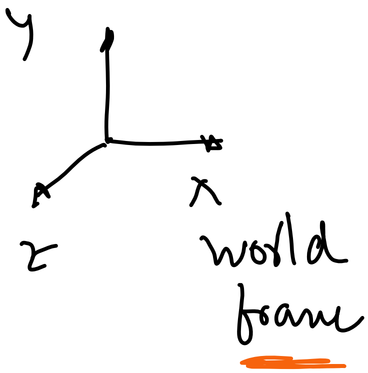
ω_b body frame $\omega_{bx}, \omega_{by}, \omega_{bz}$

x, y, z ω_b ... $\omega_{bx}, \omega_{by}, \omega_{bz}$

$$\omega_b, \ \omega = f(\underline{\dot{\phi}, \dot{\theta}, \dot{\psi}}, \underline{\phi, \theta, \psi})$$

$$T = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{1}{2}\left(\underbrace{\omega^T}_{1\times 3} (\underbrace{I}_{3\times 3} \underbrace{\omega}_{3\times 1})\right)$$





$$\begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

World frame

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$ World frame

will change with time

Textbooks / Manuals.

These give inertia in the body frame,

$$\begin{bmatrix} I_{bx} & 0 & 0 \\ 0 & I_{by} & 0 \\ 0 & 0 & I_{bz} \end{bmatrix}$$ does not change with time

It can be shown that

$$\underbrace{{}^{x}\omega^T I \, {}^{f}\omega}_{\text{complex}} = \underbrace{\omega_b^T I_b \omega_b}_{\text{simple}}$$

$$\omega_b = \begin{bmatrix} \omega_{bx} \\ \omega_{by} \\ \omega_{bz} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{bmatrix}$$

$$T = \frac{1}{2} m(\dot x^2 + \dot y^2 + \dot z^2) + \frac{1}{2} \omega_b^T I_b \omega_b$$

$$= \frac{1}{2} m(\dot x^2 + \dot y^2 + \dot z^2) + \frac{1}{2}(I_{bx}\omega_{bx}^2 + I_{by}\omega_{by}^2 + I_{bz}\omega_{bz}^2)$$

$f(\theta, \phi, \psi, \dot\theta, \dot\phi, \dot\psi)$

$$V = mgz$$

$$\mathcal{L} = T - V$$

(3) Equations of motion

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot q_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = Q_j$$

$q_j = x, y, z, \phi, \theta, \psi$; $Q_j = 0$ (no external forces)

(4) $AX = b$ $\quad X = [\ddot x, \ddot y, \ddot z, \ddot\phi, \ddot\theta, \ddot\psi]$