3-2-1

$$\mathbf{R} = \mathbf{R}_z(\psi)\mathbf{R}_y(\theta)\mathbf{R}_x(\phi)$$

$$= \begin{bmatrix} \cos(\psi)\cos(\theta) & \cos(\psi)\sin(\phi)\sin(\theta) - \cos(\phi)\sin(\psi) & \sin(\phi)\sin(\psi) + \cos(\phi)\cos(\psi)\sin(\theta) \\ \cos(\theta)\sin(\psi) & \cos(\phi)\cos(\psi) + \sin(\phi)\sin(\psi)\sin(\theta) & \cos(\phi)\sin(\psi)\sin(\theta) - \cos(\psi)\sin(\phi) \\ -\sin(\theta) & \cos(\theta)\sin(\phi) & \cos(\phi)\cos(\theta) \end{bmatrix}$$

3D rotations ✓

$$= R_n^0$$

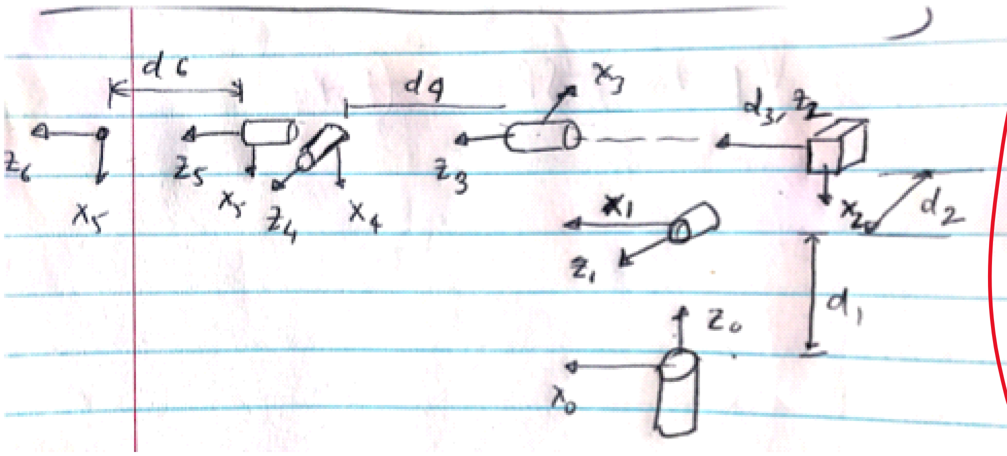
$$= \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

3x3 numeric

$R_{31} = -\sin\theta \Rightarrow \theta_{ref} = \sin^{-1}(R_{31})$ ✓

$R_{32} = \cos\theta\sin\phi \Rightarrow \phi_{ref} = \sin^{-1}\left(\dfrac{R_{32}}{\cos\theta_{ref}}\right)$

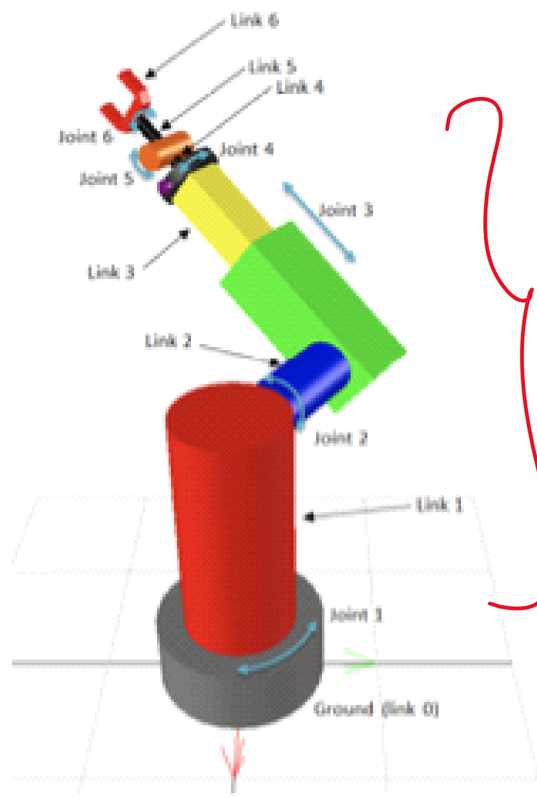
$R_{11} = \cos\psi\cos\theta \Rightarrow \psi_{ref} = \cos^{-1}\left(\dfrac{R_{11}}{\cos\theta_{ref}}\right)$



Joint i	α_i	a_i	d_i	θ_i
1	$-Pi/2$	0	d_1	θ_1
2	$Pi/2$	0	d_2	θ_2
3	0	0	d_3	$-Pi/2$
4	$-Pi/2$	0	d_4	θ_4
5	$Pi/2$	0	0	θ_5
6	0	0	d_6	θ_6

?

Stanford Manipulator



1. Compute position/orientation of end-effector
2. Animate the manipulator
3. Given a position/orientation of the end-effector, compute the inverse kinematics

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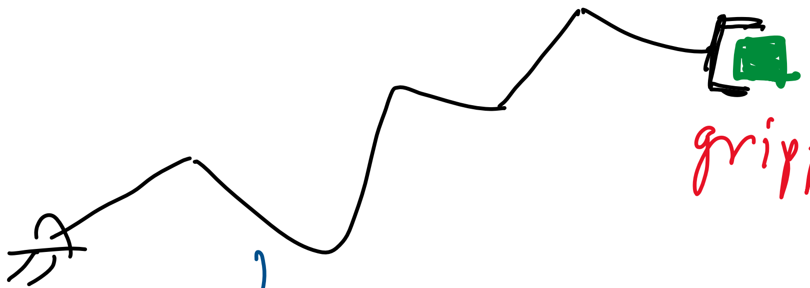
1. $$H_6^0 = H_1^0 \, H_2^1 \, H_3^2 \, H_4^3 \, H_5^4 \, H_6^5 = \begin{bmatrix} \underbrace{R_6^0}_{\text{orientation}} & \underbrace{d_6^0}_{\text{pos'}} \\ 0 & 1 \end{bmatrix}$$

Inverse kinematics of 3D manipulators



Needs atleast
6 joints or
6 degrees of freedom

$$\begin{bmatrix} X_{ref}, & \phi_{ref} \\ Y_{ref}, & \theta_{ref} \\ Z_{ref}, & \psi_{ref} \end{bmatrix}$$

6 numbers.

1. Assign co-ordinate frames
2. DH Table
3. $H_n^0 = H_1^0 H_2^1 \cdots H_n^{n-1} = \begin{bmatrix} R_n^0 & d_n^0 \\ 0 & 1 \end{bmatrix}$

$$d_n^0 = \begin{bmatrix} X_{ref} \\ Y_{ref} \\ Z_{ref} \end{bmatrix}$$

$$R_n^0 = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

Relate to $\phi_{ref}, \theta_{ref}, \psi_{ref}$