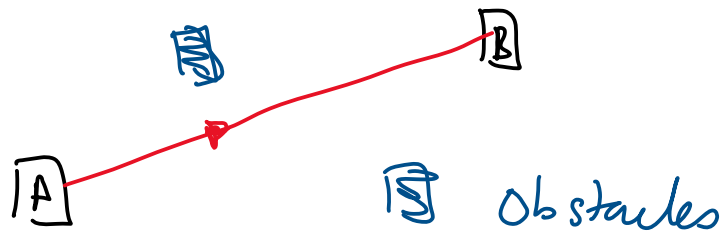


# Trajectory Optimization



## Optimization

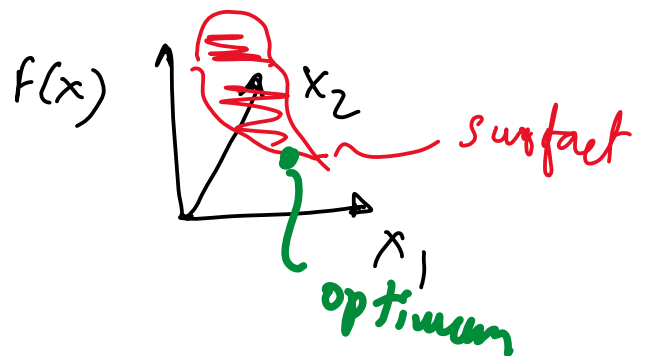
$$\min_{x_1, x_2} f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

ways to solve

(I) Graph

(II) Guess

maybe  $f(x) = 0$   
 $x_2 = x_1 = 1$



(III) Calculus

$$\frac{df}{dx} = 0 \quad (\text{extremum}) \Rightarrow \underline{\underline{x_1 = x_2 = 1}}$$

$$\frac{d^2f}{dx^2} > 0 \quad \text{minimum} \quad \parallel \quad \frac{d^2f}{dx^2} < 0 \quad \text{maximum}$$

$$\frac{d^2f}{dx^2} > 0 \quad \text{minimum} \quad \Bigg\| \quad \frac{d^2f}{dx^2} < 0 \quad \text{maximum}$$

(IV) Numerically using `scipy.optimize`.

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(I) Unconstrained optimization

e.g.  $f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$   
Cost

(II) Constrained optimization

Cost  $f(x_1, x_2) = x_1^2 + (x_1 - x_2)^3$

constraints: e.g.  $x_1 + x_2 = 5$

$$x_1^2 + e^{x_2} < 2$$

## Constrained Optimization

$$\min f(x) = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2$$

$$x = \{x_1, x_2, x_3, x_4, x_5\}$$

subject to:

$$x_1 + x_2 + x_3 = 5 \quad \text{Linear equality constraint}$$

$$x_3^2 + x_4 = 5$$

Non linear equality constraint

$$x_1 \geq 0.3$$

$$x_3 \leq 5$$

$$x_4^2 + x_5^2 \leq 5$$

Non linear Inequality constraint

### Bounds

$$0.3 \leq x_1 \leq \infty$$

$$-\infty \leq x_3 \leq 5$$

$$-\infty \leq x_2, x_4, x_5 \leq \infty$$