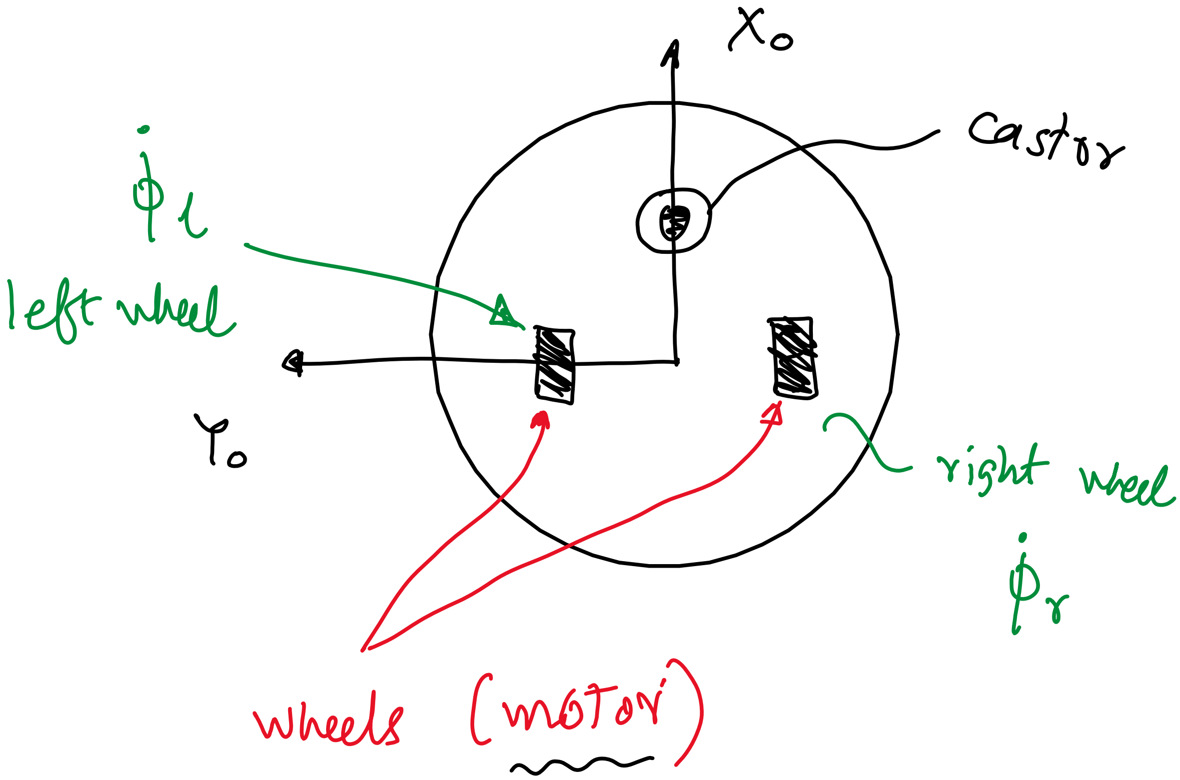
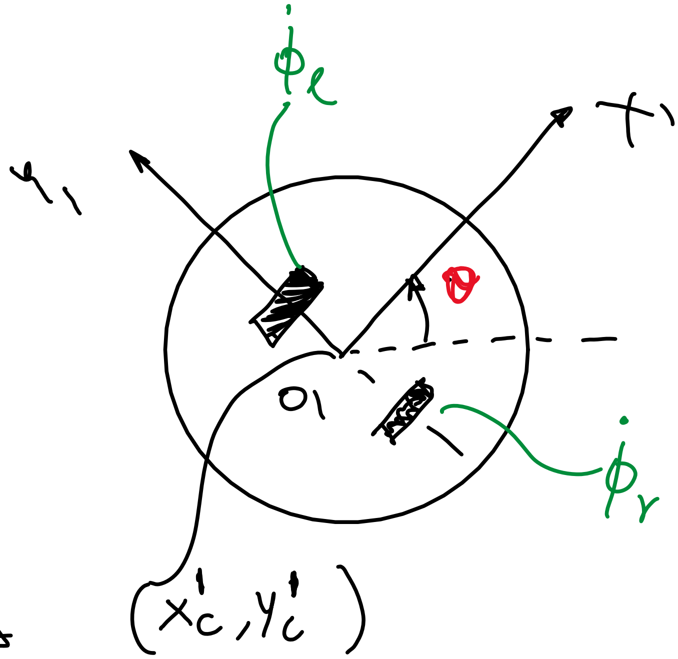
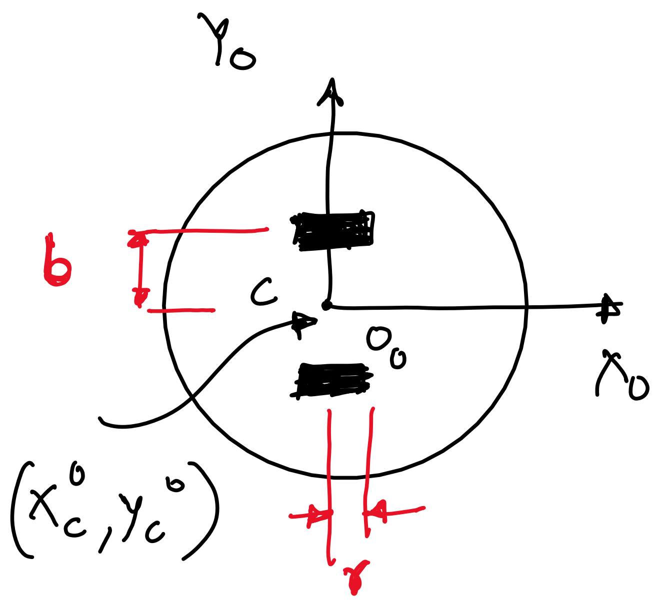
Differential Drive Car



move straight, set $\dot{\phi}_r / \dot{\phi}_l$ to the same value

turn, set $\dot{\phi}_r / \dot{\phi}_l$ to different values.



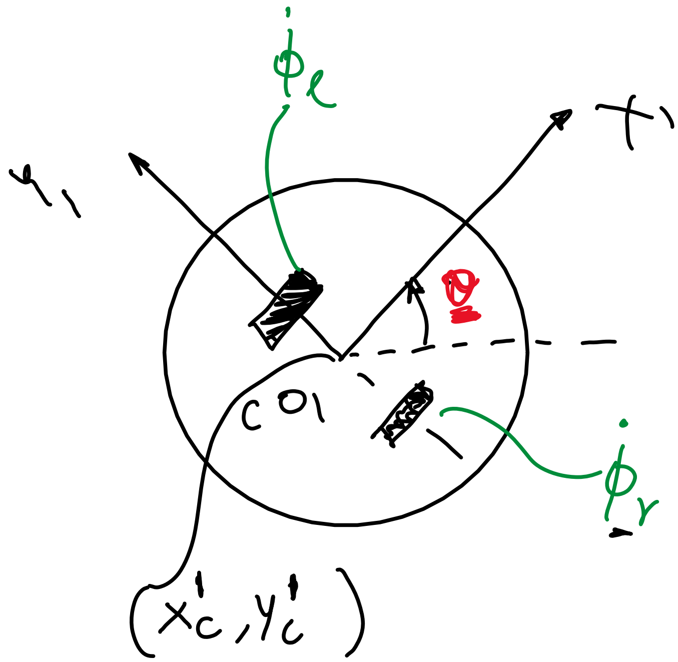


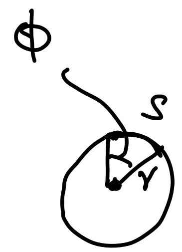
position at some time $t = t_1$

position at $t=0$

Compute $(x_c^0, y_c^0) = ?$

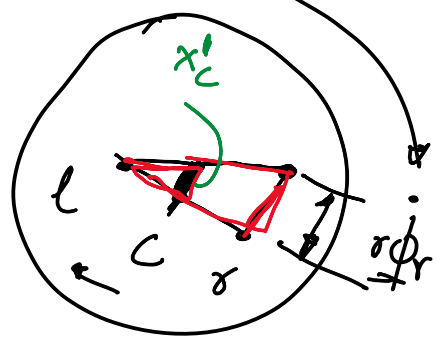
We will derive $\overbrace{(\dot{x}_c^0, \dot{y}_c^0)}^{?}$, then we will integrate this equations to comput x_c^0, y_c^0





$s = r\phi$

$\dot{s} = r\dot{\phi}$



Part 1 : Assume straight motion

If $\dot{\phi}_r \neq 0$ $\dot{\phi}_l = 0$

$$\dot{x}_c' = \frac{r\dot{\phi}_r}{2} \quad \text{— (I)} \qquad y_c' = 0$$

If $\dot{\phi}_l \neq 0$ $\dot{\phi}_r = 0$

$$\dot{x}_c' = \frac{r\dot{\phi}_l}{2} \quad \text{— (II)} \qquad y_c' = 0$$

If $\dot{\phi}_l \neq 0$ & $\dot{\phi}_r \neq 0$

$$\begin{cases} \dot{x}_c' = \dfrac{r}{2}(\dot{\phi}_r + \dot{\phi}_l) \\ \dot{y}_c' = 0 \end{cases}$$

From (I) and (II) by super position

$$c^0 = R_1^0 c^1 \qquad c^0 = \begin{bmatrix} x_c^0 \\ y_c^0 \end{bmatrix} \qquad c^1 = \begin{bmatrix} x_c^1 \\ y_c^1 \end{bmatrix}$$

$$\dot{c}^0 = R_1^0 \dot{c}^1 + \cancel{\dot{R}_1^0} c^1$$

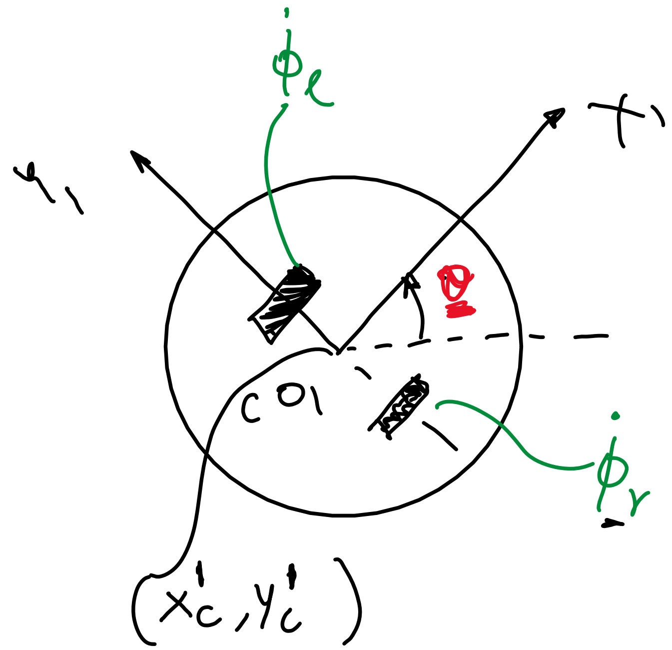
only translation

$\dot{R} = 0$

$$\begin{bmatrix} \dot{x}_c^0 \\ \dot{y}_c^0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \dot{x}_c^1 \\ \dot{y}_c^1 \end{bmatrix}$$

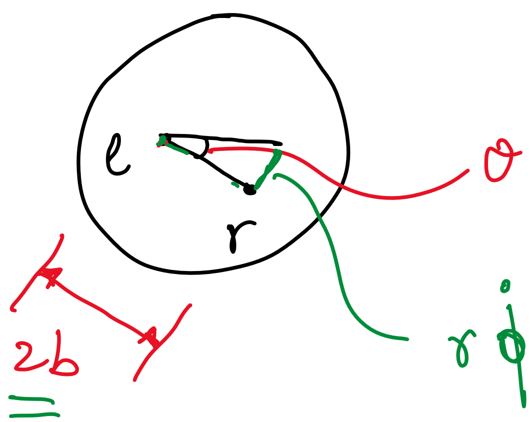
$$\begin{bmatrix} \dot{x}_c^0 \\ \dot{y}_c^0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \frac{r}{2}(\dot{\phi}_r + \dot{\phi}_l) \\ 0 \end{bmatrix}$$

$$\boxed{\begin{aligned} \dot{x}_c^0 &= \frac{r}{2}(\dot{\phi}_r + \dot{\phi}_l)\cos\theta \\ \dot{y}_c^0 &= \frac{r}{2}(\dot{\phi}_r + \dot{\phi}_l)\sin\theta \end{aligned}} \qquad *$$



Part 2: Effect of rotation due to different speeds

$\dot{\phi}_l = 0$ & $\dot{\phi}_r \neq 0$



$\dot{\theta} = ?$

$r\dot{\phi}_r = 2b\dot{\theta}$

$$\dot{\theta} = \frac{r\dot{\phi}_r}{2b} \quad \text{(Counter clockwise)} \quad \text{III}$$

$$\dot{\phi}_r = 0 \;\; \& \;\; \dot{\phi}_l \neq 0 \qquad \dot{\theta} = -\frac{r\dot{\phi}_l}{2b} \quad \text{IV}$$

If $\dot{\phi}_r \neq 0$ & $\dot{\phi}_l \neq 0$

From III and IV

$$\boxed{\dot{\theta} = \frac{r}{2b}(\dot{\phi}_r - \dot{\phi}_l)} \quad \sim$$

From * and ~

$$\dot{x}_c^o = 0.5\, r\, (\dot{\phi}_r + \dot{\phi}_l) \cos\theta$$

$$\dot{y}_c^o = 0.5\, r\, (\dot{\phi}_r + \dot{\phi}_l) \sin\theta$$

$$\dot{\theta} = 0.5 \frac{r}{b} (\dot{\phi}_r - \dot{\phi}_l)$$

$$\left.\begin{aligned} V &= 0.5\, r\, \dot{\phi}_r + \dot{\phi}_l) \\ \omega &= 0.5 \frac{r}{b} (\dot{\phi}_r - \dot{\phi}_l) \end{aligned}\right\}$$

$$\boxed{\begin{aligned} \dot{x}_c^o &= V\cos\theta \\ \dot{y}_c^o &= V\sin\theta \\ \dot{\theta} &= \omega \end{aligned}}$$

Eulers method

$$x_c^o(t_{i+1}) = x_c^o(t_i) + h\, v(t_i)\cos(\theta(t_i))$$

$$y_c^o(t_{i+1}) = y_c^o(t_i) + h\, v(t_i)\sin(\theta(t_i))$$

$$\theta(t_{i+1}) = \theta(t_i) + h\, \omega(t_i)$$

$x_c^o(t_i)$, $y_c^o(t_i)$, $\theta(t_i)$ are known

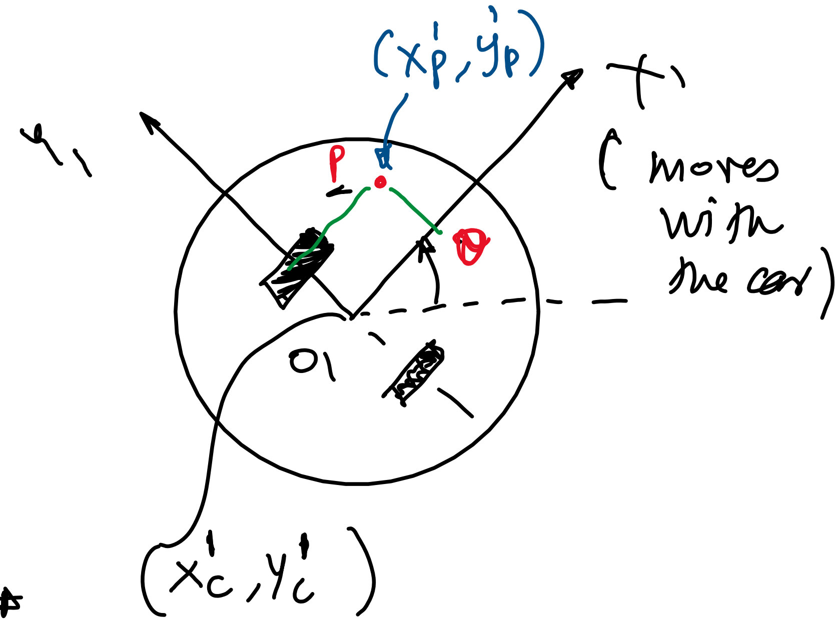
$v(t_i)$ & $\omega(t_i)$ $\longrightarrow$ controls

$$t_{i+1} = h + t_i$$

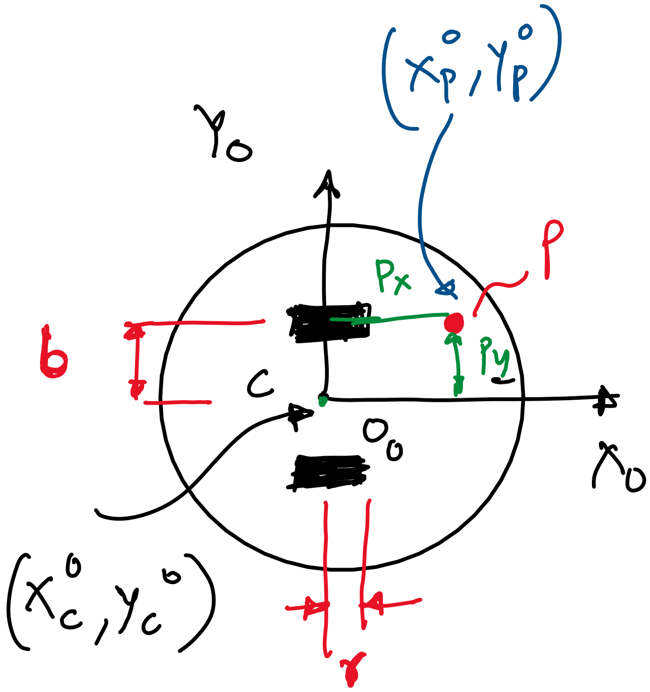
We need to know $x_c^o(t_o)$, $y_c^o(t_o)$, $\theta(t_o)$

$v(t_i)$, $\omega(t_i)$

Inverse kinematics of a differential drive car



position at some time $t = t_1$



position at $t = 0$

Goal: Get the point P to track a given reference trajectory $(x_{ref}(t), y_{ref}(t))$

$$c^0 = R_1^0 c^1 + d_1^0$$

$$p^0 = R_1^0 p^1 + d_1^0$$

$$(p^0 - c^0) = R_1^0 (p^1 - c^1)$$

$$\begin{bmatrix} x_p^o - x_c^o \\ y_p^o - y_c^o \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_p' - x_c' \\ y_p' - y_c' \end{bmatrix}$$

$$\begin{bmatrix} x_p^o - \underline{x_c^o} \\ y_p^o - \underline{y_c^o} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

Given position of p^o, compute c^o

$$\underline{\underline{\begin{bmatrix} x_c^o \\ y_c^o \end{bmatrix}}} = \underline{\underline{\begin{bmatrix} x_p^o \\ y_p^o \end{bmatrix}}} - \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

Given position of c^o, compute p^o

$$\begin{bmatrix} x_p^o \\ y_p^o \end{bmatrix} = \begin{bmatrix} x_c^o \\ y_c^o \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} p_x \\ p_y \end{pmatrix}$$

needed for IK code later

Differentiate with respect to time

$$\begin{bmatrix} \dot{x}_p^o \\ \dot{y}_p^o \end{bmatrix} = \begin{bmatrix} \dot{x}_c^o \\ \dot{y}_c^o \end{bmatrix} + \begin{bmatrix} (-\sin\theta)\dot{\theta} & (-\cos\theta)\dot{\theta} \\ (\cos\theta)\dot{\theta} & (-\sin\theta)\dot{\theta} \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_p^o \\ \dot{y}_p^o \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \end{bmatrix} + \begin{bmatrix} (-\sin\theta)\, p_x \dot{\theta} - (\cos\theta)\, p_y \dot{\theta} \\ (\cos\theta)\, p_x \dot{\theta} - (\sin\theta)\, p_y \dot{\theta} \end{bmatrix}$$

$\dot{\theta} = \omega$

$$\begin{bmatrix} \dot{x}_p^o \\ \dot{y}_p \end{bmatrix} = \begin{bmatrix} \underline{v}\cos\theta - (p_x \sin\theta + p_y \cos\theta)\,\omega \\ \underline{v}\sin\theta + (p_x \cos\theta - p_y \sin\theta)\,\omega \end{bmatrix}$$

I ~ $$\begin{bmatrix} \dot{x}_p^o \\ \dot{y}_p^o \end{bmatrix} = \begin{bmatrix} \cos\theta & -p_x \sin\theta - p_y \cos\theta \\ \sin\theta & p_x \cos\theta - p_y \sin\theta \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

II ~ $$\begin{cases} \dot{x}_p^o = k_{px}\,(x_{ref}(t) - x_p^o) \\ \dot{y}_p^o = k_{py}\,(y_{ref}(t) - y_p^o) \end{cases}$$

sensor measurements.

controller to track $x_{ref}(t), y_{ref}(t)$

Put I in II

$$\begin{bmatrix} k_{px}\,(x_{ref} - x_p^o) \\ k_{py}\,(y_{ref} - y_p^o) \end{bmatrix} = \begin{bmatrix} \cos\theta & -p_x \sin\theta - p_y \cos\theta \\ \sin\theta & p_x \cos\theta - p_y \sin\theta \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

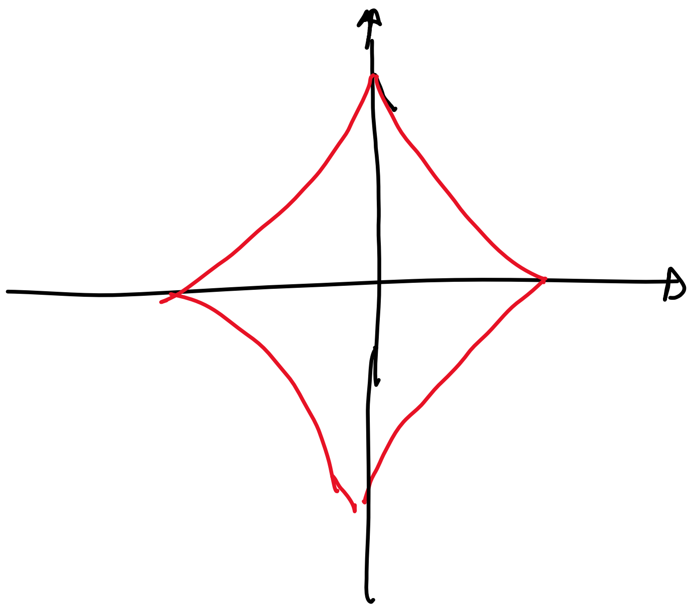
$$(b) = A\,X$$

unknown

$$X = A^{-1}\, b$$

$$A^{-1} = \begin{bmatrix} \cos\theta - \left(\frac{p_y}{p_x}\right)\sin\theta & \sin\theta + \left(\frac{p_y}{p_x}\right)\cos\theta \\ -\left(\frac{1}{p_x}\right)\sin\theta & \left(\frac{1}{p_x}\right)\cos\theta \end{bmatrix}$$

Astroid



$$x_{ref}(t) = \cos^3\left(\pi t/5\right)$$

$$y_{ref}(t) = \sin^3\left(\pi t/5\right)$$