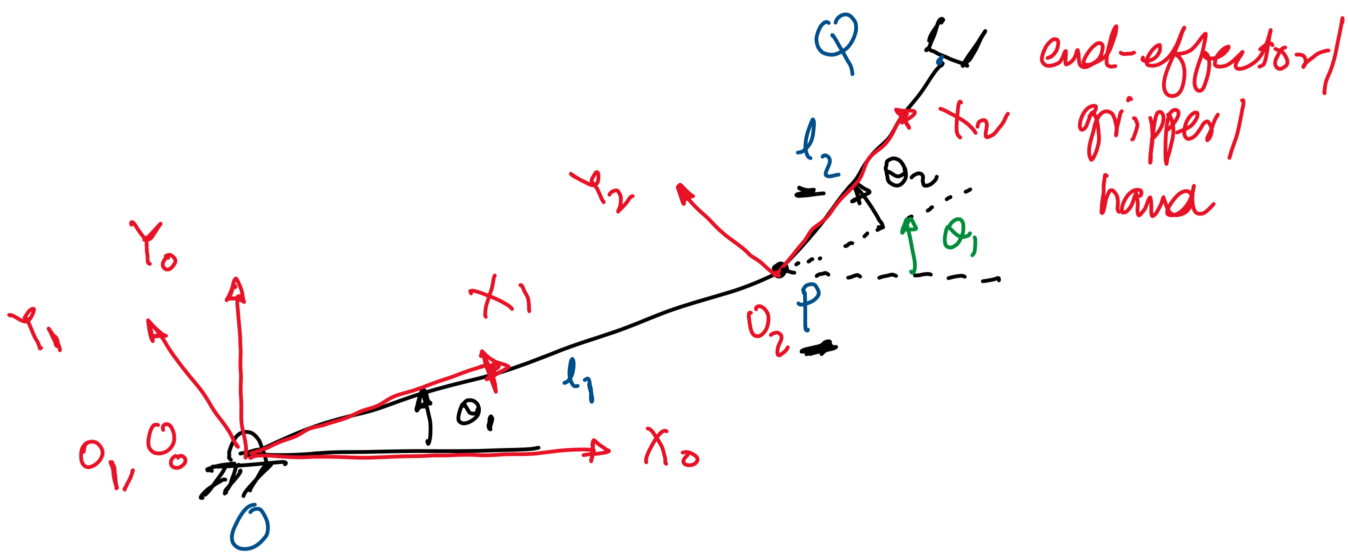
Manipulator Forward Kinematics

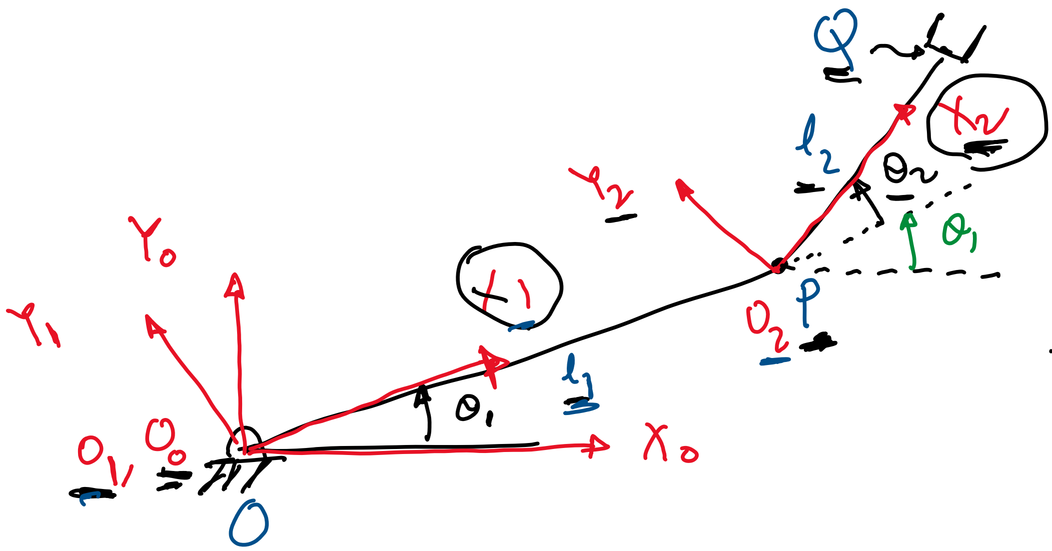
→ given the joint angles, determine the position / orientation of the $\underbrace{\text{end-effector}}_{\text{hand}}$.



Compute position of P and Q as a function of $\theta_1, \theta_2, l_1, l_2$

Method 1 : Trignometry

$$P^0 = \begin{bmatrix} l_1 \cos\theta_1 \\ l_1 \sin\theta_1 \end{bmatrix} \qquad Q^0 = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \end{bmatrix}$$



$$p^{i-1} = H_i^{i-1} \, p^i$$

$$?\; p^0 = H_1^0 \, p^1$$

$$?\; Q^0 = H_1^0 \, H_2^1 \, Q^2$$

$$H_i^{i-1} = \begin{bmatrix} R_i^{i-1} & O_i^{i-1} \\ 0 & 1 \end{bmatrix}_{3\times3}$$

$$p^1 = \begin{bmatrix} l_1 \\ 0 \\ 1 \end{bmatrix} \qquad Q^2 = \begin{bmatrix} l_2 \\ 0 \\ 1 \end{bmatrix}$$

$$H_1^0 = \begin{bmatrix} R_1^0 & O_1^0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_2^1 = \begin{bmatrix} R_2^1 & O_2^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & l_1 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P^0 = H_1^0 \, P^1$$

$$= \begin{bmatrix} l_1 \cos\theta_1 \\ l_1 \sin\theta_1 \\ 1 \end{bmatrix}$$

$$Q^0 = H_1^0 \, H_2^1 \, Q^2$$

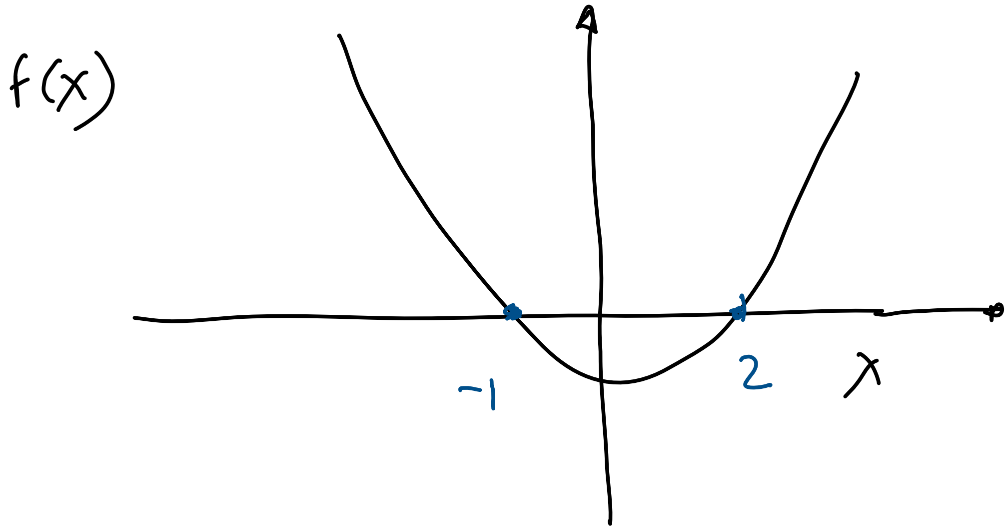
$$= \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \\ 1 \end{bmatrix}$$

Root finding - precursor to inverse kinematics

↳ given Q^0 (end-effector) compute θ_1, θ_2

$$f(x) = x^2 - x - 2$$

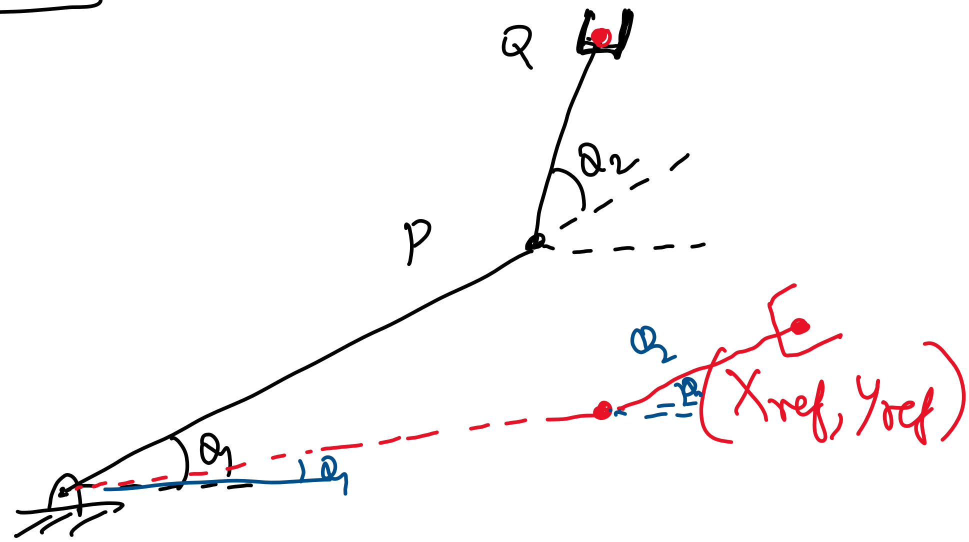
Roots : If $f(x) = 0$, compute x.



We will use fsolve to compute the roots of $f(x)$

Inverse kinematics of a 2-link manipulator

IK



Compute θ_1, θ_2 such that the end-effector, Q, is at X_{ref}, Y_{ref}

Given: $x_{ref}, y_{ref}, l_1, l_2$

Unknown: θ_1, θ_2

$\Big\}$ IK problem

$$Q^0 = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \end{bmatrix} = \begin{bmatrix} x_{ref} \\ y_{ref} \end{bmatrix}$$

$$\underbrace{F(\theta_1,\theta_2)}_{\text{Use fsolve}} = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) - x_{ref} \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) - y_{ref} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$