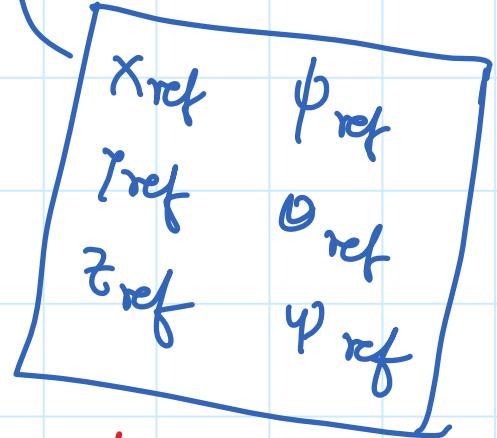
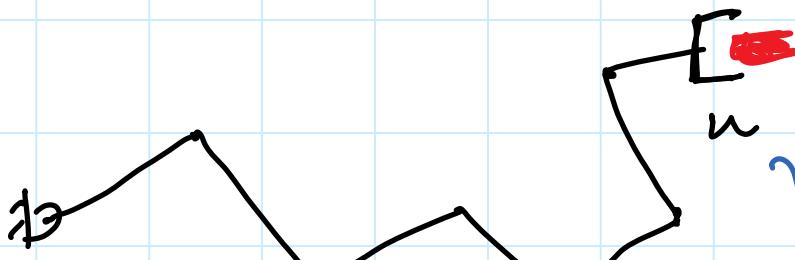


Inverse kinematics of 3D manipulators



reference
end-effector
position/orientation

Need at least 6 degrees
of freedom / 6 controlled
joints.

Given DH, we can compute end-effector
position & orientation

$$H_n^0 = H_1^0 H_2^1 \dots H_n^{n-1} = \begin{bmatrix} R_n^0 \\ d_n^0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} x_{ref} \\ y_{ref} \\ z_{ref} \end{bmatrix}$$

3x3

Diagram illustrating the transformation matrix H_n^0 as a product of DH matrices H_i^i from joint 1 to n. The resulting matrix is shown as a 3x3 matrix with columns labeled R_n^0 , d_n^0 , and 0. To its right is a 3x1 vector representing the reference end-effector position $[x_{ref}, y_{ref}, z_{ref}]$. Below the matrix are four circles labeled I, II, III, and IV, with arrows pointing to the matrix and the vector.

(V) (VI)

3-21

$$\begin{aligned} \mathbf{R} &= \mathbf{R}_z(\psi) \mathbf{R}_y(\theta) \mathbf{R}_x(\phi) \\ &= \begin{bmatrix} \underline{\cos(\psi)} \cos(\theta) & \underline{\cos(\psi)} \sin(\phi) \sin(\theta) - \cos(\phi) \sin(\psi) & \underline{\sin(\phi)} \sin(\psi) + \cos(\phi) \cos(\psi) \sin(\theta) \\ \underline{\cos(\theta)} \sin(\psi) & \underline{\cos(\phi)} \cos(\psi) + \sin(\phi) \sin(\psi) \sin(\theta) & \underline{\cos(\phi)} \sin(\psi) \sin(\theta) - \cos(\psi) \sin(\phi) \\ -\sin(\theta) & \underline{\cos(\theta)} \sin(\phi) & \underline{\cos(\phi)} \cos(\theta) \end{bmatrix} \end{aligned}$$

$$= R_n^0$$

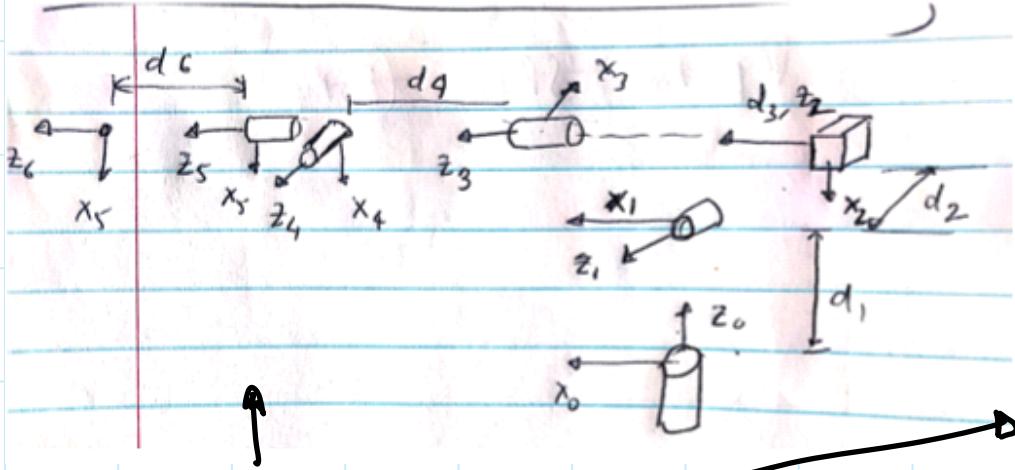
$$= \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$-\sin \theta_{ref} = r_{31} \Rightarrow \underline{\theta_{ref}} = \sin^{-1}(-r_{31}) \quad -\textcircled{I}$$

$$\cos \underline{\theta_{ref}} \sin \phi_{ref} = r_{32} \Rightarrow \phi_{ref} = \sin^{-1} \left(\frac{r_{32}}{\cos \theta_{ref}} \right) \quad -\textcircled{II}$$

$$\cos \theta_{ref} \sin \psi_{ref} = r_{21} \Rightarrow \psi_{ref} = \sin^{-1} \left(\frac{r_{21}}{\cos \theta_{ref}} \right) \quad -\textcircled{III}$$

\textcircled{I}, \textcircled{II}, \textcircled{III} gives $\phi_{ref}, \theta_{ref}, \psi_{ref}$



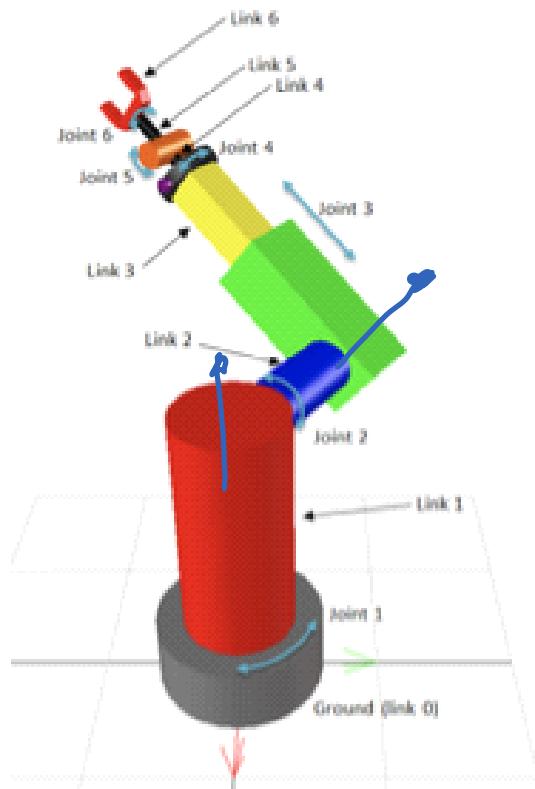
D-H table

Joint i	α_i	a_i	d_i	θ_i
1	$-\pi/2$	0	d_1	θ_1
2	$\pi/2$	0	d_2	θ_2
3	0	0	d_3	$-\pi/2$
4	$-\pi/2$	0	d_4	θ_4
5	$\pi/2$	0	0	θ_5
6	0	0	d_6	θ_6

Stanford manipulator

- ① Compute position / orientation of the end-effector
- ② Animation for given joint angles .
- ③ Inverse kinematics.

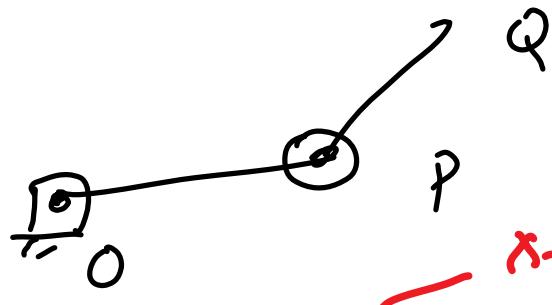
Given , the position /orientation of end - effector , compute the necessary joint angles .



$$\textcircled{1} \quad H_6^0 = H_1^0 H_2^1 H_3^2 H_4^3 H_5^4 H_6^5 \\ = =$$

Animation

2-D



line $([O(1) \quad P(1)], [O(2) \quad P(2)])$

line $(P(1) \quad Q(1)), [P(2) \quad Q(2)]$

3-D



line $([O(1) \quad P(1)), (O(2) \quad P(2)), (O(3) \quad P(3))]$

$$H_1^0 = J$$

$$H_2^0 = H_1^0 H_2^1$$

$$H_3^0 = H_1^0 H_2^1 H_3^2$$

$$O_i^0 = (\underline{x} \quad \underline{y} \quad \underline{z})$$

and so on

$$= \begin{bmatrix} R_1^0 & O_1^0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R_2^0 & O_2^0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R_3^0 & O_3^0 \\ 0 & 1 \end{bmatrix}$$

position
of end-of
link 1

end-of
link 2

end of
link 3