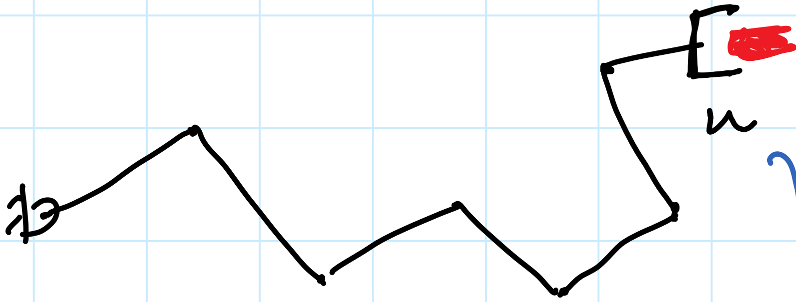
Inverse kinematics of 3D manipulators



x_{ref} ψ_{ref}
y_{ref} θ_{ref}
z_{ref} φ_{ref}

reference end-effector position / orientation

Need atleast 6 degrees of freedom / 6 controlled joints.

Given D-H, we can compute end-effector position & orientation

$$H_n^0 = H_1^0 H_2^1 \cdots H_n^{n-1} = \begin{bmatrix} \underbrace{R_n^0}_{3\times 3} & d_n^0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} x_{ref} \\ y_{ref} \\ z_{ref} \end{bmatrix}$$

IV

V VI

V VI

3-21

$$\mathbf{R} = \mathbf{R}_z(\psi)\mathbf{R}_y(\theta)\mathbf{R}_x(\phi)$$

$$= \begin{bmatrix} \cos(\psi)\cos(\theta) & \cos(\psi)\sin(\phi)\sin(\theta) - \cos(\phi)\sin(\psi) & \sin(\phi)\sin(\psi) + \cos(\phi)\cos(\psi)\sin(\theta) \\ \cos(\theta)\sin(\psi) & \cos(\phi)\cos(\psi) + \sin(\phi)\sin(\psi)\sin(\theta) & \cos(\phi)\sin(\psi)\sin(\theta) - \cos(\psi)\sin(\phi) \\ -\sin(\theta) & \cos(\theta)\sin(\phi) & \cos(\phi)\cos(\theta) \end{bmatrix}_{3\times 3}$$

$$= R_n^0$$

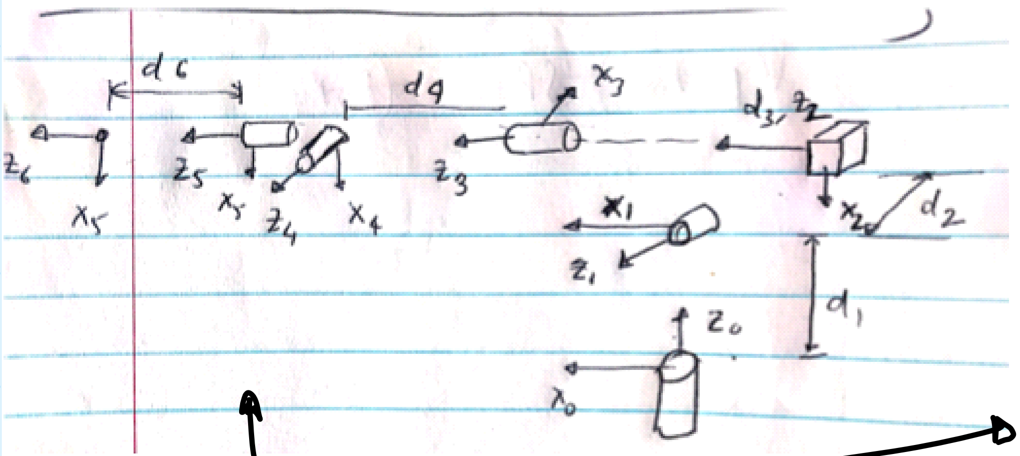
$$= \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$-\sin\theta_{ref} = r_{31} \Rightarrow \theta_{ref} = \sin^{-1}(-r_{31}) \quad \text{— I}$$

$$\cos\theta_{ref}\sin\phi_{ref} = r_{32} \Rightarrow \phi_{ref} = \sin^{-1}\left(\frac{r_{32}}{\cos\theta_{ref}}\right) \quad \text{— II}$$

$$\cos\theta_{ref}\sin\psi_{ref} = r_{21} \Rightarrow \psi_{ref} = \sin^{-1}\left(\frac{r_{21}}{\cos\theta_{ref}}\right) \quad \text{— III}$$

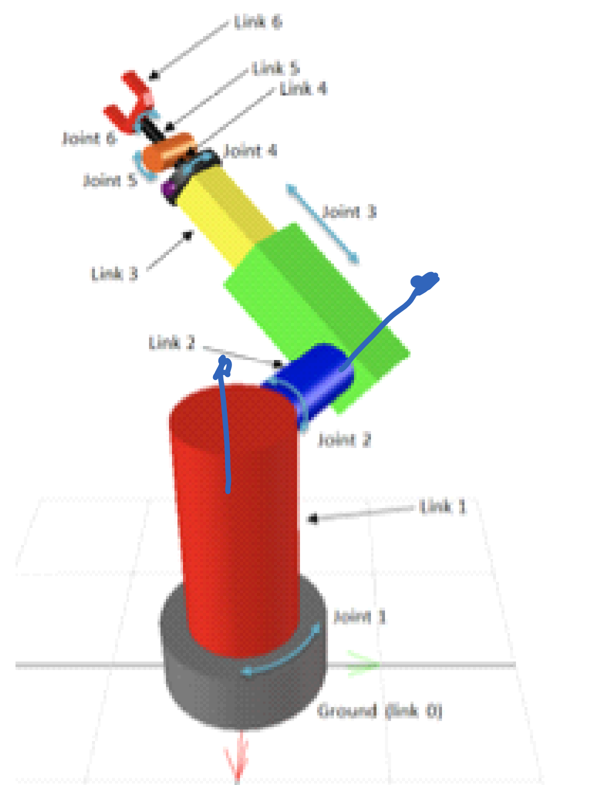
I, II, III gives $\phi_{ref}, \theta_{ref}, \psi_{ref}$



Joint i	α_i	a_i	d_i	θ_i
1	-Pi/2	0	d_1	θ_1
2	Pi/2	0	d_2	θ_2
3	0	0	d_3	-Pi/2
4	-Pi/2	0	d_4	θ_4
5	Pi/2	0	0	θ_5
6	0	0	d_6	θ_6

D-H table

Stanford manipulator



① Compute position/orientation of the end-effector

② Animation for given joint angles.

③ Inverse kinematics.

Given, the position/orientation of end-effector, compute the necessary joint angles.

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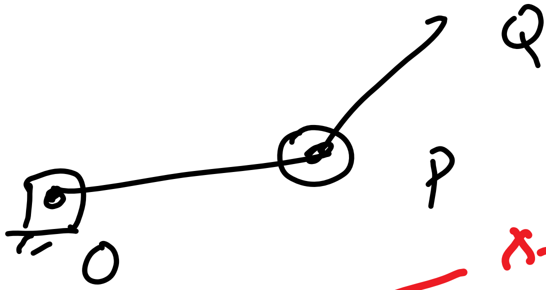
① $H_6^0 = H_1^0 \, H_2^1 \, H_3^2 \, H_4^3 \, H_5^4 \, H_6^5$

$=$ $=$

Animation

2-D



x-coordinate   y-coordinate

line ([O(1) P(1)], [O(2) P(2)])

line ([P(1) Q(1)], [P(2) Q(2)])

3-D

$$\text{line}\left(\underbrace{[O(1)\;\; P(1)]}_{x},\; \underbrace{[O(2)\;\; P(2)]}_{y},\; \underbrace{[O(3)\;\; P(3)]}_{z}\right)$$

$$H_1^0 = \checkmark = \begin{bmatrix} R_1^0 & o_1^0 \\ 0 & 1 \end{bmatrix}$$ position of end-of link 1

$$H_2^0 = H_1^0 H_2^1 = \begin{bmatrix} R_2^0 & o_2^0 \\ 0 & 1 \end{bmatrix}$$ end-of link 2

$$H_3^0 = H_1^0 H_2^1 H_3^2 = \begin{bmatrix} R_3^0 & o_3^0 \\ 0 & 1 \end{bmatrix}$$ end of link 3

$$o_1^0 = (x \;\; y \;\; z)$$

and so on