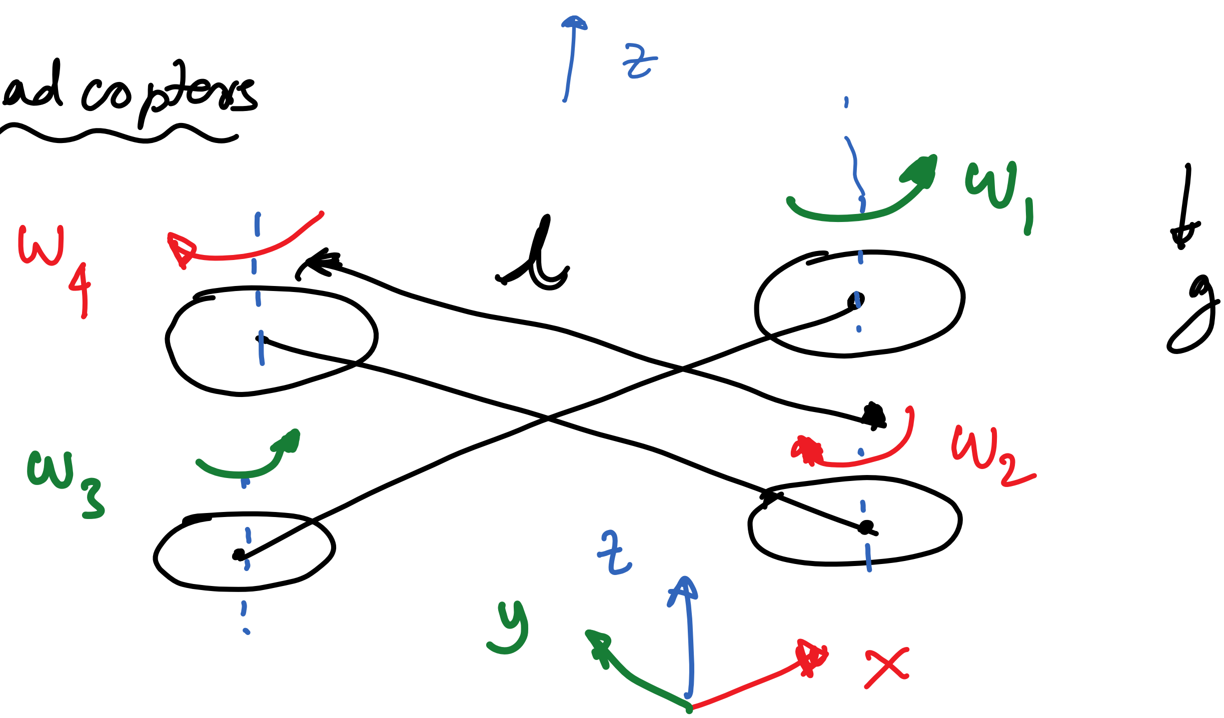
Quadcopters



ω_1, ω_3 — CCW

ω_2, ω_4 — CW

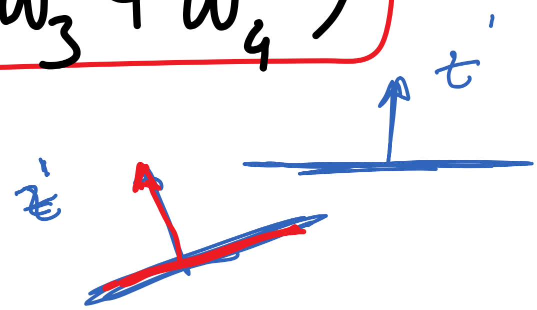
Rotors produce a lift force at right angle: Thrust $\propto \omega^2$

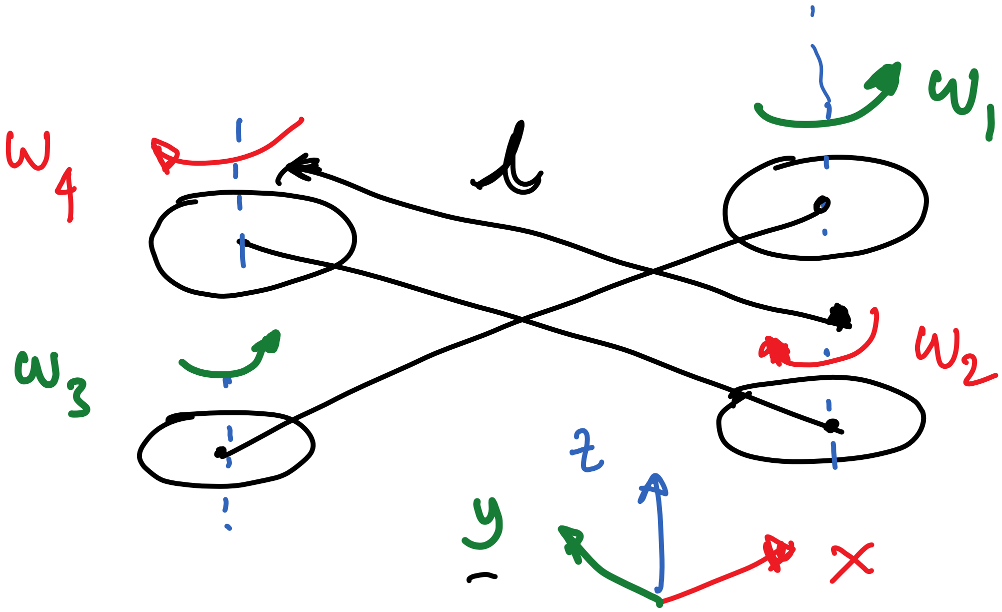
$$\underline{\underline{\text{Thrust}}} = k\omega^2 \qquad k: N/(rad/s)^2$$

↑ lift constant

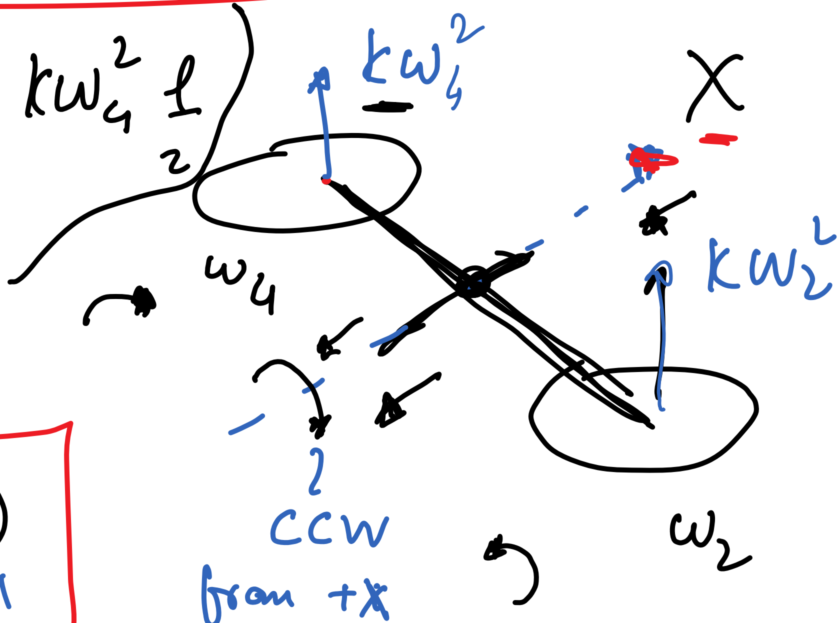
$$\boxed{F^{t}_{z'} = k(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)}$$

↑ body frame



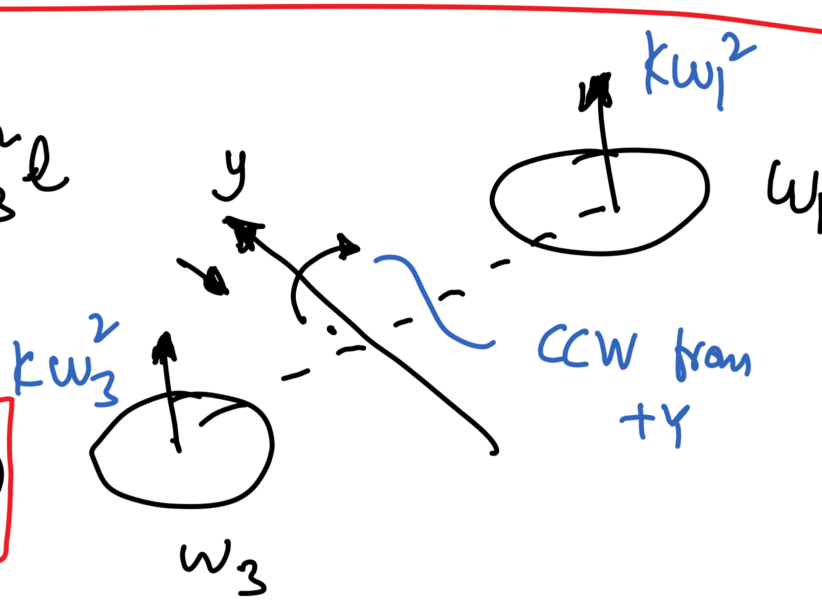


$$\tau_x = -k\,\omega_2^2\,\frac{\ell}{2} + k\,\omega_4^2\,\frac{\ell}{2}$$

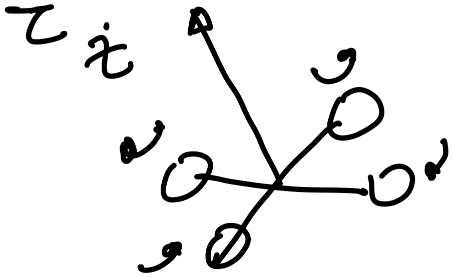


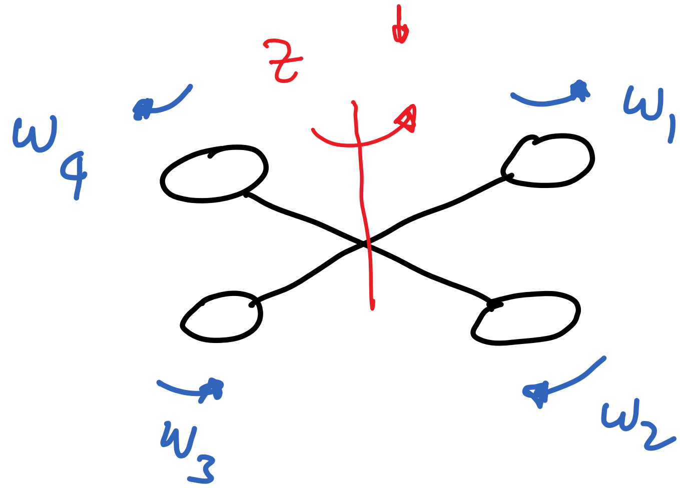
$$\boxed{\tau_x = \frac{k\ell}{2}\left(+\omega_4^2 - \omega_2^2\right)}$$

$$\tau_y = -k\,\omega_1^2\,\frac{\ell}{2} + \frac{k\,\omega_3^2\,\ell}{2}$$



$$\boxed{\tau_y = +\frac{k\ell}{2}\left(+\omega_3^2 - \omega_1^2\right)}$$





$$\boxed{\tau_z = b\left(+\omega_1^2 + \omega_3^2 - \omega_2^2 - \omega_4^2\right)}$$

drag constant

$Nm/(rad/s)^2$

Summary

1. F_z - force in body z-direction
2. τ_x, τ_y, τ_z - torques in body x, y, z direction

We cannot induce forces in x, y direction

- system is underactuated.

6 state variables: $x, y, z, \theta, \phi, \psi$

4 control variables: $F_z, \tau_x, \tau_y, \tau_z$

or

$\omega_1, \omega_2, \omega_3, \omega_4$

Equations of motion of a quadcopter

1) Positions: $x, y, z, \phi, \theta, \psi$

Velocities: $\dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\psi}$

Body frame angular velocity

$$\omega_b = \begin{bmatrix} \omega_{bx} \\ \omega_{by} \\ \omega_{bz} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta \sin\phi \\ 0 & -\sin\phi & \cos\theta \cos\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

2) $T = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{1}{2} \omega_b^T I \omega_b$

$$\begin{bmatrix} \omega_{bx} \\ \omega_{by} \\ \omega_{bz} \end{bmatrix}^T \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \begin{bmatrix} \omega_{bx} \\ \omega_{by} \\ \omega_{bz} \end{bmatrix}$$

$V = mgz$

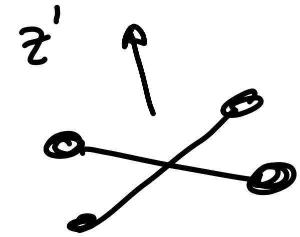
$\mathcal{L} = T - V$

3) $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = Q_j$ ← external forces

✓ $q_j = \{\underline{x}, \underline{y}, \underline{z}, \underline{\dot{\phi}, \dot{\theta}, \dot{\psi}}\}$ ✓

✓ $Q_j = \begin{bmatrix} F_{ext} \\ \tau_{ext} \end{bmatrix}_{6\times1}$?

$$\underline{\underline{F_{ext}}} = R \cdot \begin{bmatrix} 0 \\ 0 \\ F_{z'} \end{bmatrix} - \begin{bmatrix} A_x \dot{x} \\ A_y \dot{y} \\ A_z \dot{z} \end{bmatrix}$$



z-y-x rotation matrix

$$R = R_z(\psi)\, R_y(\theta)\, R_x(\phi)$$

A_x, A_y, A_z drag force constant ($\propto$ velocity)

$$\tau_{ext} = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} kl\,(\omega_4^2 - \omega_2^2) \\ kl\,(\omega_3^2 - \omega_1^2) \\ b\,(\omega_1^2 + \omega_3^2 - \omega_2^2 - \omega_4^2) \end{bmatrix}$$

$$Q_j = \begin{bmatrix} R \begin{bmatrix} 0 \\ 0 \\ k\,(\underline{\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2}) \end{bmatrix} - \begin{pmatrix} A_x \dot{x} \\ A_y \dot{y} \\ A_z \dot{z} \end{pmatrix} \\ 0.5\, kl\,(\omega_4^2 - \omega_2^2) \\ 0.5\, kl\,(\omega_3^2 - \omega_1^2) \\ b\,(\omega_1^2 + \omega_3^2 - \omega_2^2 - \omega_4^2) \end{bmatrix}$$

6×1

4) Using Euler-Lagrange we can write

$$\underset{6\times 6}{A}\ \underset{6\times 1}{X} = \underset{6\times 1}{b}$$

$$X = [\ddot{x} \quad \ddot{y} \quad \ddot{z} \quad \ddot{\phi} \quad \ddot{\theta} \quad \ddot{\psi}]^T$$

$$F_x = F_z \sin 0.5$$