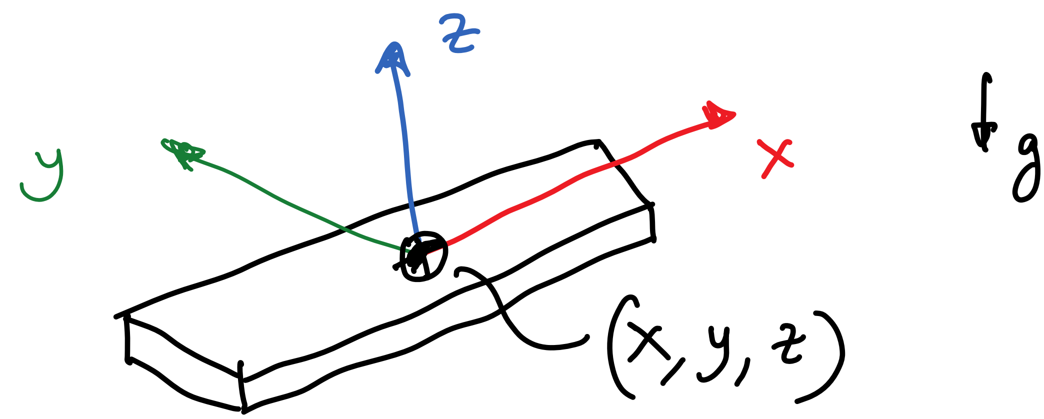
3 D dynamics



Given an initial position/orientation and linear/angular velocity describe the motion of the object

- Equations
- simulate (ode)
- animate

Equations using Euler-Lagrange method.

1) Position / Orientation

↙ x, y, z

↘ Euler angles 3-2-1

$z-y-x$

$\psi-\theta-\phi$

Linear / Angular velocity

↙ $[\dot{x}, \dot{y}, \dot{z}]$

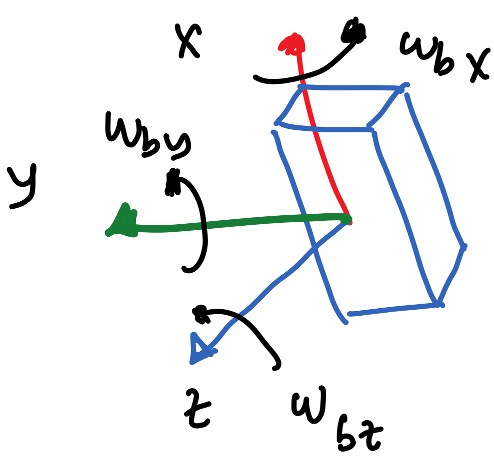
world frame

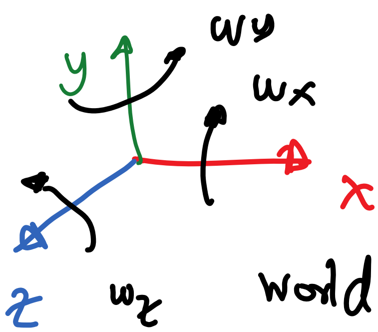
linear velocity

↘ ω in world frame X → $\omega_x, \omega_y, \omega_z$

[ω in body frame]

$$\omega_b = \begin{bmatrix} \omega_{bx} \\ \omega_{by} \\ \omega_{bz} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$





world frame

2) $L = T - V$

$V = mgz$

$$T = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{1}{2}\left(\omega^T (I \omega)\right)$$

$$\begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix}$$

Inertia about world frame

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$ – world frame

$$= \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{1}{2}\left(\omega_b^T (I^b \omega_b)\right)$$

$$\begin{bmatrix} \bar{I}_1 & 0 & 0 \\ 0 & \bar{I}_2 & 0 \\ 0 & 0 & \bar{I}_3 \end{bmatrix}$$

symmetric

$$\begin{bmatrix} \omega_{bx} \\ \omega_{by} \\ \omega_{bz} \end{bmatrix}$$

$$\begin{bmatrix} I_{11}^b & I_{12}^b & I_{13}^b \\ I_{21}^b & I_{22}^b & I_{23}^b \\ I_{31}^b & I_{32}^b & I_{33}^b \end{bmatrix}$$

$$\mathcal{L} = T - V$$

$$= \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{1}{2}(\bar{I}_1 \omega_{bx}^2 + \bar{I}_2 \omega_{by}^2 + \bar{I}_3 \omega_{bz}^2)$$

$$- mgz$$

3) Equation

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = Q_j$$

$q_j = x, y, z, \phi, \theta, \psi$

6 equations

$Q_j = 0$ (no external forces)

4) Simplify as

$$\underset{6\times 6}{A}\ \underset{6\times 1}{X} = \underset{6\times 1}{b}$$

unknowns $[\ddot{x}\ \ \ddot{y}\ \ \ddot{z}\ \ \ddot{\phi}\ \ \ddot{\theta}\ \ \ddot{\psi}]$