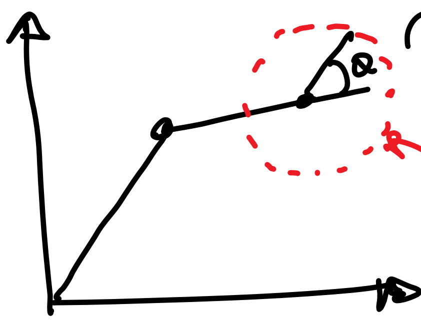
Trajectory generation



$x = r\cos(\theta)$

$y = r\sin(\theta)$

trajectory

$\theta(t)$

Caveat: static problem

Look at dynamic trajectory

$x(t),\ y(t),\ z(t)$

$\theta_1(t),\ \theta_2(t),\ \theta_3(t),\ \ldots$

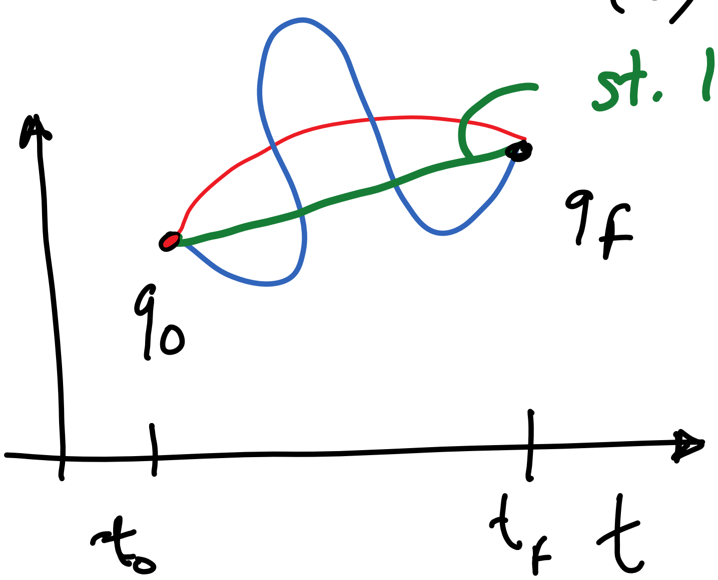
$\Big\}\ q(t)$

(x, y, z) cartesian or

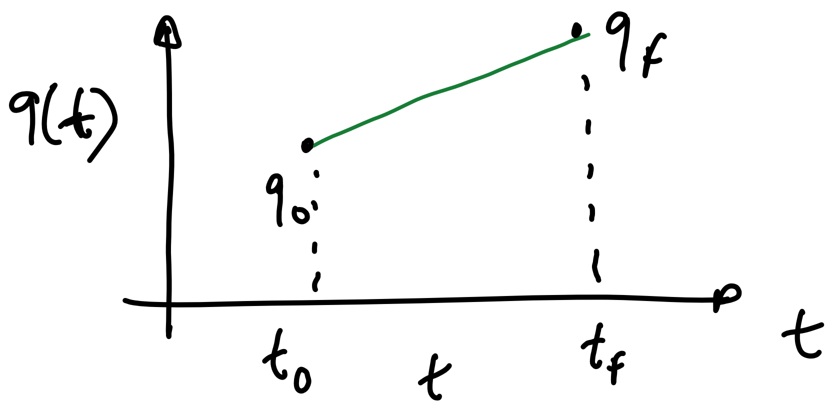
(θ) joint

st. line

$q(t)$



1) Connect q_0 & q_f with a st. line



Find $q(t)$

$$q = a_0 + a_1 t$$

$$q(t = t_0) = a_0 + a_1 t_0 = \checkmark q_0$$

$$q(t = t_f) = a_0 + a_1 t_f = \checkmark q_f$$

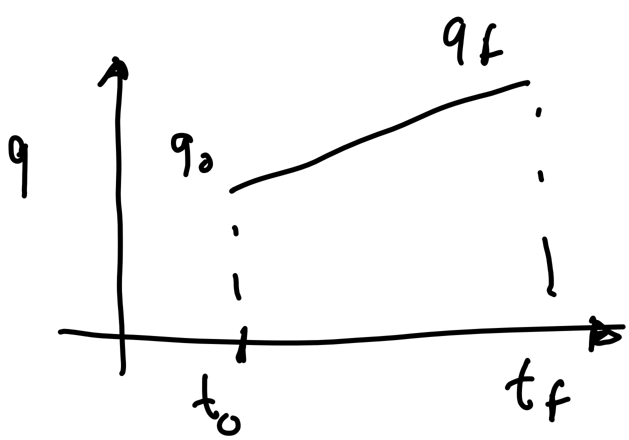
$$\begin{bmatrix} 1 & t_0 \\ 1 & t_f \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} q_0 \\ q_f \end{bmatrix}$$

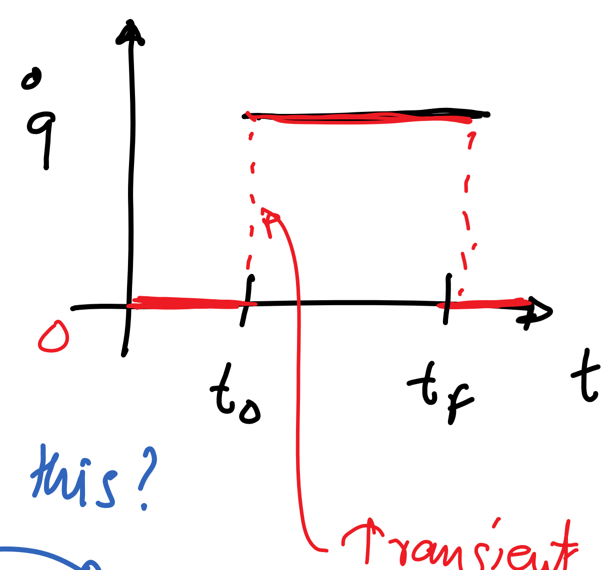
$$\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{bmatrix} 1 & t_0 \\ 1 & t_f \end{bmatrix}^{-1} \begin{bmatrix} q_0 \\ q_f \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \frac{1}{(t_f - t_0)} \begin{bmatrix} q_0 t_f - q_f t_0 \\ q_f - q_0 \end{bmatrix}$$

$$q(t) = \underbrace{\left[\frac{q_0 t_f - q_f t_0}{t_f - t_0}\right]}_{a_0} + \underbrace{\left[\frac{q_f - q_0}{t_f - t_0}\right]}_{a_1} t$$

$$\dot{q}(t) = \left(\frac{q_f - q_0}{t_f - t_0}\right)$$





How to avoid this?

may damage the motors

$q(t=t_0)=q_0$; $\dot{q}(t=t_0)=0$

$q(t=t_f)=q_f$; $\dot{q}(t=t_f)=0$

4 conditions

$$q(t)=a_0+a_1 t+a_2 t^2+a_3 t^3$$

4 constants

$$\dot{q}(t)=a_1+2a_2 t+3a_3 t^2$$

$$q_0=\underline{a_0}+\underline{a_1} t_0+a_2 t_0^2+a_3 t_0^3$$

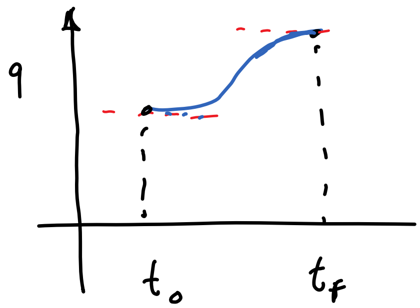
$$q_f=a_0+a_1 t_f+a_2 t_f^2+a_3 t_f^3$$

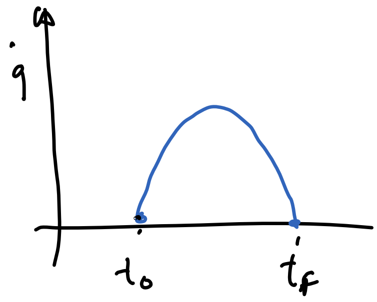
✓ $$0=\underline{a_1}+2\underline{a_2} t_0+3a_3 t_0^2$$

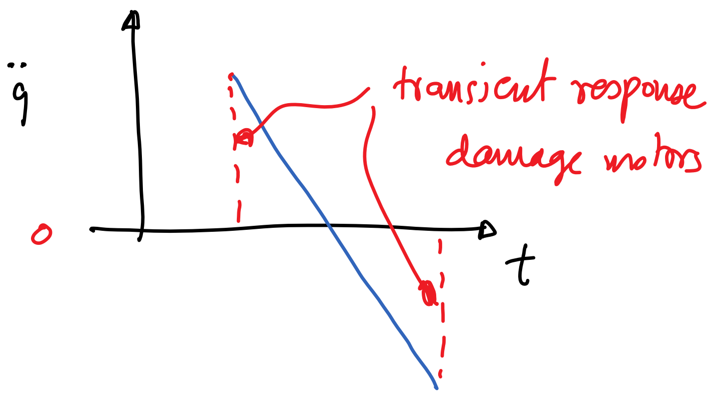
$$0=a_1+2a_2 t_f+3a_3 t_f^2$$

$$\begin{bmatrix} 1 & t_0 & t_0^2 & t_0^3 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_0 & 3t_0^2 \\ 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} q_0 \\ q_f \\ 0 \\ 0 \end{bmatrix}$$

invert







To avoid this:

$q(t_0) = q_0$; $\dot{q}(t_0) = 0$; $\ddot{q}(t_0) = 0$

$q(t_f) = q_f$; $\dot{q}(t_f) = 0$; $\ddot{q}(t_f) = 0$

6 conditions; 5^{th} order polynomial

$$q = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

5^{th} order poly: $\dddot{q}(t)$ = jerk is discontinuous

7^{th} order poly: $\frac{d^4 q}{dt^4}$ = snap

9^{th} $\frac{d^5 q}{dt^5}$ = crackle

11^{th} $\frac{d^6 q}{dt^6}$ = pop

θ

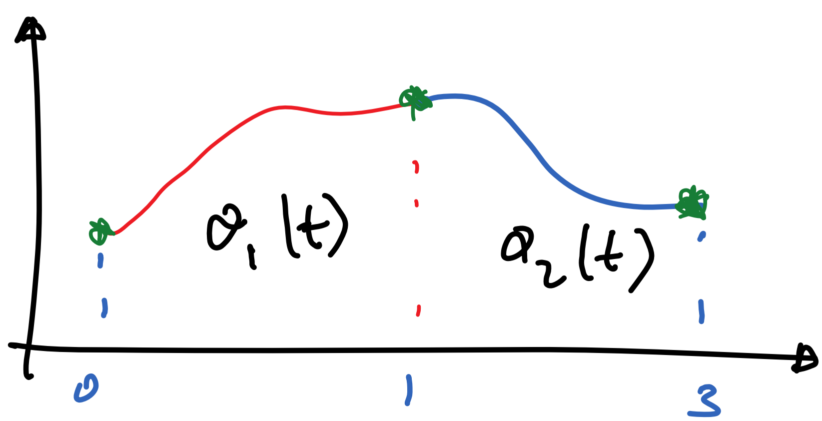
Example 1: Find a time based parameterization for a revolute joint of a manipulator. The joint should move from 0 to 0.5 rad from time t= 0 to t=1 sec followed by movement from 0.5 rad to 1 rad in from t=1 to t=3 secs. Also, the velocity of the joint at the start of motion (t=0) and end of motion (t=3) should be 0 and the velocity of the joint at the intermediate point (t=1) should be 0.2 rad/s. Assume two minimal order polynomials of time, one for each movement.

$$\boxed{\dot{\theta}_1(1) = \dot{\theta}_2(1) = 0.2}$$

$\theta(t=0) = 0$ $\qquad$ $\dot{\theta}(t=0) = 0$ ✓

$\theta(t=1) = 0.5$ $\qquad$ $\dot{\theta}(t=3) = 0$

$\theta(t=3) = 1$ $\qquad$ $\dot{\theta}(t=1) = 0.2$



$\theta_1(t=0) = 0$ $\qquad$ $\theta_2(t=1) = 0.5$

$\theta_1(t=1) = 0.5$ $\qquad$ $\theta_2(t=3) = 1$

$\dot{\theta}_1(t=0) = 0$ $\qquad$ $\dot{\theta}_2(t=3) = 0$

$\dot{\theta}_1(t=1) = 0.2$ $\qquad$ $\dot{\theta}_2(t=1) = 0.2$

3rd order $\qquad$ 3rd order

$$\theta_1(t) = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3 \quad 0 \leq t \leq 1$$

$$\theta_2(t) = a_{20} + a_{21}t + a_{22}t^2 + a_{23}t^3 \quad 1 \leq t \leq 3$$

✓ $\dot{\theta}_1(t) = a_{11} + 2a_{12}t + 3a_{13}t^2$

$\dot{\theta}_2(t) = a_{21} + 2a_{22}t + 3a_{23}t^2$

$\theta_1(t=0) = 0$	$\theta_2(t=1) = 0.5$
$\theta_1(t=1) = 0.5$	$\theta_2(t=3) = 1$
$\dot{\theta}_1(t=0) = 0$	$\dot{\theta}_2(t=3) = 0$
$\dot{\theta}_1(t=1) = 0.2$	$\dot{\theta}_2(t=1) = 0.2$

$\theta_1(t=0) = a_{10} = 0$

$\theta_1(t=1) = a_{10} + a_{11} + a_{12} + a_{13} = 0.5$

$\dot{\theta}_1(t=0) = a_{11} = 0$

$\dot{\theta}_1(t=1) = a_{11} + 2a_{12} + 3a_{13} = 0.2$

$\theta_2(t=1) = a_{20} + a_{21} + a_{22} + a_{23} = 0.5$

$\theta_2(t=3) = a_{20} + 3a_{21} + 9a_{22} + 27a_{23} = 1$

$\dot{\theta}_2(t=3) = a_{21} + 6a_{22} + 27a_{23} = 0$

$\dot{\theta}_2(t=1) = a_{21} + 2a_{22} + 3a_{23} = 0.2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 & 9 & 27 \\ 0 & 0 & 0 & 0 & 0 & 1 & 6 & 27 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ a_{20} \\ a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 0 \\ 0.2 \\ 0.5 \\ 1 \\ 0 \\ 0.2 \end{bmatrix}$$

$$\underset{8\times 8}{A}\ \underset{8\times 1}{x} = \underset{8\times 1}{b} \Rightarrow x = A^{-1} b$$

✓ ? ✓

$a_{10} = 0$; $a_{11} = 0$; $a_{12} = 1.3$, $a_{13} = -0.8$

$a_{20} = 0.55$, $a_{21} = -0.375$; $a_{22} = 0.4$;

$a_{23} = -0.075$

python code

Example 2: Find a time based parameterization for a revolute joint of a manipulator. The joint should move from 0 to 0.5 rad from time t= 0 to t=1 sec followed by movement from 0.5 rad to 1 rad in from t=1 to t=3 secs. Also, the velocity of the joint at the start of motion (t=0) and end of motion (t=3) should be 0 and the acceleration of the joint at the intermediate point (t=1) should be continuous. Assume two minimal order polynomials of time, one for each movement.

$$\ddot{\theta}_1(t=1) = \ddot{\theta}_2(t=1) \quad — \; 1$$

$$\left.\begin{aligned} \theta_1(t=0) &= 0 \\ \theta_1(t=1) &= 0.5 \\ \dot{\theta}_1(t=0) &= 0 \end{aligned}\right\} 3 \qquad \left.\begin{aligned} \theta_2(t=1) &= 0.5 \\ \theta_2(t=3) &= 1 \\ \dot{\theta}_2(t=3) &= 0 \end{aligned}\right\} 3$$

$$\dot{\theta}_1(t=1) = \dot{\theta}_2(t=1) \quad — \; 1$$

8 conditions ; 3^{rd} order polynomial

$$\tilde{A}X = \tilde{b}$$

python