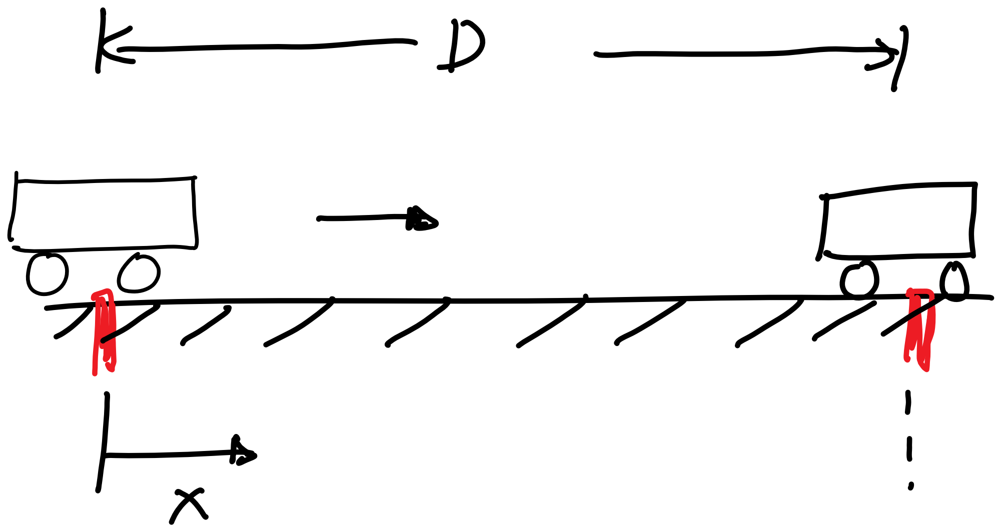
Trajectory optimization



$x(t=0)=0$ $\qquad$ $x(t=T)=D$

$\dot{x}(t=0)=0$ $\qquad$ $\dot{x}(t=T)=0$

Model $\qquad \ddot{x} = u \qquad -5 \leq u \leq 5$

Goal : Minimize the time

Formulation

$$\min_{\underline{T,\, u}} \int_{t=0}^{t=T} dt = T$$ Cost

optimization variables

constraints

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u \end{cases}$$

x_1 = position

x_2 = velocity

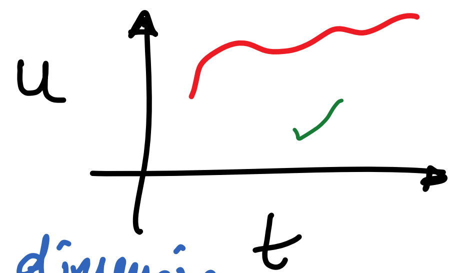
$$-5 \le u \le 5$$ Bounds

$$\left.\begin{array}{l} x_1(t=0) = 0 \\ x_1(t=T) = D = 5 \\ x_2(t=0) = 0 \\ x_2(t=T) = 0 \end{array}\right\}$$ Constraints

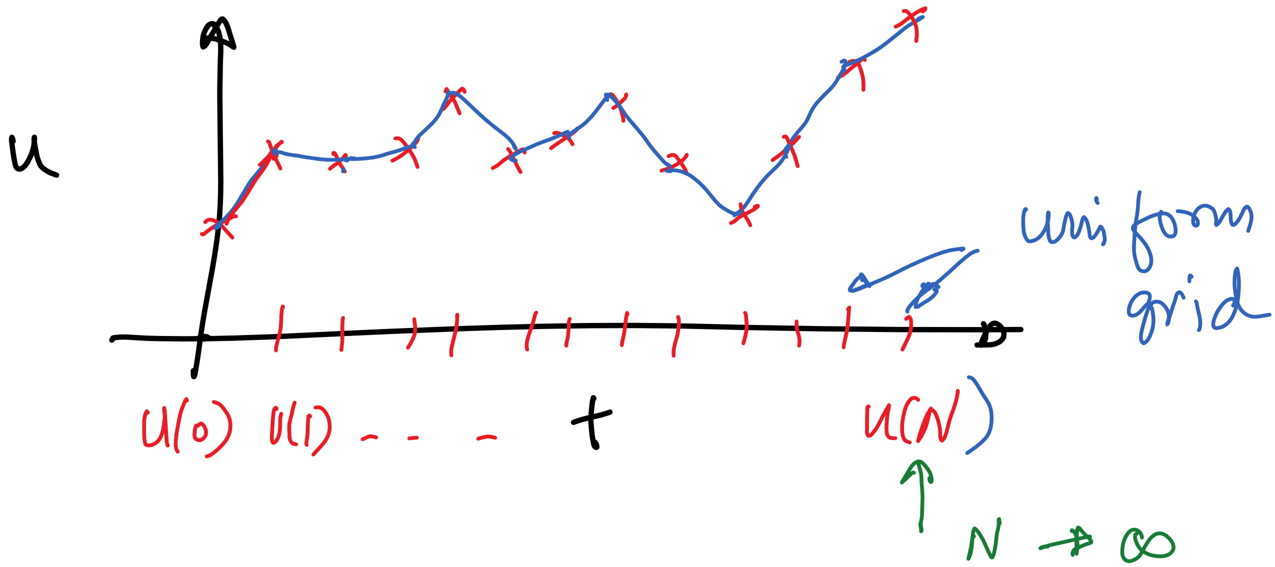
$$\left.\begin{array}{l} x_1(t) = ? \\ x_2(t) = ? \end{array}\right\}$$ Trajectory optimization

We need to compute T, u — infinite dimensional

T: scalar; $u \to u(t)$



convert the problem from <u>infinite</u> TO <u>finite</u> dimension



Two methods:

1. Collocation method
2. Shooting method

① Collocation method

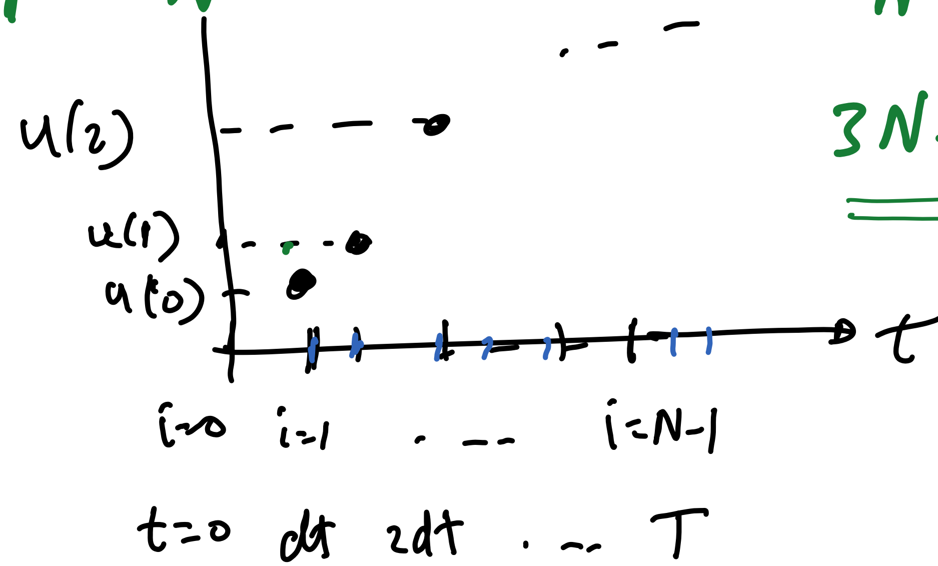
satisfy the system dynamics at grid points

$\dot{x}_1 = x_2$; $\dot{x}_2 = u$

a) Optimization variables

T, $u(i)$ $i = 0, 1, \ldots N-1$ $x_1(i)$, $x_2(i)$

1 N N N



3N+1 variables

b) Objective : T ✓

c) Constraints :

③ Constraints

$$-5 \leq u(i) \leq 5 \qquad N$$

$$\left.\begin{aligned} &- x_1(0) = 0 \\ &- x_1(T) = 5 \\ &- x_2(0) = 0 \\ &- x_2(T) = 0 \end{aligned}\right\} 4$$

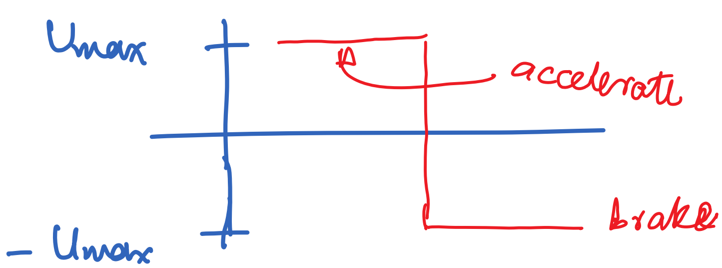
$$\Rightarrow \dot{x}_1 = x_2 \Rightarrow \frac{x_1(t+\Delta t) - x(t)}{\Delta t} = x_2(t)$$

Euler's method $\longrightarrow$

$$x_1(i+1) = x_1(i) + \Delta t \, x_2(i) \qquad N$$

$$x_2(i+1) = x_2(i) + \Delta t \, u(i) \qquad N$$

$\underline{\underline{3N+4}}$ equations



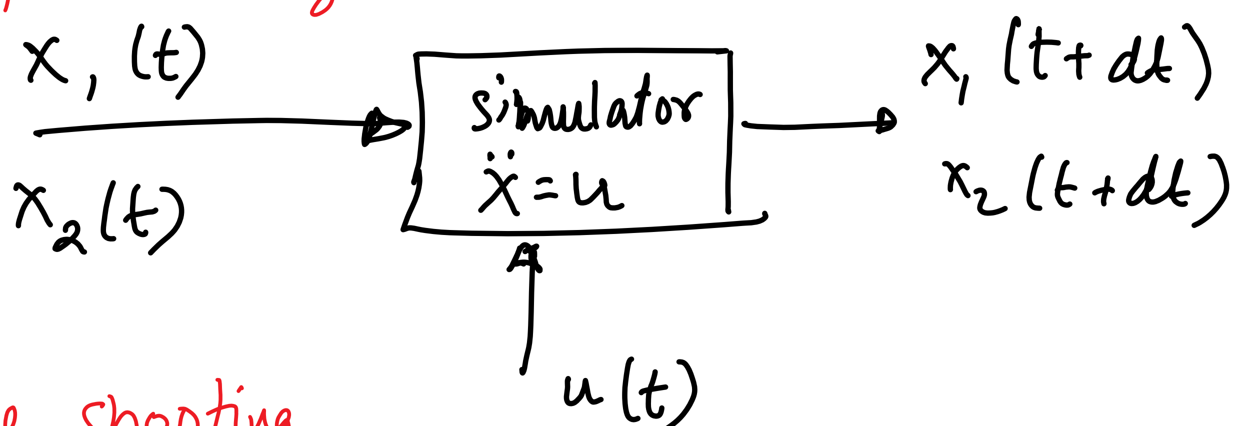
b) Single shooting method

- Treats the dynamics as a black-box

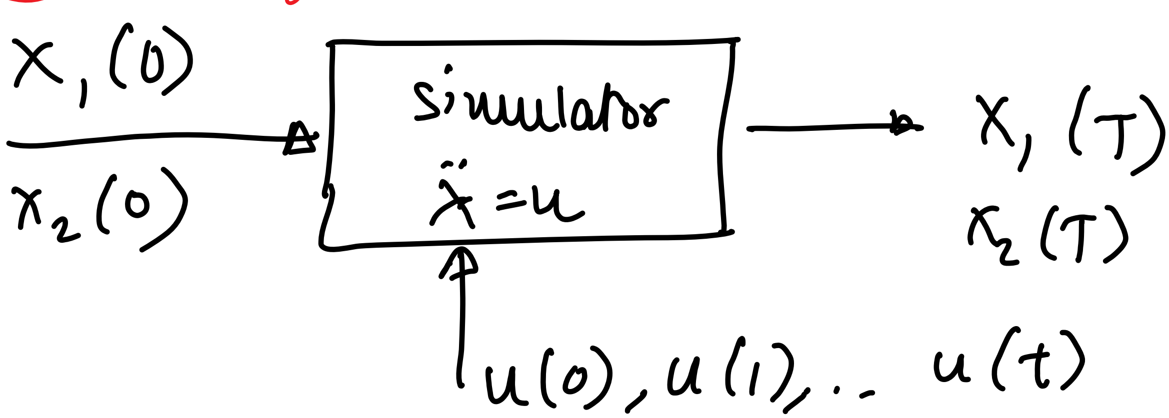
useful when you don't have access to the equations of the system (e.g. simulator)

e.g. $\left.\dot{x}_1 = x_2 \; ; \; \dot{x}_2 = u \right\} \rightarrow \ddot{x} = u$

Multiple shooting

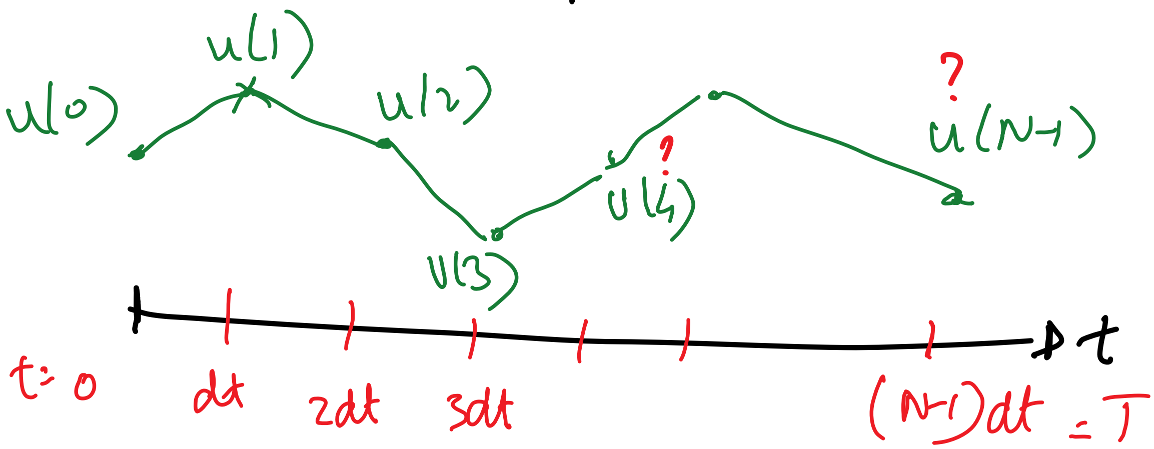


Single shooting



Formulation

① Choose N grid points (~ collocation)



Optimization variables

$\underbrace{T}_{1}, \quad \underbrace{u[0], u[1], \ldots \quad u[N-1]}_{N}$ $\quad N+1$

② Cost : $\min T$

③ Constraints

✓ $x_1(0) = 0$

✓ $x_2(0) = 0$ ensured in simulator

$x_1(T) = D = 5$

$x_2(T) = 0$ $\Big\}$ simulator (output)

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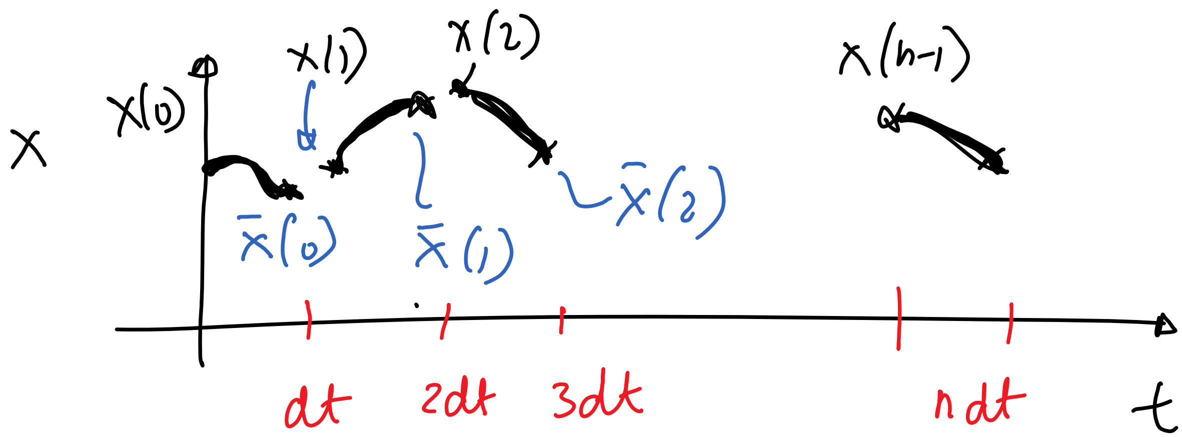
Multiple shooting



$x(0)$ — $x_1(0)$, $x_2(0)$

$x(1)$

$x(2)$

opt. variables

$\bar{x}(i)$ is obtained by integrating equation starting from $x(i)$, $u(i)$

Constraints

$$\bar{x}(0) = x(1)$$
$$\bar{x}(1) = x(2)$$
$$\vdots$$