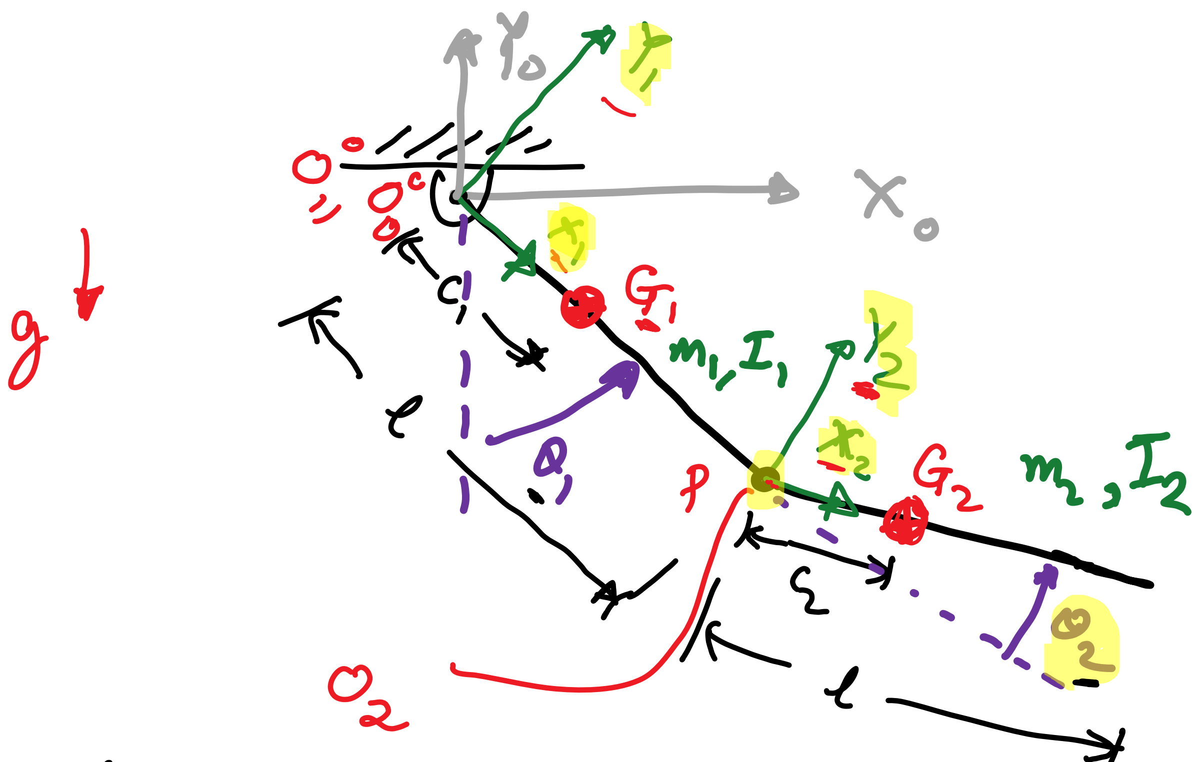
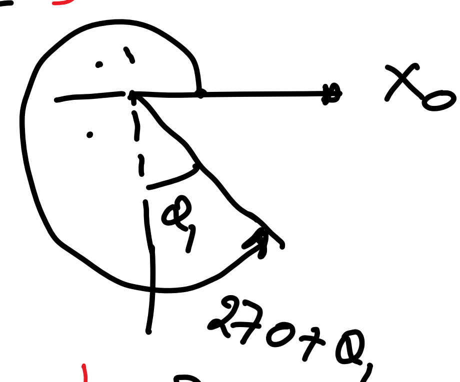
Double pendulum: derivation, simulation



① Position of the center of mass in frame $O X_0 Y_0$

$$G_1^0 = H_1^0 \, G_1^1 \qquad G_1^1 = \begin{bmatrix} c_1 \\ 0 \\ 1 \end{bmatrix},$$



$$H_1^0 = \begin{bmatrix} R_1^0 & O_1^0 \\ 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{cc|c} \cos(270+\theta_1) & -\sin(270+\theta_1) & 0 \\ \sin(270+\theta_1) & \cos(270+\theta_1) & 0 \\ \hline 0 & 0 & 1 \end{array}\right]$$

$$G_1^0 = \begin{bmatrix} x^0_{G_1} \\ y^0_{G_1} \\ 1 \end{bmatrix} = \begin{bmatrix} \sin Q_1 & \cos Q_1 & 0 \\ -\cos Q_1 & \sin Q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 \sin Q_1 \\ -c_1 \cos Q_1 \\ 1 \end{bmatrix}$$

$$G_2^0 = H_2^0 \, G_2^2$$

$$= H_1^0 \, H_2^1 \, G_2^2, \qquad G_2^2 = \begin{bmatrix} c_2 \\ 0 \\ 1 \end{bmatrix}$$

position of point P in frame 1

$$H_2^1 = \begin{array}{c} \begin{array}{cc} R_2^1 & o_2^1 \end{array} \\ \begin{bmatrix} \cos Q_2 & -\sin Q_2 & l \\ \sin Q_2 & \cos Q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$G_2^0 = \begin{bmatrix} x^0_{G_2} \\ y^0_{G_2} \\ 1 \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} \begin{bmatrix} c_2 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} l \sin Q_1 + c_2 \sin(Q_1 + Q_2) \\ -l \cos Q_1 - c_2 \cos(Q_1 + Q_2) \\ 1 \end{bmatrix}$$

$$V^0_{G_1} = \begin{bmatrix} \dot{x}^0_{G_1} \\ \dot{y}^0_{G_1} \end{bmatrix} = \begin{bmatrix} c_1 \cos Q_1 \dot{Q}_1 \\ c_1 \sin Q_1 \dot{Q}_1 \end{bmatrix} = \begin{bmatrix} c_1 \cos Q_1 \omega_1 \\ c_1 \sin Q_1 \omega_1 \end{bmatrix}$$

$$V^0_{G_2} = \begin{bmatrix} \dot{x}^0_{G_2} \\ \dot{y}^0_{G_2} \end{bmatrix} = \begin{bmatrix} \omega_1 (c_2 \cos(Q_1+Q_2) + l\cos Q_1) + \omega_2 c_2 \cos(Q_1+Q_2) \\ \omega_1 (c_2 \sin(Q_1+Q_2) + l\sin Q_1) + \omega_2 c_2 \sin(Q_1+Q_2) \end{bmatrix}$$

$$\omega_1 = \dot{Q}_1 \quad ; \quad \omega_2 = \dot{Q}_2$$

2) $$T = \frac{1}{2} m_1 (V^0_{G_1})^2 + \frac{1}{2} m_2 (V^0_{G_2})^2 + \frac{1}{2} I_1 \omega_1^2 + \ldots \frac{1}{2} I_2 (\omega_1 + \omega_2)^2$$

$$(\dot{x}^0_{G_1})^2 + (\dot{y}^0_{G_1})^2$$

$$(\dot{x}^0_{G_2})^2 + (\dot{y}^0_{G_2})^2$$

$$V = m_1 g y^0_{G_1} + m_2 g y^0_{G_2}$$

$$\mathcal{L} = T - V$$

3) Euler- lagrange Equations

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_j \qquad (q_j = Q_1, Q_2)$$

$$dt\left(\frac{}{\partial \dot{q}_j}\right) \qquad \frac{}{\partial q_j}$$

$$Q_j = 0$$

When simplified:

$$\left.\begin{aligned} A_{11}\ddot{\theta}_1 + A_{12}\ddot{\theta}_2 &= b_1 \\ A_{21}\ddot{\theta}_1 + A_{22}\ddot{\theta}_2 &= b_2 \end{aligned}\right\} \text{ simplify}$$

A's are functions of θ_1, θ_2

b's are functions of $\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2$

$$A\ddot{\theta} = b$$

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} \qquad \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\ddot{\theta} = A^{-1} b$$