2D Dynamics

Newton's law

$F = ma$ $\quad$ a: acceleration

$T = I\alpha$ $\quad$ α = angular acceleration

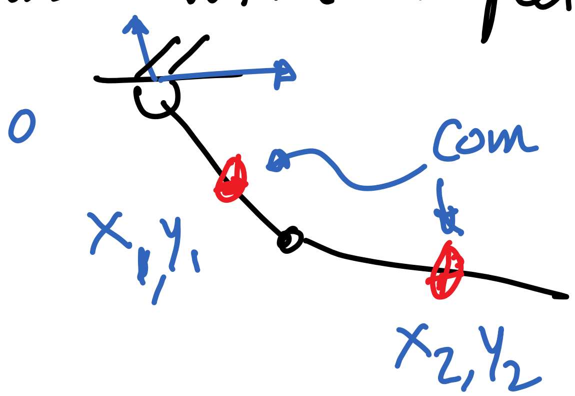
1. Free Body Diagram
2. $F = ma$ or $T = I\alpha$
3. Given F, m, T, I solve for a, α

Euler-lagrange method

- Get equations of motion without drawing Free Body Diagrams.

Procedure

① Write positions of the center of mass with respect to the fixed frame



② $\mathcal{L} = T - V$ → Potential energy

→ Kinetic energy

lagrangian

$$T = \frac{1}{2} \sum_{i=1}^{n} \left(m_i v_i^2 + I_i \omega_i^2 \right)$$

m_i - mass ; v_i - linear speed

I_i - inertia ; ω_i - angular speed

$$V = \sum_{i=1}^{n} m_i g_i y_i + 0.5 \sum_{i=1}^{q} k_p (r_p - r_{p0})^2$$

g_i - gravity

y_i - y position of center of mass

r_p - spring length

r_{p0} - rest length

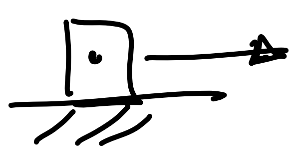
k_p - spring constant

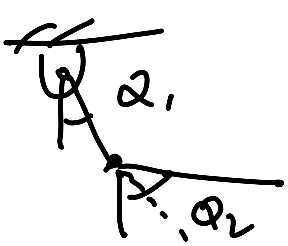
(3) Equations of motion

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = Q_j}$$

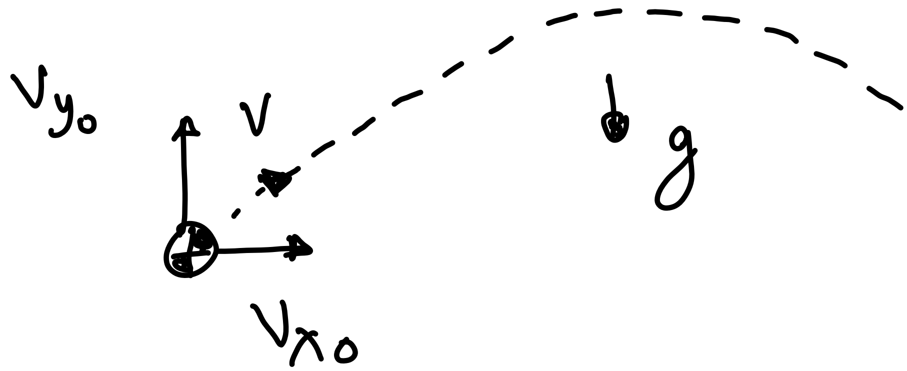
Newtons law

q_j - degrees of freedom

e.g.  $q_j = x$

 $q_j = \varphi_1, \varphi_2$

Example: projectile motion



There is a quadratic drag force

$$F_d = -c\,v^2\,\hat{v}$$

$$-c\left(\dot{x}^2+\dot{y}^2\right)\frac{\dot{x}\,\hat{i}+\dot{y}\,\hat{j}}{\sqrt{\dot{x}^2+\dot{y}^2}}$$

$$F_{dx} = -c\sqrt{\dot{x}^2+\dot{y}^2}\;\dot{x}$$

$$F_{dy} = -c\sqrt{\dot{x}^2+\dot{y}^2}\;\dot{y}$$

1) $x, y,$ - position
$\dot{x}, \dot{y},$ - velocity

2) $\mathcal{L} = T - V$

$T = 0.5\, m v^2 = 0.5\, m(\dot{x}^2 + \dot{y}^2)$

$V = m g y$

$\mathcal{L} = T - V = 0.5\, m(\dot{x}^2 + \dot{y}^2) - m g y$

3 $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = Q_j$

$q_j = x$

$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) - \frac{\partial \mathcal{L}}{\partial x} = F_{dx}$

$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{y}}\right) - \frac{\partial \mathcal{L}}{\partial y} = F_{dy}$

$$\mathcal{L} = T - V = 0.5\,m(\underline{\dot{x}^2} + \dot{y}^2) - mgy$$

$$\frac{d}{dt}\left(\underline{\underline{0.5\,m(2\dot{x})}} \right) - 0 = -c\dot{x}\sqrt{\dot{x}^2+\dot{y}^2}$$

$$\boxed{m\ddot{x} = -c\dot{x}\sqrt{\dot{x}^2+\dot{y}^2}}$$

$$\frac{d}{dt}\left(0.5\,m(2\dot{y}) \right) + mg = -c\dot{y}\sqrt{\dot{x}^2+\dot{y}^2}$$

$$\boxed{m\ddot{y} + mg = -c\dot{y}\sqrt{\dot{x}^2+\dot{y}^2}}$$

$$\boxed{\begin{aligned} \ddot{x} &= -\frac{c}{m}\,\dot{x}\sqrt{\dot{x}^2+\dot{y}^2} \\ \ddot{y} &= -g - \frac{c}{m}\,\dot{y}\sqrt{\dot{x}^2+\dot{y}^2} \end{aligned}}$$

(4) Simulate and animate in python

→ integrate the equations.

* i) Euler's method

✓ ii) Runge-kutta method

✓ (iii) Adaptive Runge-Kutta method

odeint in python

z = odeint(projectile_rhs, z_0, t, arguments)

return $\dot{x}, \ddot{x}, \dot{y}, \ddot{y}$

$-\frac{c}{m}\dot{x}\sqrt{\dot{x}^2+\dot{y}^2}$

initial condition

x_0, y_0

$\vdots \quad \vdots$

times at which we need data

m, g, c

$-g - \frac{c}{m}\dot{y}\sqrt{\dot{x}^2+\dot{y}^2}$ x_0, y_0