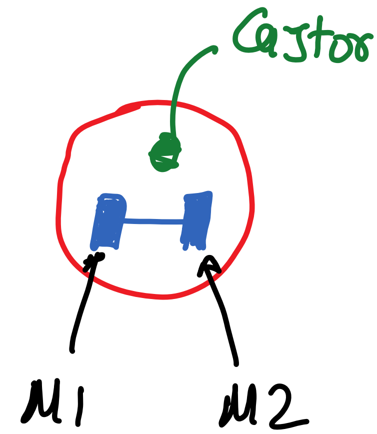
# Differential Drive car




## Forward Kinematics

cannot go sideways.

Go straight
M1, M2 same speed

Turn
$M_1, M_2$ different speeds

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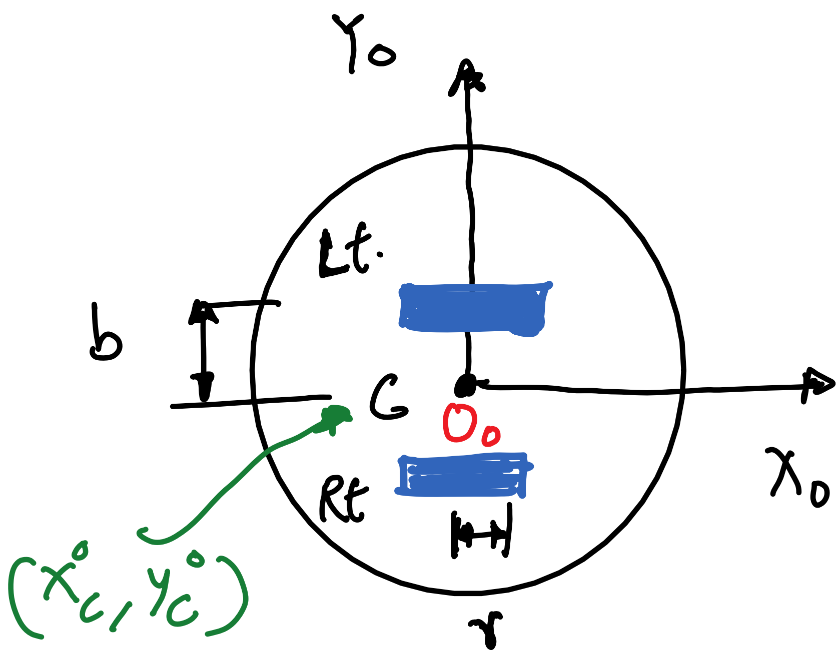
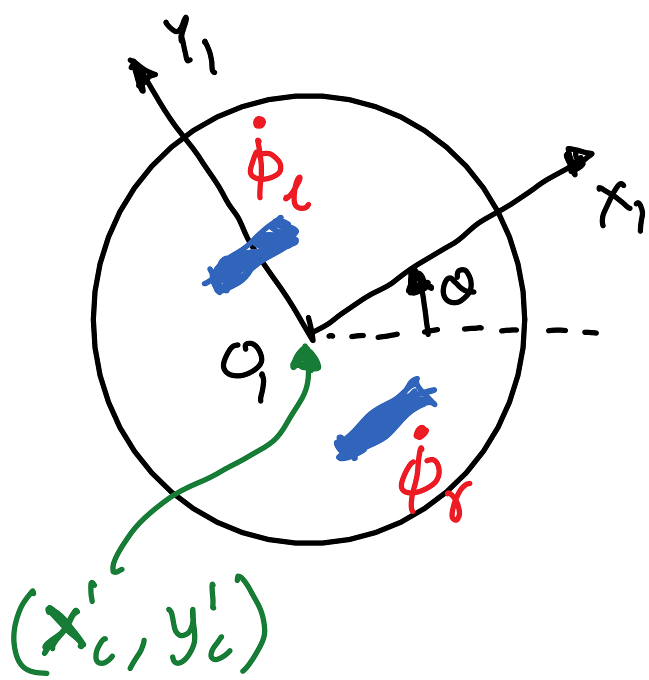


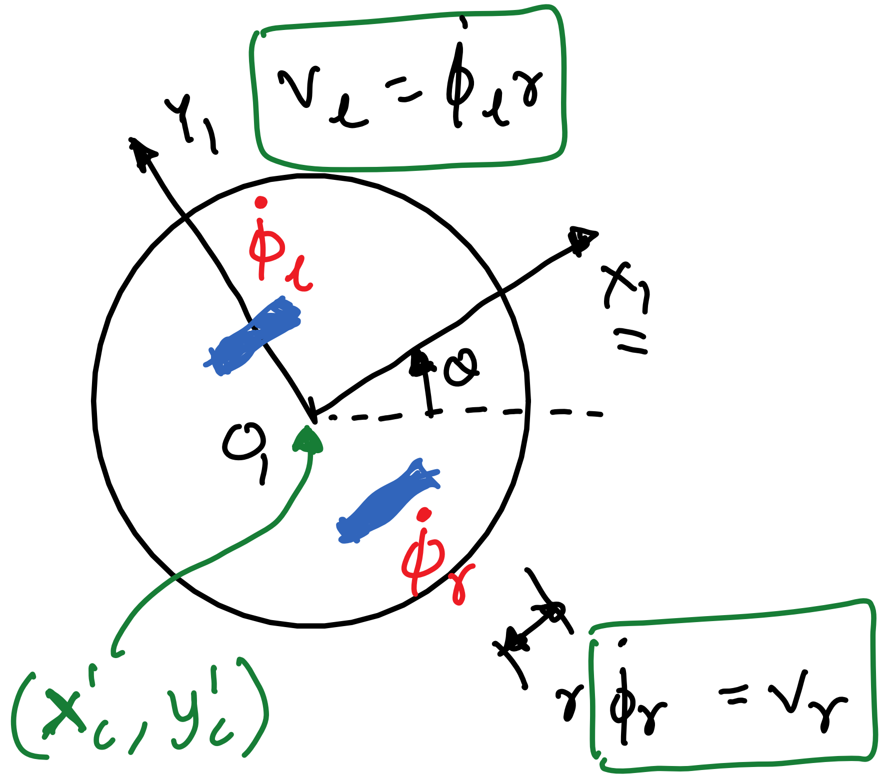

t = 0



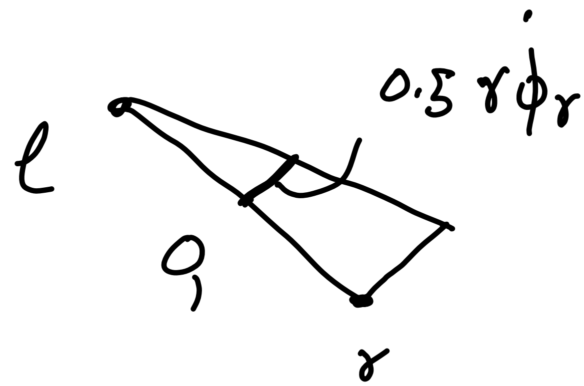

time t

$t = 0$

Time : $t$




time $t$




$$\dot x'_c = 0.5 r\dot\phi_r + 0.5 r\dot\phi_l$$

$$\dot x'_c = 0.5 r(\dot\phi_r + \dot\phi_l)$$

$$\dot y'_c = 0$$

$$\begin{bmatrix} \dot{x}_c^0 \\ \dot{y}_c^0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \dot{x}_c^1 \\ \dot{y}_c^1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0.5r(\dot{\phi}_l + \dot{\phi}_r) \\ 0 \end{bmatrix}$$

$$\boxed{\begin{aligned} \dot{x}_c^0 &= \overbrace{0.5r(\dot{\phi}_r + \dot{\phi}_l)}^{v} \cos\theta \\ \dot{y}_c^0 &= 0.5r(\dot{\phi}_r + \dot{\phi}_l) \sin\theta \end{aligned}} \quad \text{— I}$$

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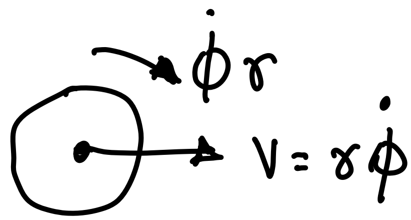
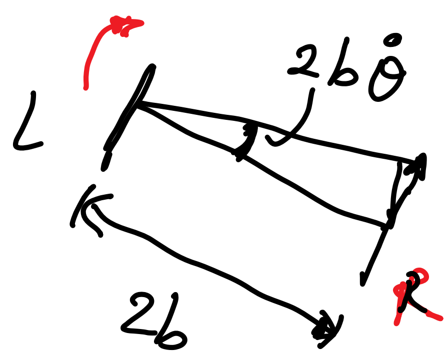
$\dot{\theta} = ?$




$$2b\,\dot{\theta} = r\dot{\phi}_r$$

$\dot{\phi}_l = 0$ $$\dot{\theta} = \frac{r\dot{\phi}_r}{2b}$$




$\dot{\phi}_r = 0$ $$\dot{\theta} = \frac{-r\dot{\phi}_l}{2b}$$

$$\boxed{\dot{\theta} = \frac{r}{2b}\underbrace{(\dot{\phi}_r - \dot{\phi}_l)}_{\omega}} \quad \text{— II}$$

$\alpha b$

$$\boxed{\dot{\theta} = \frac{r}{2b}\underbrace{(\dot{\phi}_r - \dot{\phi}_l)}_{\omega}} \quad \text{II}$$

$$\boxed{\begin{aligned} \dot{x}_c^o &= v\cos\theta \\ \dot{y}_c^o &= v\sin\theta \\ \dot{\theta} &= \omega \end{aligned} \quad \Bigg\| \quad \begin{aligned} v &= 0.5r(\dot{\phi}_r + \dot{\phi}_l) \\ \omega &= \frac{0.5r}{b}(\dot{\phi}_r - \dot{\phi}_l) \end{aligned}}$$

Forward kinematics: Compute $x_c^o$, $y_c^o$, $\theta$

At $t=0$ ; $x_c = x_c^o(0)$; $y_c = y_c^o(0)$

We know $v(t)$, $\omega(t)$

We can integrate the equations to compute $x_c^o(t)$, $y_c^o(t)$, $\theta(t)$

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We use Euler's method to integrate the equations of motion

$$\dot{x}_c^o = v\cos\theta$$

$$\frac{x_c^o(t_{i+1}) - x_c^o(t_i)}{h} = v(t_i)\cos(\theta(t_i))$$

$$\boxed{\begin{aligned} x_c^o(t_{i+1}) &= x_c^o(t_i) + h\,v(t_i)\cos(\theta(t_i)) \\ y_c^o(t_{i+1}) &= y_c^o(t_i) + h\,v(t_i)\sin(\theta(t_i)) \\ \theta(t_{i+1}) &= \theta(t_i) + h\,\omega(t_i) \\ h &= t_{i+1} - t_i \end{aligned}} \quad \text{III}$$

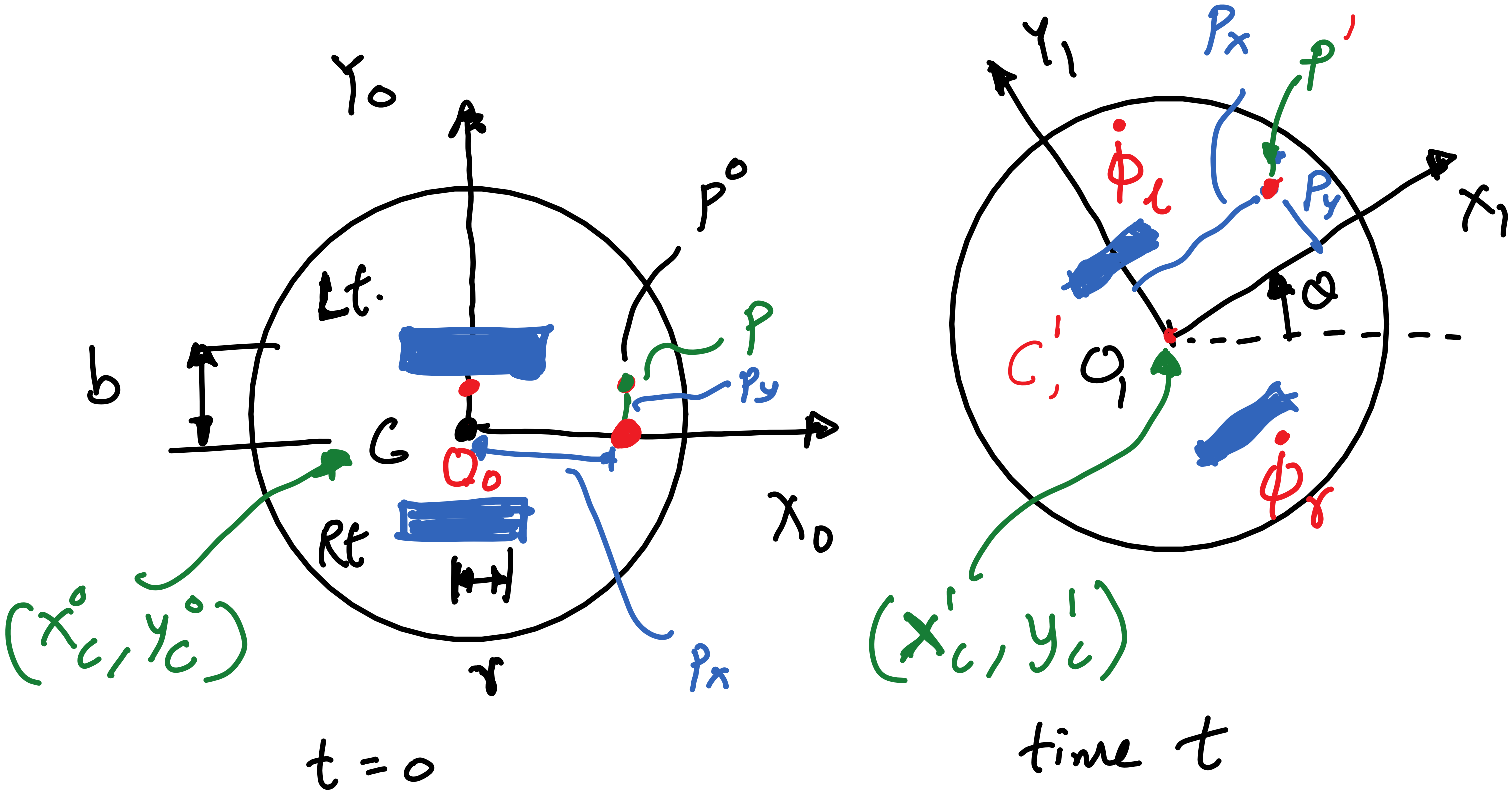
Given $x_c^o(t_0)$, $y_c^o(t_0)$, $\theta(t_0)$

$v(t_i)$, $\omega(t_i)$

we use III to compute

$x_c^o(t_i)$; $y_c^o(t_i)$; $\theta(t_i)$

# Inverse kinematics of a differential drive car




Goal : To get $p$ to track a reference curve

$$p^0 = R_1^0 \, p^1$$

$$c^0 = R_1^0 \, c^1$$

$$p^0 - c^0 = R_1^0 \, (p^1 - c^1) \equiv \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$\begin{bmatrix} x_p^o - x_c^o \\ y_p^o - y_c^o \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix} \quad \text{— I}$$

$$x_p^o = \underline{x_c^o} + p_x \cos\theta - p_y \sin\theta$$

$$y_p^o = \underline{y_c^o} + p_x \sin\theta + p_y \cos\theta$$

Diff. with respect to time

$$\dot{x}_p^o = \dot{x}_c^o + p_x (-\sin\theta)\dot{\theta} - p_y \cos\theta (\dot{\theta})$$

$$\dot{y}_p^o = \dot{y}_c^o + p_x (\cos\theta)\dot{\theta} + p_y (-\sin\theta)(\dot{\theta})$$

II

$$\dot{x}_p^o = v\cos\theta - p_x \underline{\sin\theta}\, \omega - p_y \underline{\cos\theta}\, \omega$$

$$\dot{y}_p^o = v\sin\theta + p_x \cos\theta\, \omega - p_y \sin\theta\, \omega$$

$$\begin{pmatrix} \dot{x}_p^o \\ \dot{y}_p^o \end{pmatrix} = \begin{bmatrix} \cos\theta & (-p_x\sin\theta - p_y\cos\theta) \\ \sin\theta & (p_x\cos\theta - p_y\sin\theta) \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

We will use feedback linearization for inverse kinematics

$$\dot{x}_p^o = \dot{x}_{ref} + k_{p_x}(x_{ref} - x_p^o)$$
$$\dot{y}_p^o = \dot{y}_{ref} + k_{p_y}(y_{ref} - y_p^o) \quad \text{(III)}$$

sensor

discussed in detail later

Put III in II

$$\underbrace{\begin{bmatrix} \cos\theta & (-p_x\sin\theta - \cos\theta\, p_y) \\ \sin\theta & (p_x\cos\theta - \sin\theta\, p_y) \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} v \\ \omega \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} \dot{x}_{ref} + k_{p_x}(x_{ref} - x_p^o) \\ \dot{y}_{ref} + k_{p_y}(y_{ref} - y_p^o) \end{bmatrix}}_{b}$$

$$A\,X = b$$
$$X = \begin{bmatrix} v \\ \omega \end{bmatrix} = A^{-1} b$$

$$A^{-1} = \begin{bmatrix} \cos\theta - \left(\frac{p_y}{p_x}\right)\sin\theta & \sin\theta + \left(\frac{p_y}{p_x}\right)\cos\theta \\ -\left(\frac{1}{p_x}\right)\sin\theta & \left(\frac{1}{p_x}\right)\cos\theta \end{bmatrix}$$

$$b = \begin{bmatrix} \dot{x}_{ref} + k_{px}(x_{ref} - x_p^{o}) \\ \dot{y}_{ref} + k_{py}(y_{ref} - y_p^{o}) \end{bmatrix}$$

(i) $p_x \neq 0$

(ii) can ignore $\dot{x}_{ref} = \dot{y}_{ref} = 0$

(iii) Feedback $k_{p_x}\underbrace{(x_{ref} - x_p)}_{\text{error in } x}$ & $k_{p_y}\underbrace{(y_{ref} - y_p)}_{\text{error in } y}$

user chosen constants.

Recipe for IK

① Get $x_{ref}$, $y_{ref}$ and optionally $\dot{x}_{ref}$, $\dot{y}_{ref}$

② compute $x_p^o$, $y_p^o$ using I

③ Solve for $\begin{bmatrix} v \\ w \end{bmatrix} = A^{\dagger} b$

④ $\dot{x}_c$, $\dot{y}_c$ and then compute $x_c$, $y_c$

⑤ Use I to compute $x_p$, $y_p$