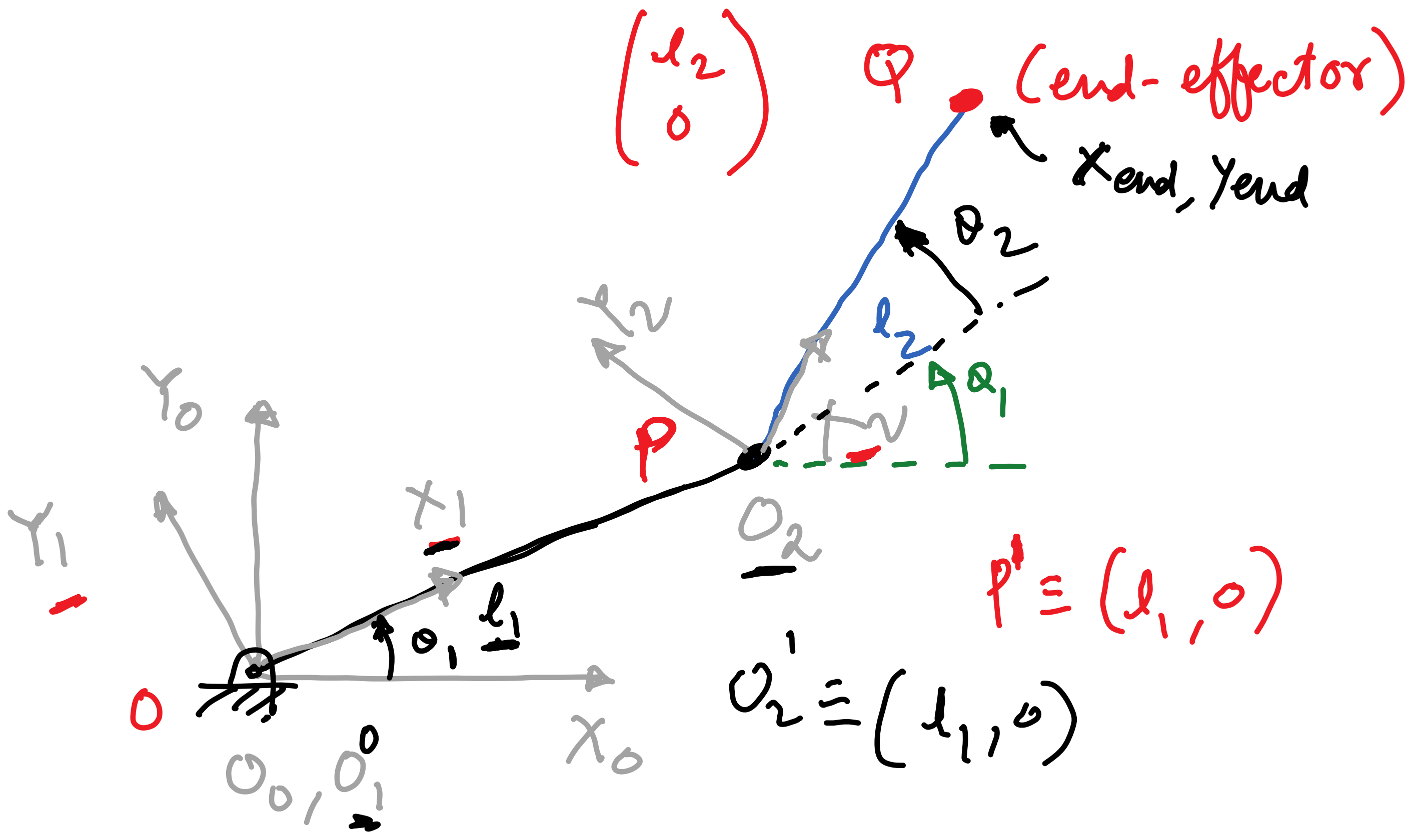
Manipulator Forward Kinematics



Forward Kinematics : (Easy)

Given θ_1, θ_2 Compute X_{end}, Y_{end}

Inverse Kinematics (Harder)

Given X_{end}, Y_{end}, Compute θ_1, θ_2

$$H_i^{i-1} = \begin{bmatrix} R_i^{i-1} & o_i^{i-1} \\ 0 & 1 \end{bmatrix}$$

p in frame 1

$$p^0 = H_1^0 \, p^1$$

$$\begin{array}{cc} & \begin{array}{cccc} & & o_1^0 & \qquad p^1 \end{array} \\ = & \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \end{array}$$

$$p^0 = \begin{bmatrix} l_1 \cos\theta_1 \\ l_1 \sin\theta_1 \\ 1 \end{bmatrix}$$

$$\left.\begin{aligned} Q^1 &= H_2^1 Q^2 \\ Q^0 &= H_1^0 Q^1 \end{aligned}\right\} \quad Q^0 = H_1^0 H_2^1 \underline{\underline{Q^2}}$$

$$\begin{array}{cc} & \begin{array}{cc} \phantom{\begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \end{bmatrix}\begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \end{bmatrix}} o_2^1 & \quad Q^2 \end{array} \\ Q^0 = & \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & l_1 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ 1 \end{bmatrix} \end{array}$$

$$P_2^0 = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ l_1 \sin\theta_1 + l_1 \sin(\theta_1 + \theta_2) \\ 1 \end{bmatrix} = \begin{bmatrix} x_{end} \\ y_{end} \\ 1 \end{bmatrix}$$

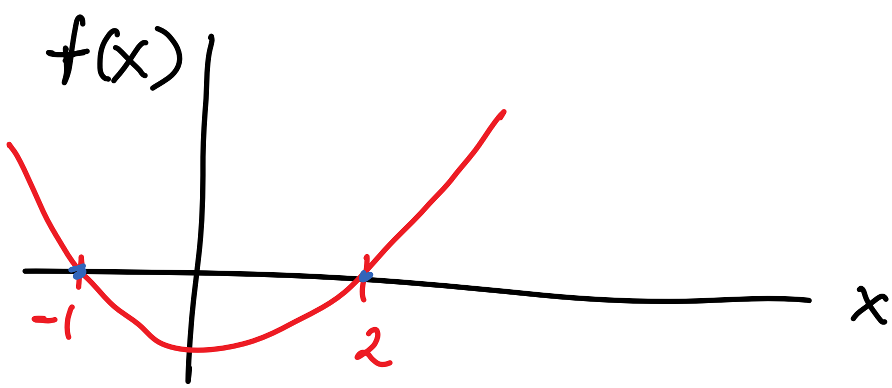
Root finding

Compute x, such that $f(x)=0$

$$f(x) = x^2 - x - 2 = 0$$

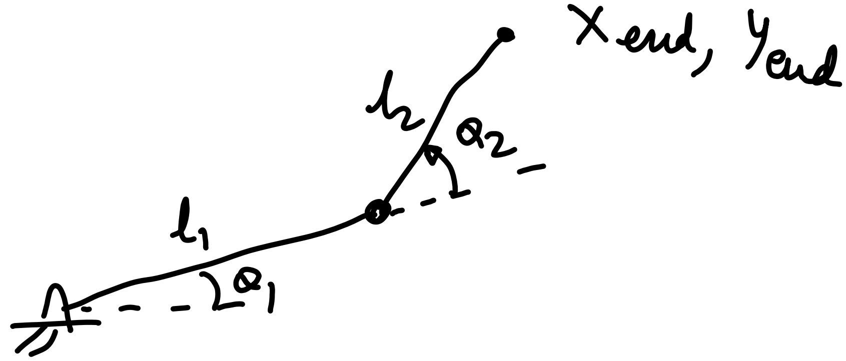
1) Sol^{n} $x = -1, 2$ guess

2) Graph $f(x)$ vs x



3) Numerical root solving: Fsolve

Inverse kinematics of a 2D manipulator



$$x_{end} = l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) = x_{ref}$$
$$y_{end} = l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) = y_{ref}$$

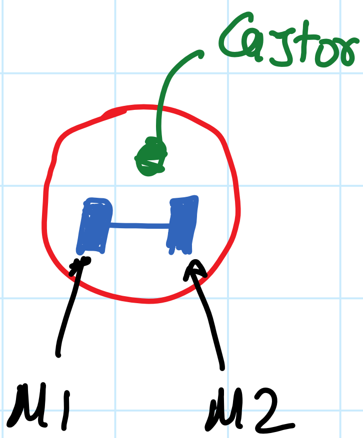
these are given

$$f(\theta_1,\theta_2) \begin{cases} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) - x_{ref} = 0 \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) - y_{ref} = 0 \end{cases}$$

Compute θ_1, θ_2 such that

$f(\theta_1,\theta_2) = 0$ 2 equations and 2 unknowns

Differential Drive car



Forward Kinematics

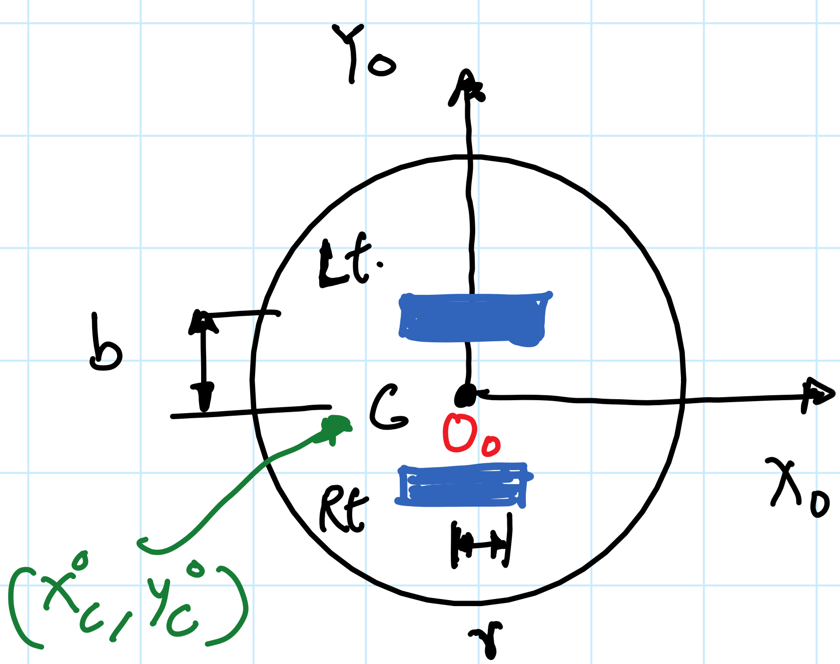
cannot go sideways.

Go straight

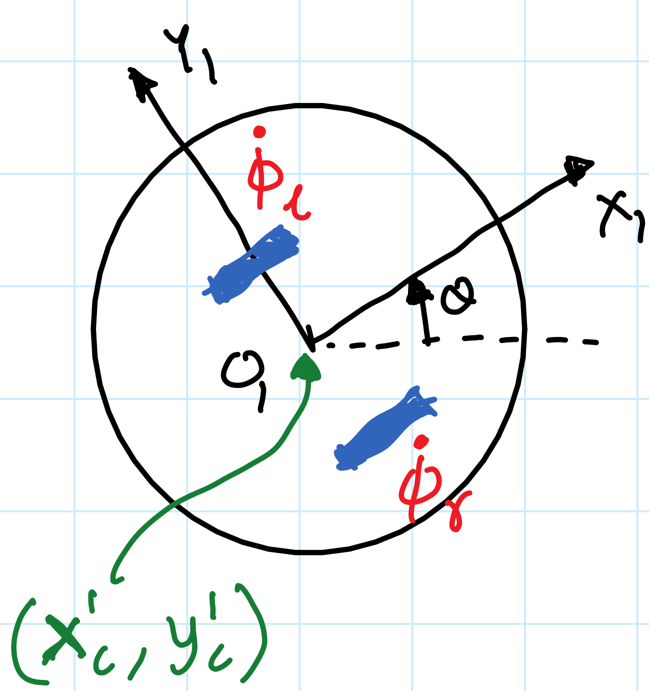
M1, M2 same speed

Turn

M_1, M_2 different speeds

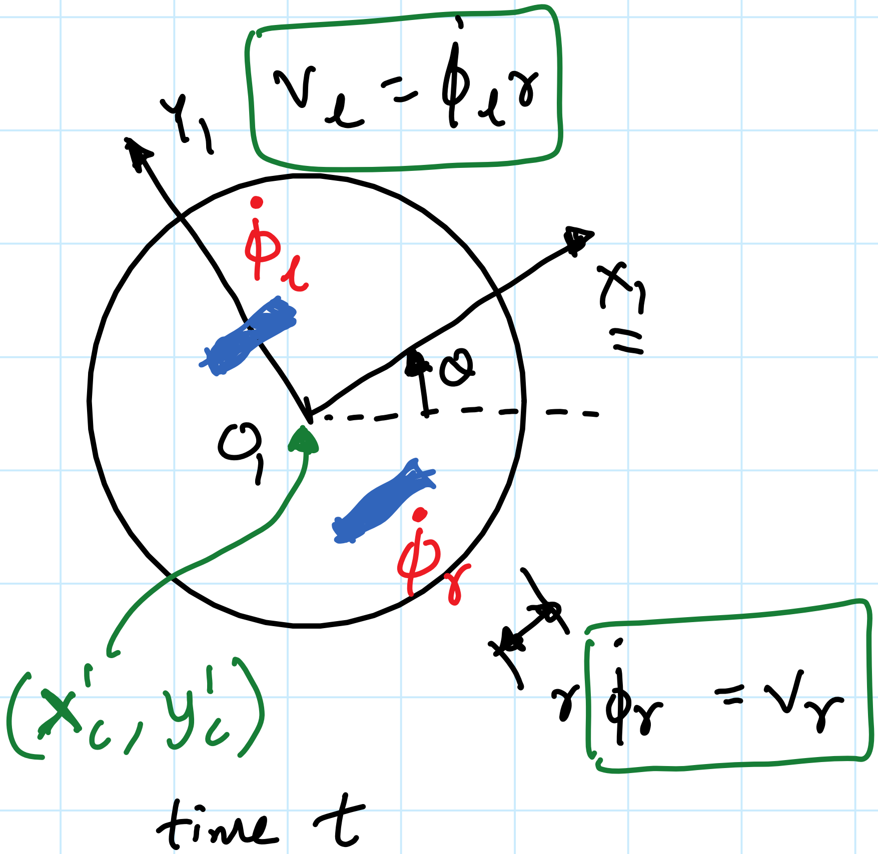


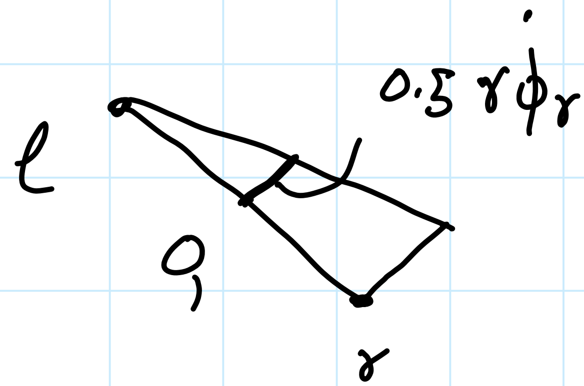
t = 0



time t

$t = 0$ Time : t





$$\dot{x}_c' = 0.5\, r\dot{\phi}_r + 0.5\, r\dot{\phi}_l$$

$$\dot{x}_c' = 0.5\, r(\dot{\phi}_r + \dot{\phi}_l)$$

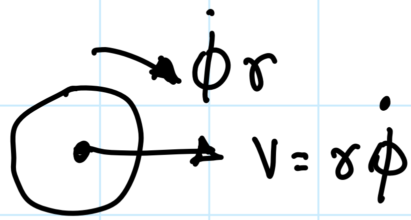
$$\dot{Y}_c' = 0$$

$$\begin{bmatrix} \dot{x}_c^0 \\ \dot{y}_c^0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \dot{x}_c^1 \\ \dot{y}_c^1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0.5r(\dot{\phi}_l + \dot{\phi}_r) \\ 0 \end{bmatrix}$$

$$\boxed{\begin{aligned} \dot{x}_c^0 &= 0.5(\dot{\phi}_r + \dot{\phi}_l)\cos\theta \\ \dot{y}_c^0 &= 0.5(\dot{\phi}_r + \dot{\phi}_l)\sin\theta \end{aligned}} \quad \text{—} \mathrm{I}$$

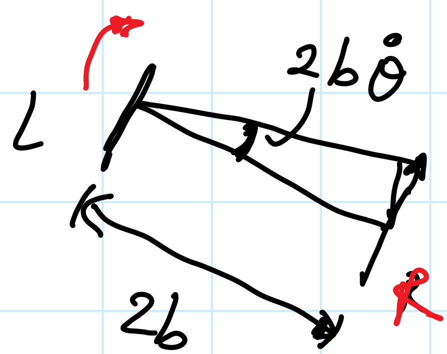
$\dot{\theta} = ?$



$$2b\,\dot{\theta} = r\dot{\phi}_r$$

$\dot{\phi}_l = 0$ $$\dot{\theta} = \frac{r\dot{\phi}_r}{2b}$$

$\dot{\phi}_r = 0$ $$\dot{\theta} = \frac{-r\dot{\phi}_l}{2b}$$



$$\boxed{\dot{\theta} = \frac{r}{2b}(\dot{\phi}_r - \dot{\phi}_l)} \quad \text{—} \mathrm{II}$$

$2b$

$$\boxed{\dot{\theta} = \frac{r}{2b}\left(\dot{\phi}_r - \dot{\phi}_\ell\right)} \quad \text{(II)}$$