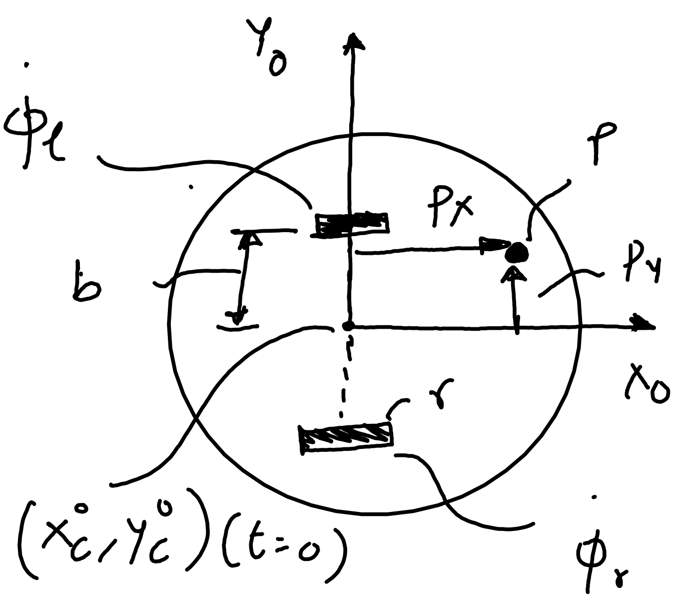
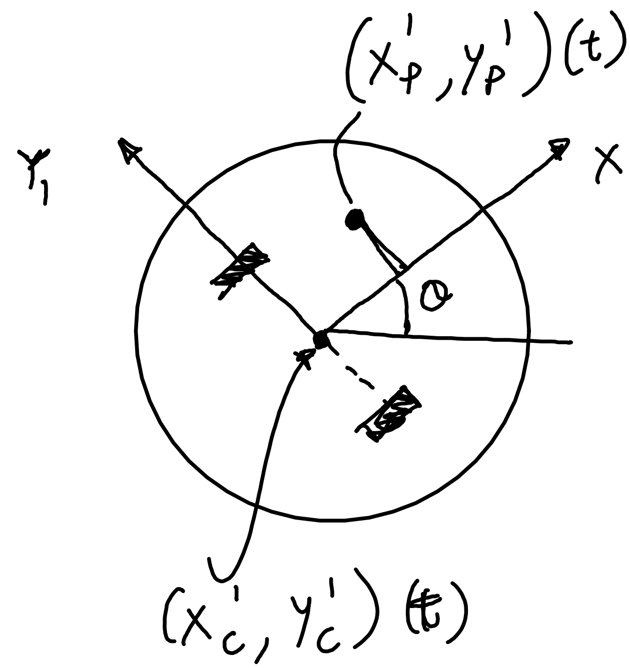
Inverse Kinematics



Position at $t=0$



Position at time t

Given x_p^0, y_p^0 as a function of time, find $\dot\phi_r$ & $\dot\phi_l$. $\xrightarrow[\text{same as}]{}$ v, ω (Definition of the inverse kinematics problem)

Goal: Find position of P in frame 0.

$$p^0 = R_1^0 \, p^1$$

$$c^0 = R_1^0 \, c^1$$

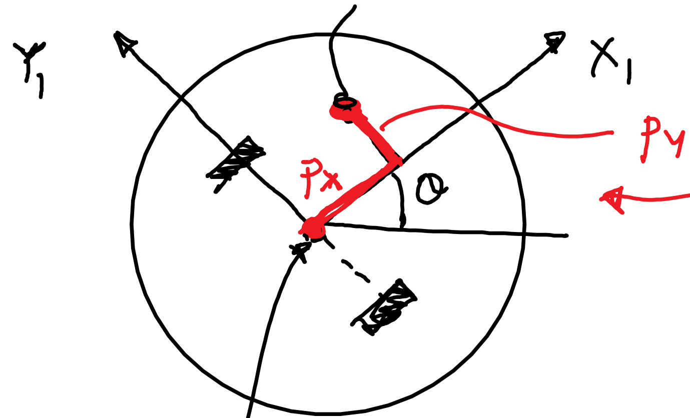
$$p^0 - c^0 = R_1^0 (p^1 - c^1)$$

$$(p^0 - c^0) = R_1^0 (p^1 - c^1)$$

$$\begin{bmatrix} x_p^0 - x_c^0 \\ y_p^0 - y_c^0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} (x_p^1 - x_c^1) \\ (y_p^1 - y_c^1) \end{bmatrix}$$

$$= \begin{bmatrix} p_x \\ p_y \end{bmatrix} = \text{fixed}$$

$(x_p^1, y_p^1)(t)$



$(x_c^1, y_c^1)(t)$

This is needed for sim

$$\begin{bmatrix} x_p^0 \\ y_p^0 \end{bmatrix} = \begin{bmatrix} x_c^0 \\ y_c^0 \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} x_c^0 \\ y_c^0 \end{bmatrix} = \begin{bmatrix} x_p^0 \\ y_p^0 \end{bmatrix} - \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix} \quad (2)$$

(1) differentiate wrt. time

$$\begin{bmatrix} \dot{x}_p^0 \\ \dot{y}_p^0 \end{bmatrix} = \begin{bmatrix} \dot{x}_c^0 \\ \dot{y}_c^0 \end{bmatrix} + \begin{bmatrix} -\sin\theta\,\dot{\theta} & -\cos\theta\,\dot{\theta} \\ \cos\theta\,\dot{\theta} & -\sin\theta\,\dot{\theta} \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_p^o \\ \dot{y}_p^o \end{bmatrix} = \begin{bmatrix} \dot{x}_c^o \\ \dot{y}_c^o \end{bmatrix} + \begin{bmatrix} -\sin\theta\,\dot\theta & -\cos\theta\,\dot\theta \\ \cos\theta\,\dot\theta & -\sin\theta\,\dot\theta \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_p^o \\ \dot{y}_p^o \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \end{bmatrix} + \begin{bmatrix} -\sin\theta\,\omega & -\cos\theta\,\omega \\ \cos\theta\,\omega & -\sin\theta\,\omega \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_p^o \\ \dot{y}_p^o \end{bmatrix} = \begin{bmatrix} v\cos\theta - \sin\theta\,\omega p_x - \cos\theta\,\omega p_y \\ v\sin\theta + \cos\theta\,\omega p_x - \sin\theta\,\omega p_y \end{bmatrix}$$

$$\underline{\underline{\begin{bmatrix} \dot{x}_p^o \\ \dot{y}_p^o \end{bmatrix}}} = \begin{bmatrix} \cos\theta & (-p_x\sin\theta - \cos\theta\, p_y) \\ \sin\theta & (p_x\cos\theta - \sin\theta\, p_y) \end{bmatrix} \underline{\underline{\begin{bmatrix} v \\ \omega \end{bmatrix}}}$$

$$\left.\begin{aligned} \dot{x}_p^o &= k_{p_x}(x_{ref} - x_p^o) \\ \dot{y}_p^o &= k_{p_y}(y_{ref} - y_p^o) \end{aligned}\right\}$$

k_{p_x}, k_{p_y} are user chosen gains

x_{ref}, y_{ref} are desired reference motion

$$\begin{bmatrix} k_{p_x}(x_{ref} - x_p^o) \\ k_{p_y}(y_{ref} - y_p^o) \end{bmatrix} = \begin{bmatrix} \cos\theta & (-p_x\sin\theta - \cos\theta\, p_y) \\ \sin\theta & (p_x\cos\theta - \sin\theta\, p_y) \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

Set v, ω such that x_p^o, y_p^o follows x_{ref}, y_{ref}

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \cos\theta - \left(\frac{p_y}{p_x}\right)\sin\theta & \sin\theta + \left(\frac{p_y}{p_x}\right)\cos\theta \\ -\left(\frac{1}{p_x}\right)\sin\theta & \left(\frac{1}{p_x}\right)\cos\theta \end{bmatrix} \begin{bmatrix} k_{px}(x_{ref} - x_p^o) \\ k_{py}(x_{ref} - y_p^o) \end{bmatrix} \quad \text{—③}$$

Also from forward kinematics

$$\left.\begin{aligned} \dot{x}_c^o &= v\cos\theta \\ \dot{y}_c^o &= v\sin\theta \\ \dot{\theta} &= \omega \end{aligned}\right\} \quad \text{—④}$$

Simulation

(i) x_{ref}, y_{ref} is given

(ii) Use ③ to solve for $\begin{bmatrix} v \\ \omega \end{bmatrix}$

(iii) Use ④ to get $\dot{x}_c^o$, $\dot{y}_c^o$, $\dot{\theta}$

(iv) Integrate ④ to get x_c^o, y_c^o

(v) Use ① to get x_p^o, y_p^o