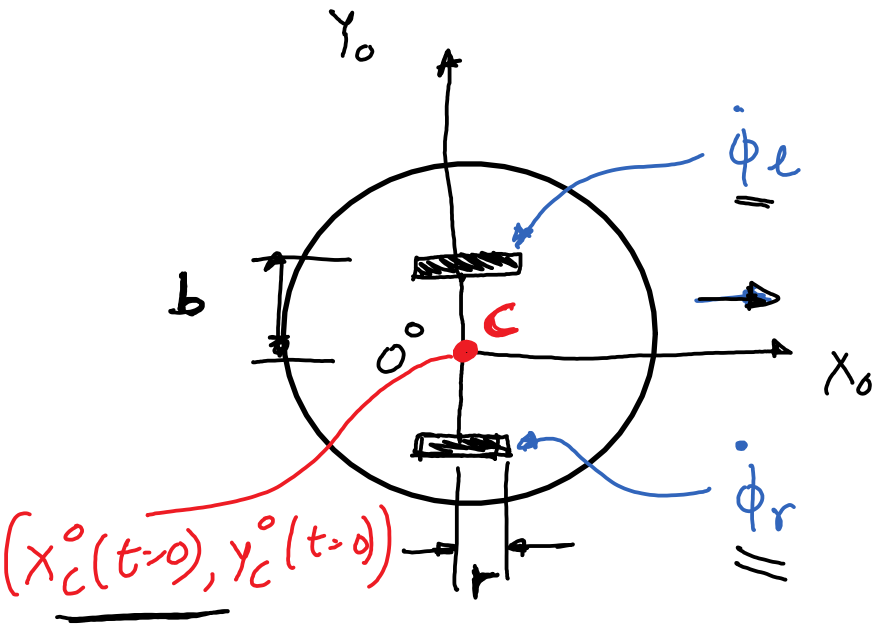
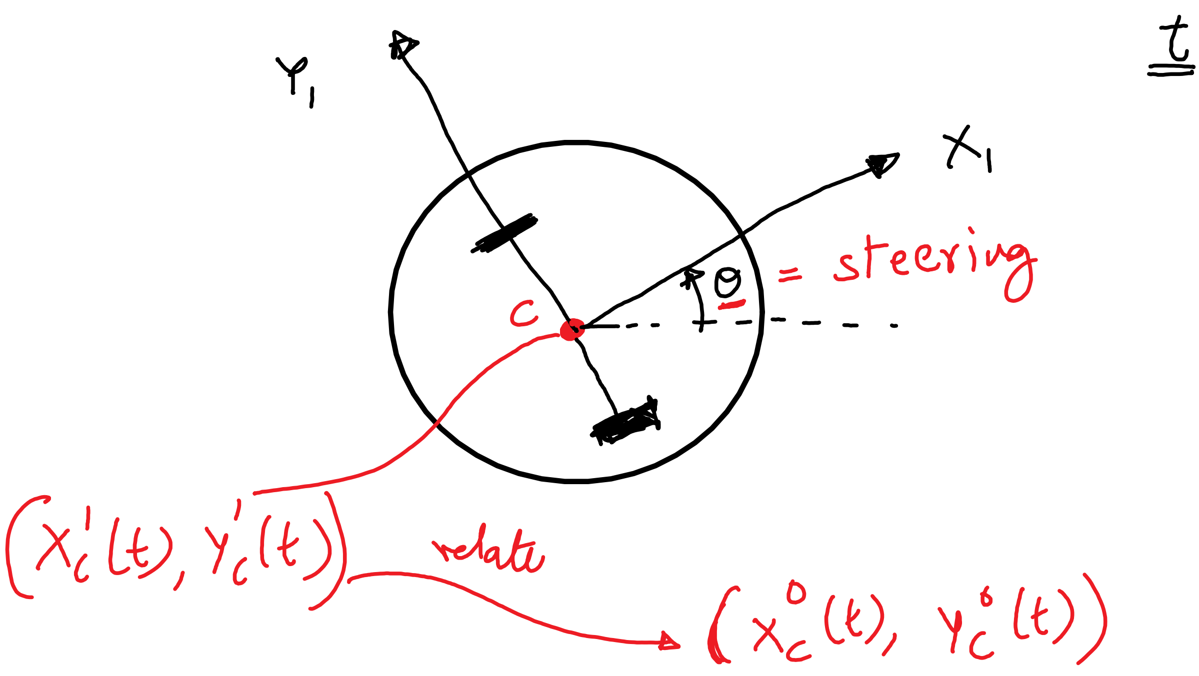
Differential Drive car



Position of the car at time $t=0$



Position of the car at time t

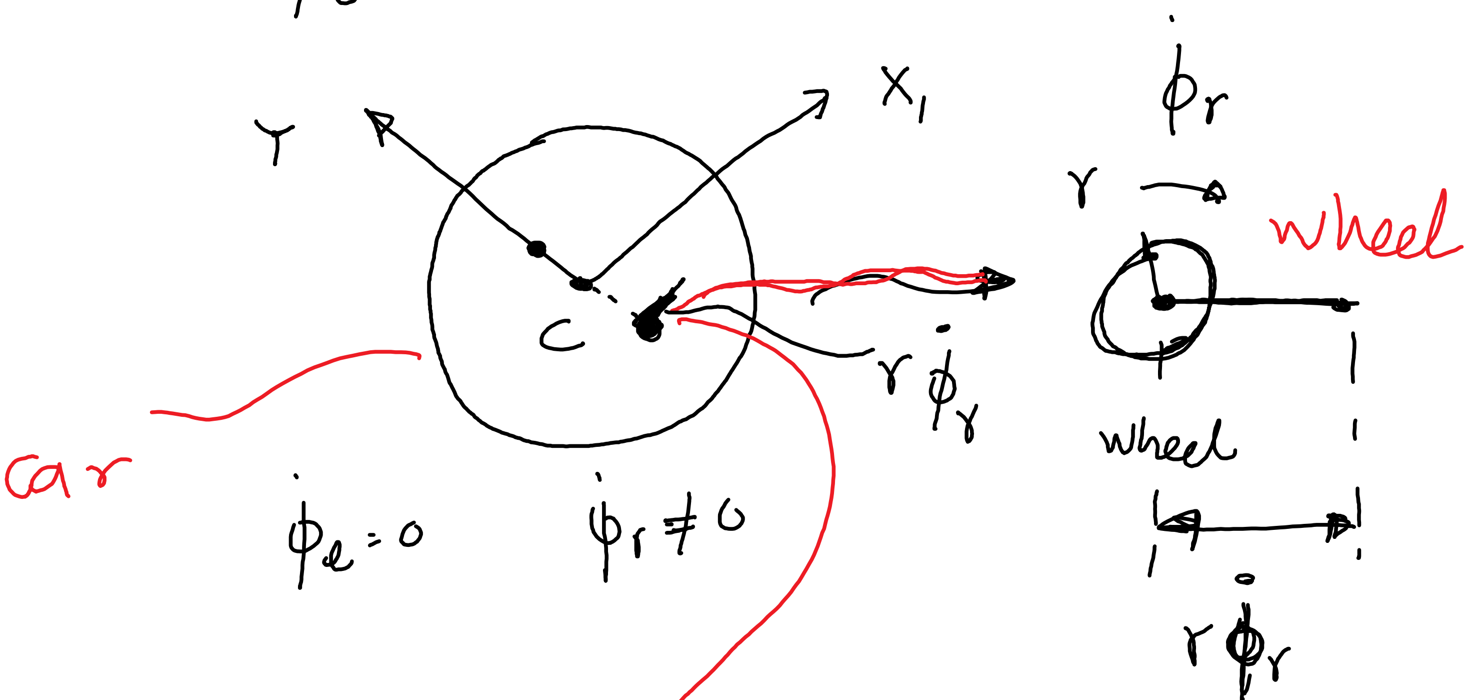
Forward kinematics

Find $x_c^0(t)$ & $y_c^0(t)$ given b, r, driving history $\dot{\phi}_r(t)$, $\dot{\phi}_l(t)$, and the initial position $x_c^0(t=0)$ & $y_c^0(t=0)$

- Unlike a manipulator where we got x_q^0, y_q^0 as a function of θ_1 & θ_2 in case of a car we will only be able to find $\dot{x}_c^0(t)$, $\dot{y}_c(t)$. We can integrate this differential equation to then solve for $x_c^0(t)$ & $y_c^0(t)$.

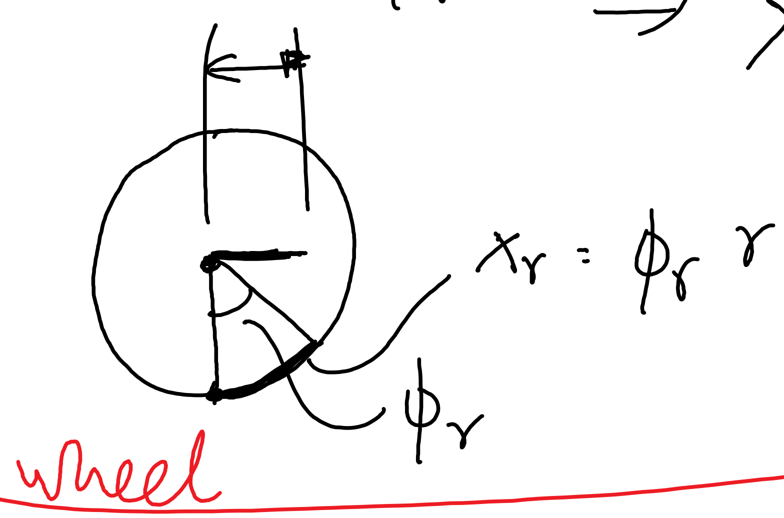
$\dot{x}_c^1(t) =$

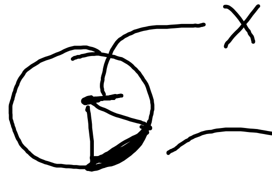
$\dot{y}_c^1(t) =$



arc length = $r\phi_r$

$$x = \phi_r r \implies \dot{x} = r\dot{\phi}_r$$

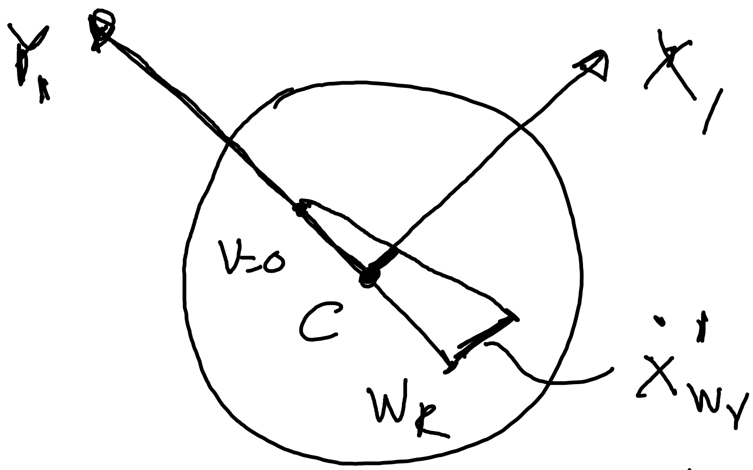




arc length: $r\phi$

$x = r\phi$

$\dot{x} = r\dot{\phi}$



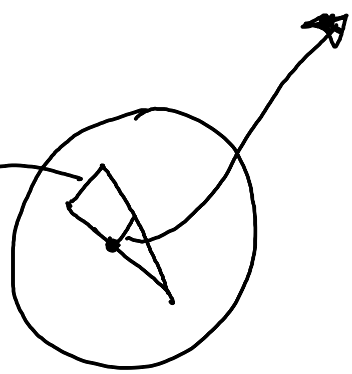
$\dot{x}_{w_r} = r\dot{\phi}_r$

$\Downarrow$

$$\begin{cases} \dot{\phi}_L = 0 \\ \dot{\phi}_R \neq 0 \end{cases} \longrightarrow \dot{x}'_c = 0.5 r \dot{\phi}_r$$

from geometry

$$\begin{cases} \dot{\phi}_R = 0 \\ \dot{\phi}_L \neq 0 \end{cases} \longrightarrow \dot{x}'_c = 0.5 r \dot{\phi}_L$$



$\dot{x}_{w_L} = r\dot{\phi}_L$

$$\Rightarrow \left.\begin{aligned} \dot{x}'_c &= 0.5 r(\dot{\phi}_r + \dot{\phi}_L) \\ \dot{y}'_c &= 0 \end{aligned}\right\}$$

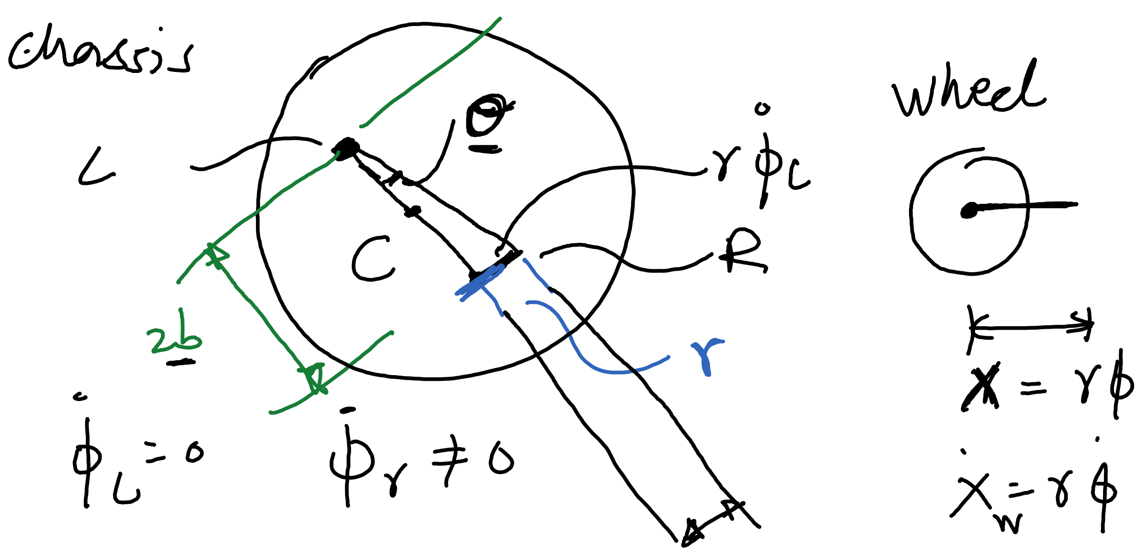
$$\dot{c}^0 = R_1^0 \, \dot{c}^1$$

$$\begin{bmatrix} \dot{x}_c^0 \\ \dot{y}_c^0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \dot{x}_c^1 \\ \dot{y}_c^1 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_c^0 \\ \dot{y}_c^0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0.5\, r\, (\dot{\phi}_r + \dot{\phi}_L) \\ 0 \end{bmatrix}$$

$$\boxed{\begin{aligned} \dot{x}_c^0(t) &= 0.5\, r\, (\dot{\phi}_r + \dot{\phi}_L) \cos\theta \\ \dot{y}_c^0(t) &= 0.5\, r\, (\dot{\phi}_r + \dot{\phi}_L) \sin\theta \end{aligned}}$$

② Derive expression for steering $\dot{\theta}$

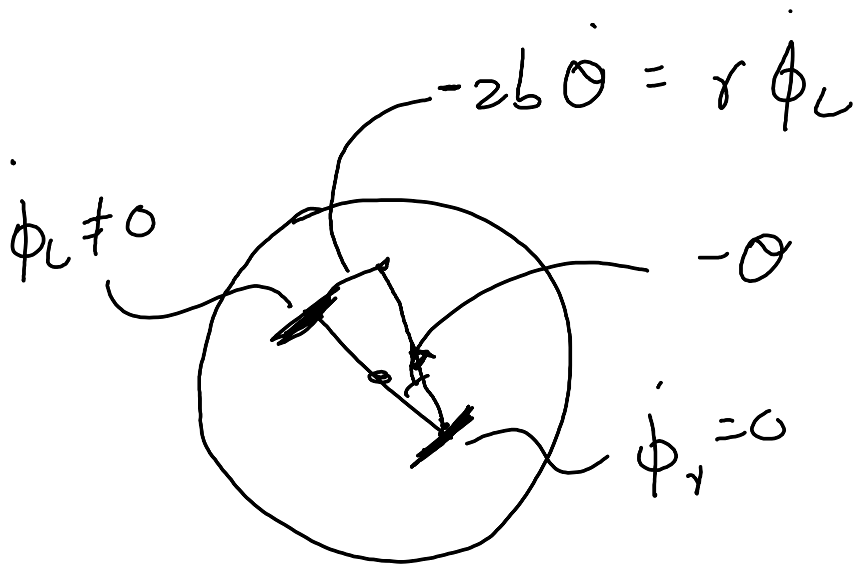


$2b\,\theta$ = arc length

$\dot{x}_w = 2b\,\dot{\theta}$

<u>wheel</u> <u>chassis</u>

$$\dot{x}_w = r\dot{\phi}_r = 2b\,\dot{\theta} \implies \dot{\theta} = \left(\frac{r}{2b}\right)\dot{\phi}_r$$



$$-2b\dot{\theta} = r\dot{\phi}_L$$

$$\dot{\theta} = \frac{-r\dot{\phi}_L}{2b}$$

$$\boxed{\dot{\theta} = \frac{r}{2b}\left(\dot{\phi}_r - \dot{\phi}_L\right)}$$

<u>Summary</u>

$$\dot{x}_c^{\circ} = 0.5r(\dot{\phi}_r + \dot{\phi}_L)\cos\theta$$

$$\dot{y}_c^{\circ} = 0.5r(\dot{\phi}_r + \dot{\phi}_L)\sin\theta$$

$$\dot{\theta} = \frac{0.5r}{b}(\dot{\phi}_r - \dot{\phi}_L)$$

$$\underline{\underline{V}} = 0.5r(\underset{?}{\dot{\phi}_r} + \underset{?}{\dot{\phi}_L}); \quad \underline{\underline{w}} = \frac{0.5r}{b}(\underset{?}{\dot{\phi}_r} - \underset{?}{\dot{\phi}_L})$$

$$\boxed{\begin{aligned} \dot{x}_c^o &= v\cos\theta \\ \dot{y}_c^o &= v\sin\theta \\ \dot{\theta} &= \omega \end{aligned}}$$

We will use this formula for simulation

To find position x_c^o, y_c^o, θ we need to integrate the equations.

$$\dot{x}_c^o = \frac{x_c(t_{i+1}) - x_c(t_i)}{t_{i+1} - t_i} = v(t_i)\cos(\theta(t_i))$$

$$\dot{y}_c^o = \frac{y_c(t_{i+1}) - y_c(t_i)}{t_{i+1} - t_i} = v(t_i)\sin(\theta(t_i))$$

$$\dot{\theta} = \frac{\theta(t_{i+1}) - \theta(t_i)}{(t_{i+1} - t_i) = h} = \omega(t_i)$$

$$\boxed{\begin{aligned} x_c^o(t_{i+1}) &= x_c^o(t_i) + h\,v(t_i)\cos(\theta(t_i)) \\ y_c^o(t_{i+1}) &= y_c^o(t_i) + h\,v(t_i)\sin(\theta(t_i)) \\ \theta(t_{i+1}) &= \theta(t_i) + h\,\omega(t_i) \end{aligned}}$$

given $x_c(t=0)$, $y_c^o(t=0)$, $\theta(t=0)$