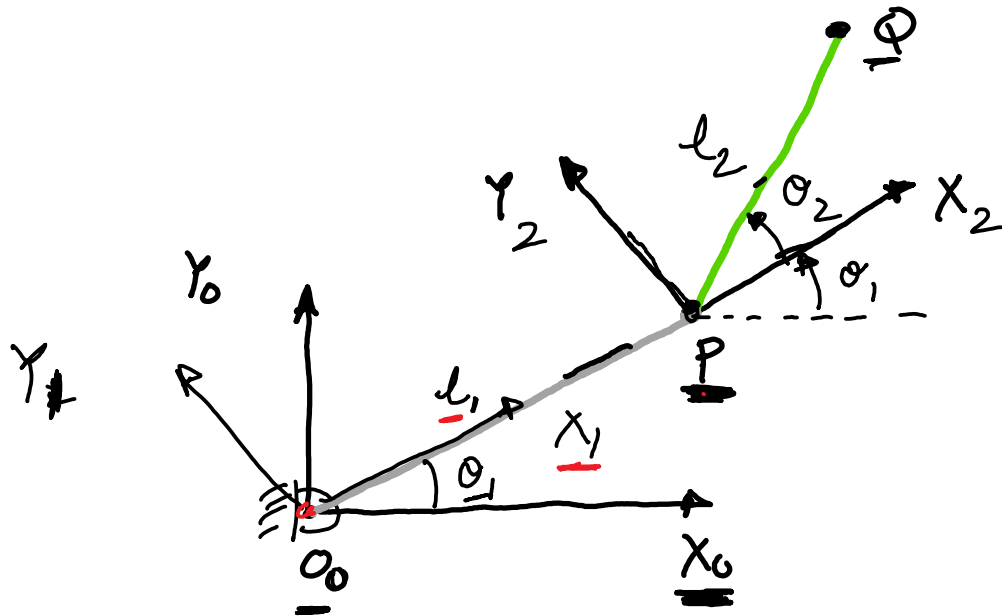


2D manipulator



Forward Kinematics

- Find the position of the end-effector (Q) given the joint angles θ_1 and θ_2

$$P^0 = R_1^0 P^1$$

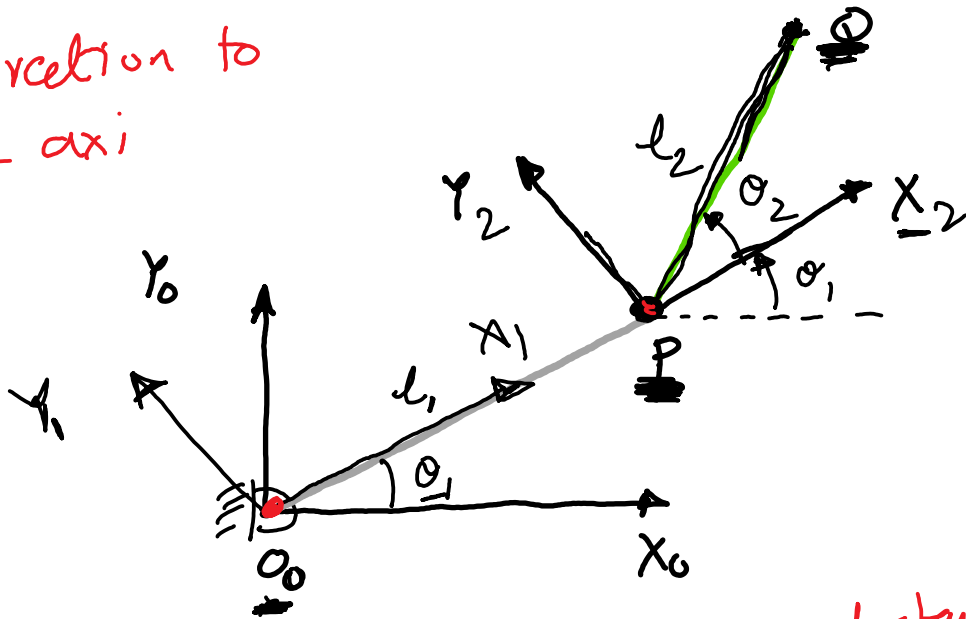
$$\begin{bmatrix} x_p^0 \\ y_p^0 \end{bmatrix} = \begin{bmatrix} \cos \theta_1 \\ \sin \theta_1 \end{bmatrix} \begin{bmatrix} x_p^1 \\ y_p^1 \end{bmatrix}$$

$$\begin{bmatrix} -\sin \theta_1 \\ \cos \theta_1 \end{bmatrix} \begin{bmatrix} x_p^1 \\ y_p^1 \end{bmatrix} \begin{matrix} \leftarrow \text{distance} \\ \text{of } P \\ \text{from origin} \\ O_0 \end{matrix} = \begin{bmatrix} l_1 \\ 0 \end{bmatrix}$$

$$\checkmark x_p^0 = l_1 \cos \theta_1$$

$$\checkmark y_p^0 = l_1 \sin \theta_1$$

Correction to the axis



$$\underline{q}^1 = R_2^1 \underline{q}^2$$

distance of Q from origin of $X_2 Y_2$ or P

$$\begin{bmatrix} x_{q^1} \\ y_{q^1} \end{bmatrix} = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 \\ \sin \theta_2 & \cos \theta_2 \end{bmatrix} \begin{bmatrix} x_q^2 \\ y_q^2 \end{bmatrix} = \begin{bmatrix} l_2 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_{q^1} \\ y_{q^1} \end{bmatrix} = \begin{bmatrix} l_2 \cos \theta_2 \\ l_2 \sin \theta_2 \end{bmatrix}$$

$$\underline{q}^0 = \underline{p}^0 + R_1^0 \underline{q}^1$$

$$\begin{bmatrix} x_{q^0} \\ y_{q^0} \end{bmatrix} = \begin{bmatrix} l_1 \cos \theta_1 \\ l_1 \sin \theta_1 \end{bmatrix} + \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 \\ \sin \theta_1 & \cos \theta_1 \end{bmatrix} \begin{bmatrix} l_2 \cos \theta_2 \\ l_2 \sin \theta_2 \end{bmatrix}$$

$$\begin{bmatrix} x_g^o \\ y_g^o \end{bmatrix} = \begin{bmatrix} l_1 \cos \theta_1 + l_2 \cos (\theta_1 + \theta_2) \\ l_1 \sin \theta_1 + l_2 \sin (\theta_1 + \theta_2) \end{bmatrix}$$

$$\begin{bmatrix} x_p^o \\ y_p^o \end{bmatrix} = \begin{bmatrix} l_1 \cos \theta_1 \\ l_1 \sin \theta_1 \end{bmatrix}$$

MATLAB program to animate the manipulator.