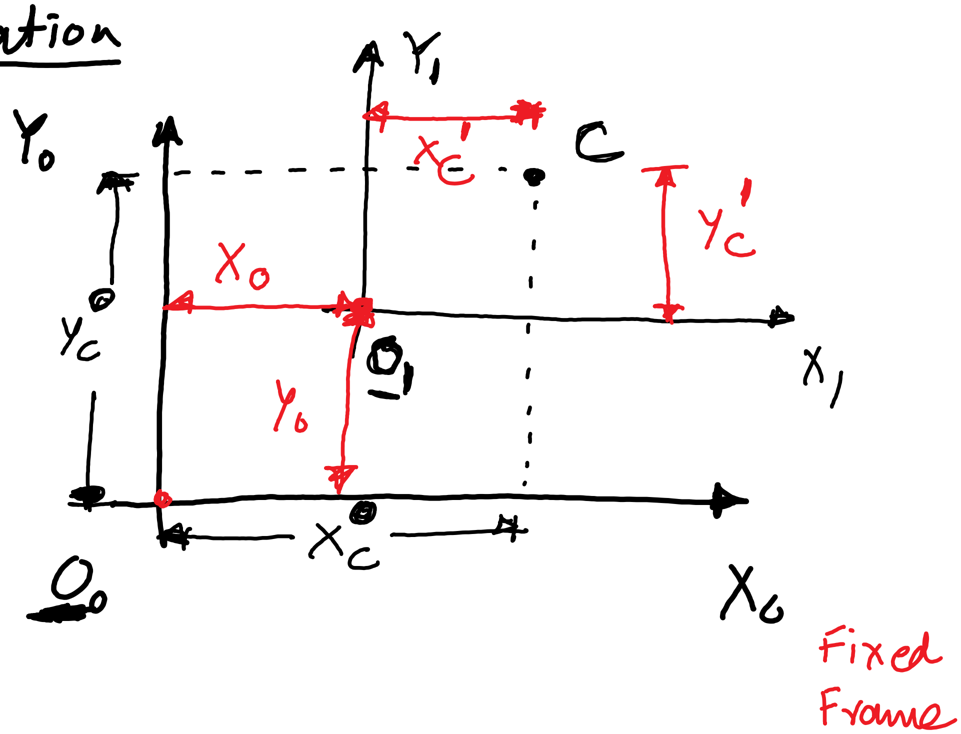
Coordinate Frames

1 Translation



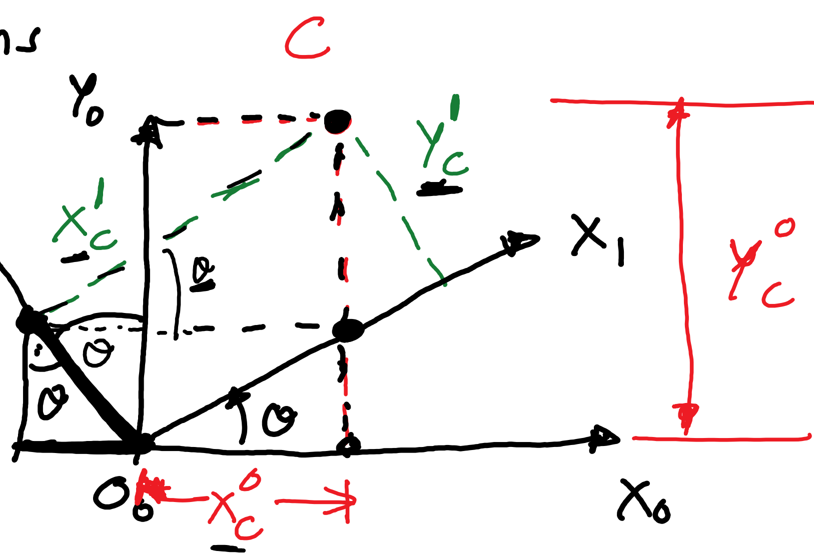
$$C^0 = \begin{bmatrix} x_c^0 \\ y_c^0 \end{bmatrix}$$ (Position of C wrt. frame $O_0 - X_0 - Y_0$)

$$C^1 = \begin{bmatrix} x_c^1 \\ y_c^1 \end{bmatrix}$$ (Position of C wrt. frame $O_1 - X_1 - Y_1$)

$$O_1^0 = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$ (Position of O_1 wrt frame $O_0 - X_0 - Y_0$)

$$O_0^1 = \begin{bmatrix} -x_0 \\ -y_0 \end{bmatrix}$$ (position of O_0 wrt frame $O_1 - X_1 - Y_1$)

2 Rotations



$$x_c^0 = x_c^1 \cos\theta - y_c^1 \sin\theta$$

$$y_c^0 = x_c^1 \sin\theta + y_c^1 \cos\theta$$

$$\begin{bmatrix} x_c^0 \\ y_c^0 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}\begin{bmatrix} x_c^1 \\ y_c^1 \end{bmatrix}$$

$$\boxed{c^0 = R_1^0 \; c^1}$$

$$\begin{bmatrix} x_c^1 \\ y_c^1 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}}\begin{bmatrix} x_c^0 \\ y_c^0 \end{bmatrix}$$

$$\boxed{c^1 = R_0^1 \; C^0}$$

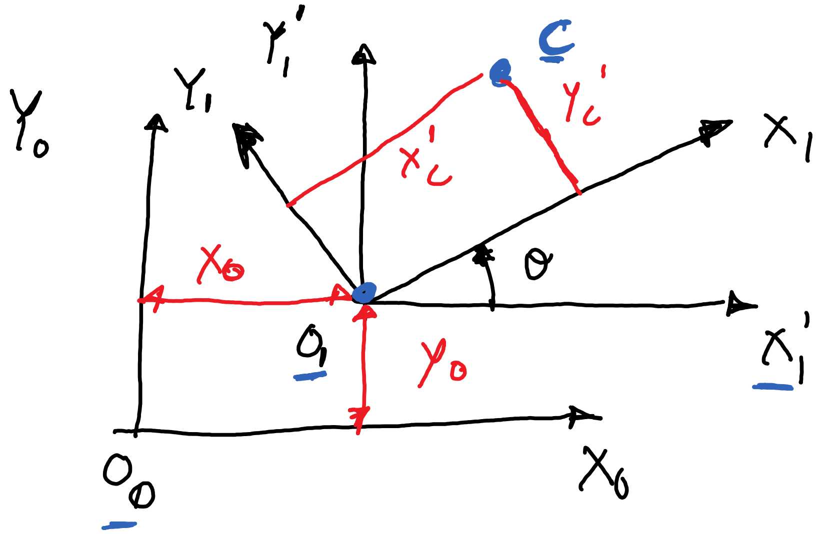
Naming

$P^{\text{frame}}_{\text{point of interest}}$

NOTE: $\boxed{R_1^0 = (R_0^1)^T}$ ✓

Property $R_1^0 (R_1^0)^T = I \Rightarrow \boxed{(R_1^0)^T = (R_1^0)^{-1}}$ ✓

3 combined translation and rotation



$$\underline{c}^{1'} = R_1^0 \, \underline{c}^1$$

position of C in frame $O_1 - X_1' - Y_1'$

$$c^0 = o_1^0 + c^{1'}$$

$$\boxed{c^0 = \underbrace{o_1^0}_{\text{translation}} + \underbrace{R_1^0 c^1}_{\text{Rotation}}}$$