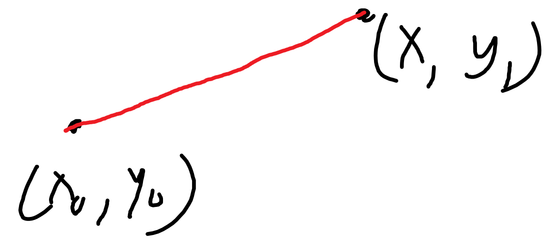
Animations of 3D manipulators

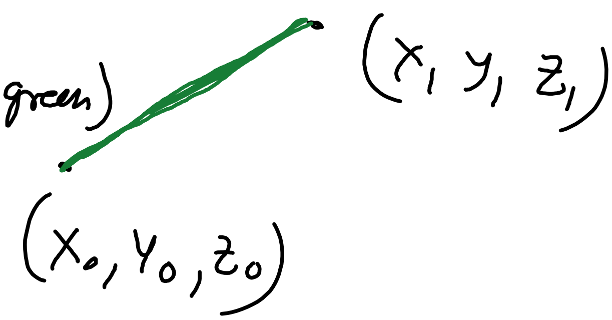
Recap: 2D

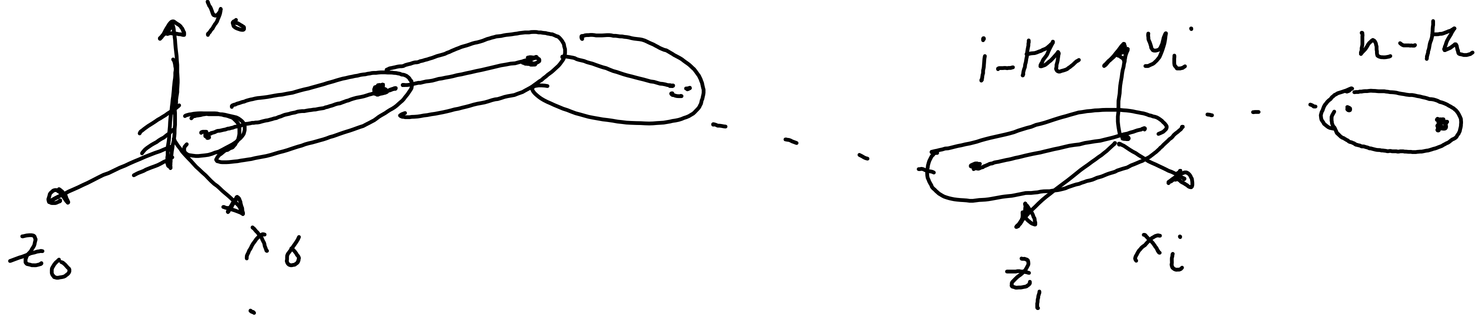
```
line ( [x0 x1], [y0, y1], 'color', 'red');
```



In 3D

```
line ([x0 x1], [y0 y1], [z0 z1], 'color', green)
```





$$H_i^{i-1} = \ldots$$

$$H_n^0 = H_1^0\, H_2^1 \ldots H_n^{n-1} = \begin{bmatrix} R_n^0 & d_n^0 \\ 0 & 1 \end{bmatrix}$$

$\rightarrow$ $d_n^0 = \{x_n, y_n\ z_n\}$

$H_i^0 = H_1^0 H_2^1 \ldots H_i^{i-1} = \begin{bmatrix} R_i^0 & d_i^0 \\ 0 & 1 \end{bmatrix}$

$d_i^0 = \{x_i, y_i, z_i\}$

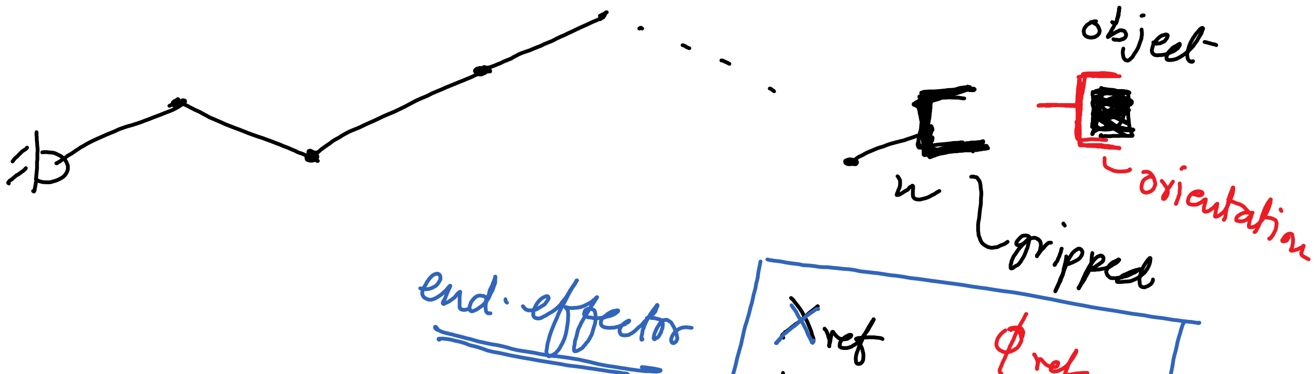
H_i^{i-1} ✓

$i = 0, 1, 2, \ldots, n$

$$\left.\begin{aligned} d_0^0 &= \{x_0, y_0, z_0\} = \{0, 0, 0\} \\ d_1^0 &= \{x_1, y_1, z_1\} \\ d_2^0 &= \{x_2, y_2, z_2\} \\ &\vdots \\ d_n^0 &= \{x_n, y_n, z_n\} \end{aligned}\right]$$

line([x_{i-1}, x_i], [y_{i-1} y_i], [z_{i-1} z_i]);

Inverse kinematics of 3D manipulator



x_{ref}	ϕ_{ref}
y_{ref}	θ_{ref}
z_{ref}	ψ_{ref}

Given $x_{ref}, y_{ref}, z_{ref}$
$\underline{\phi}_{ref}, \underline{\theta}_{ref}, \underline{\psi}_{ref}$

Find the joint angles $\rightarrow$ NEEDS atleast a 6 degree of freedom robot.

$$H_n^0 = \begin{bmatrix} R_n^0 & d_n^0 \\ 0 & 1 \end{bmatrix}$$

$$d_n^0 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \checkmark$$

$$\underline{\underline{R_n^0}} = \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}$$

9 numbers

but these should match the $\underline{\underline{\theta, \phi, \psi}}$

3-2-1

$$\mathbf{R} = \mathbf{R}_z(\psi)\mathbf{R}_y(\theta)\mathbf{R}_x(\phi)$$
$$= \begin{bmatrix} \cos(\psi)\cos(\theta) & \cos(\psi)\sin(\phi)\sin(\theta) - \cos(\phi)\sin(\psi) & \sin(\phi)\sin(\psi) + \cos(\phi)\cos(\psi)\sin(\theta) \\ \cos(\theta)\sin(\psi) & \cos(\phi)\cos(\psi) + \sin(\phi)\sin(\psi)\sin(\theta) & \cos(\phi)\sin(\psi)\sin(\theta) - \cos(\psi)\sin(\phi) \\ -\underline{\sin(\theta)} & \boxed{\underline{\cos(\theta)\sin(\phi)}} & \cos(\phi)\cos(\theta) \end{bmatrix}$$

$$= R_n^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ \boxed{r_{21}} & r_{22} & r_{23} \\ r_{31} & \boxed{\underline{r_{32}}} & r_{33} \end{bmatrix}$$

$$-\sin\theta = r_{31}$$

$$\boxed{\underline{\theta} = \sin^{-1}(-r_{31})}$$

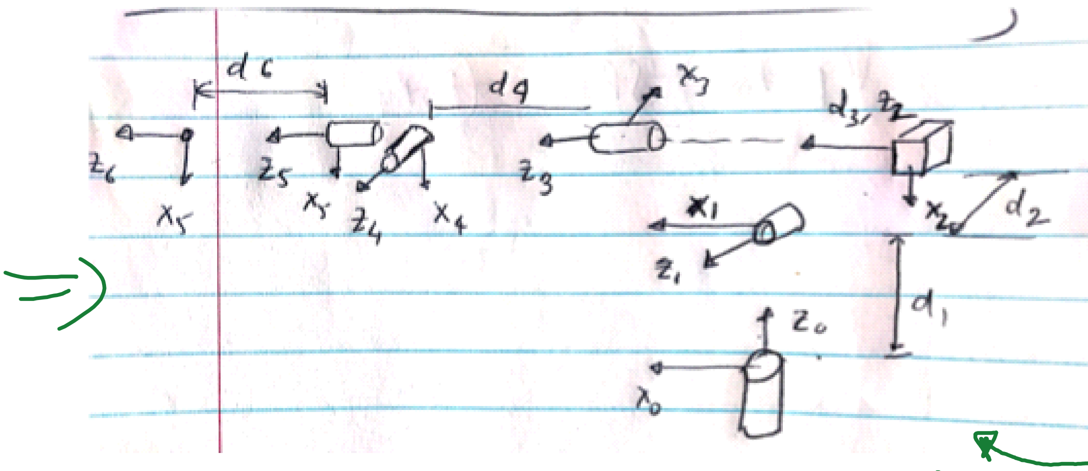
✓

$$\underline{\underline{\cos\theta}} \sin\phi = \boxed{r_{32}}$$

$$\boxed{\phi = \sin^{-1}\left(\frac{r_{32}}{\cos\theta}\right)}$$

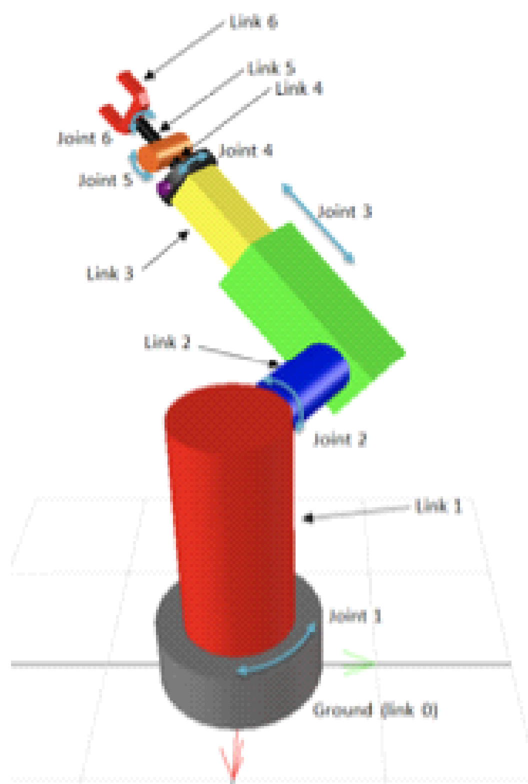
$$\underline{\underline{\cos\theta}} \sin\psi = \boxed{r_{21}}$$

$$\boxed{\psi = \sin^{-1}\left(\frac{r_{21}}{\cos\theta}\right)}$$



Joint i	α_i	a_i	d_i	θ_i
1	$-\pi/2$	0	d_1	θ_1
2	$\frac{\pi}{2}$	0	d_2	θ_2
3	0	0	d_3	$-\pi/2$
4	$-\pi/2$	0	d_4	θ_4
5	$\frac{\pi}{2}$	0	0	θ_5
6	0	0	d_6	θ_6

Stanford manipulator



1. Find the position & orientation of the end-effector
2. Forward kinematics & animation
3. Inverse Kinematics given $x_{ref}, y_{ref}, z_{ref}$, $\theta_{ref}, \phi_{ref}, \psi_{ref}$

find the joint angles