Algorithm for using DH for forward kinematics There are three steps.

1. **Assign coordinate frames:**
   (a) Assign z_i along the axis of actuation for each link, where $i = 0, 1, 2, ...(n-1)$.
   (b) Assign the base frame $o_0 - x_0 - y_0 - z_0$. The z_0 has already been assigned. Assign x_0 arbitrarily. Assign y_0 based on x_0 and z_0 using right hand rule.

   (c) Now assign coordinate frames $o_i - x_i - y_i - z_i$ for $i = 1, 2, ..., n-1$. z_i is already attached in first step. Next we assign x_i using these rules.
      i. **z_{i-1} and z_i are not coplanar:** In this case, there is a unique shorted distance segment that is perpendicular to z_{i-1} and z_i. Choose this as x_i axis. The origin o_i is where x_i intersects z_i. The y_i is found from right hand rules.
      ii. **z_{i-1} and z_i parallel:** In this case, there infinitely many perpendiculars. Choose any of these perpendiculars for x_i. Furthermore, where x_i intersects z_i we draw the origin x_i. Finally, y_i is found from the right hand rule. To make equations simpler, choose x_i such that is passes through o_{i-1}. This will make $d_i = 0$. Also, since z_{i-1} is parallel to z_i, $\alpha_i = 0$.
      iii. **z_{i-1} and z_i intersect:** In this case, x_i is chosen to be normal to the plane formed by z_{i-1} and z_i. There will be two possible directions for x_i, one of them is chosen arbitrarily and o_i is obtained by the intersection of $z - i$ and x_i. Finally y_i is obtained from right hand rule. Also, since z_{i-1} intersects z_i, $a_i = 0$.

   (d) Finally we need to attach an end effector frame, $o_n - x_n - y_n - z_n$. Attach z_n to be the same direction as z_{n-1}. Now depending on the relation between z_n and z_{n-1}, attach frame x_n. Finally, attach y_n using the right hand rule.

2. **Generate a table for DH parameter:** Now generate the DH table as follows.

Link	a_i	α_i	d_i	θ_i
1				
2				
.				
.				
.				
n				

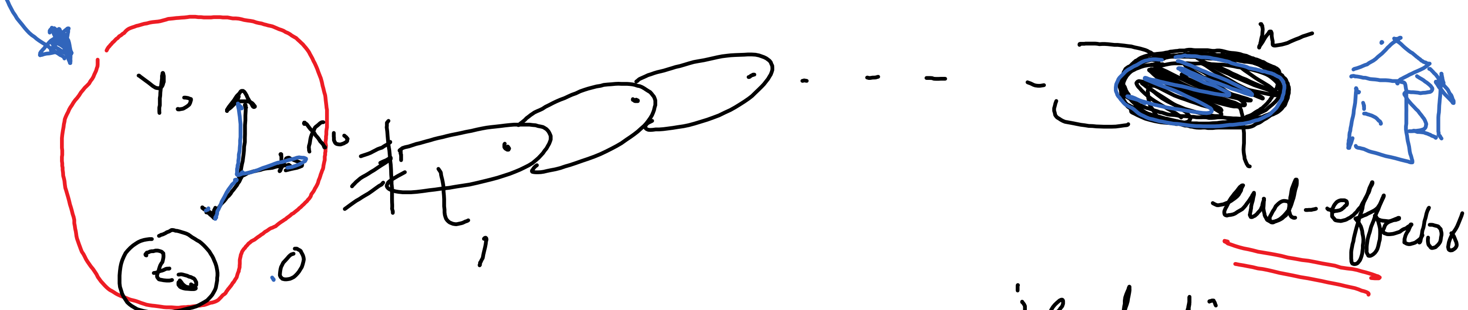
3. **Apply DH transformation to evaluate forward kinematics:** Finally, use the DH formulate to link two adjacent frames

$$\mathbf{H}_i^{i-1} = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} = H_z(\theta_i) H_z(d_i) \ldots H_x(a_i) H_x(\alpha_i)$$

The position and orientation of the end-effector is found using the formula

$$\mathbf{H}_n^0 = \mathbf{H}_1^0 \mathbf{H}_2^1 \mathbf{H}_3^2 \ldots \mathbf{H}_n^{n-1} = \begin{bmatrix} \mathbf{R}_n^0 & \mathbf{d}_n^0 \\ 0 & 1 \end{bmatrix} \quad 4 \times 4$$

The position of the end-effector is $\mathbf{d}_n^0$ and the orientation is $\mathbf{R}_n^0$. From $\mathbf{R}_n^0$, we can recover the Euler angles for the end-effector frame.

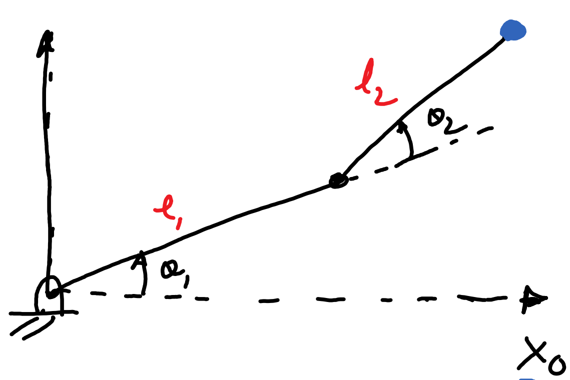


fixed frame

orientation R_n^0

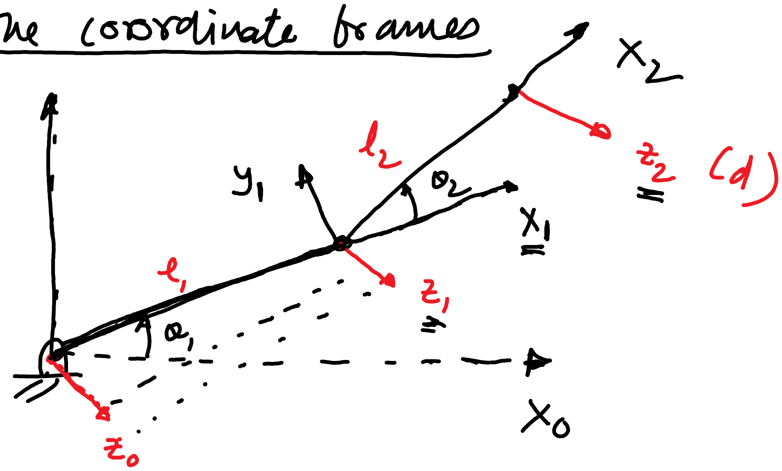
position d_n^0

1) Example 1: 2 link planar manipulator



→ Find the position & orientation of the end-effector

① Assign the coordinate frames



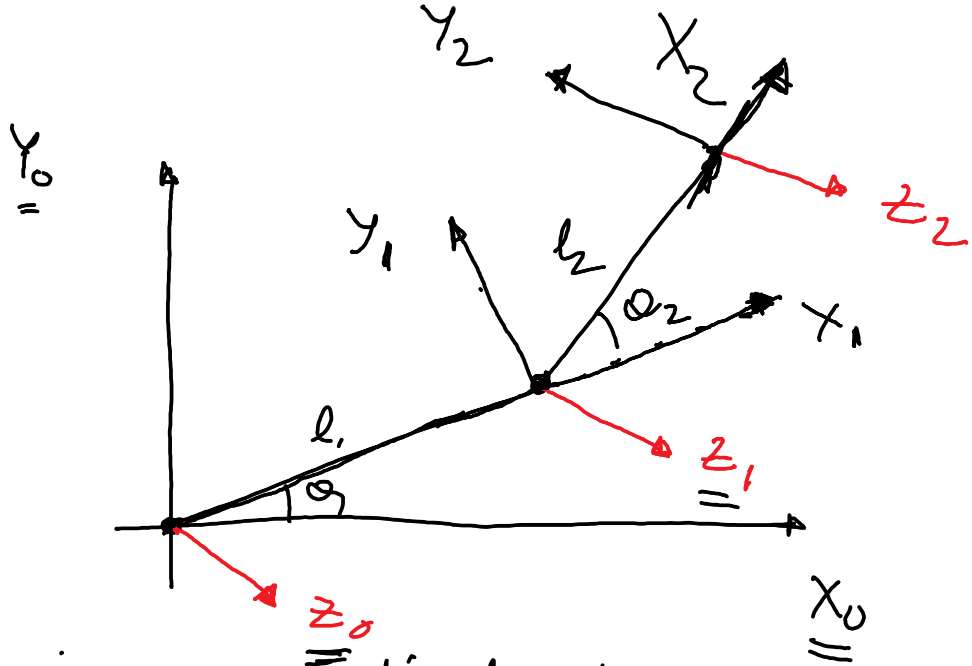
(a) Assign z_i

(b) Assign base frame $O_0\ x_0 Y_0, z_0$

(c) (i), (ii), (iii) applies

(d) Attach z_2 along z_1

z_n $\quad$ z_{n-1}



(1) Assign coordinate frames

(a) ✓

(b) Already done for you

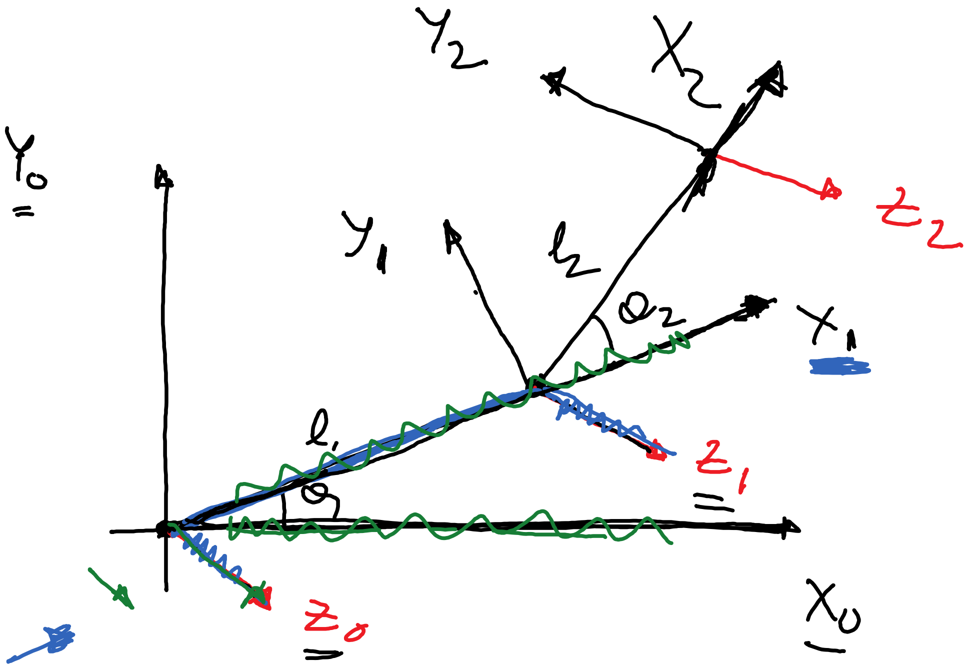
→ (c) ~~(i)~~ (ii) ✓ z_0 & z_1 are parallel

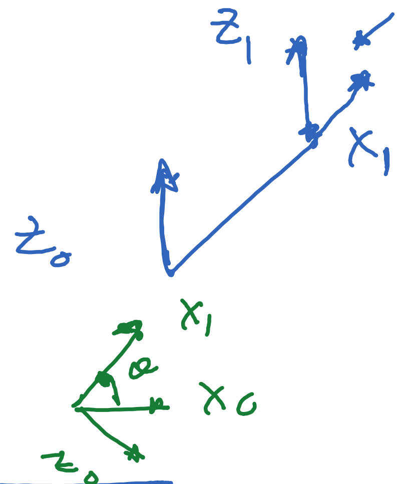
(d) z_2 is attached

X_2 (1)(c) ~~(i)~~ (ii) ✓

(2)

Link i	a_i	α_i	d_i	θ_i
1				
2				



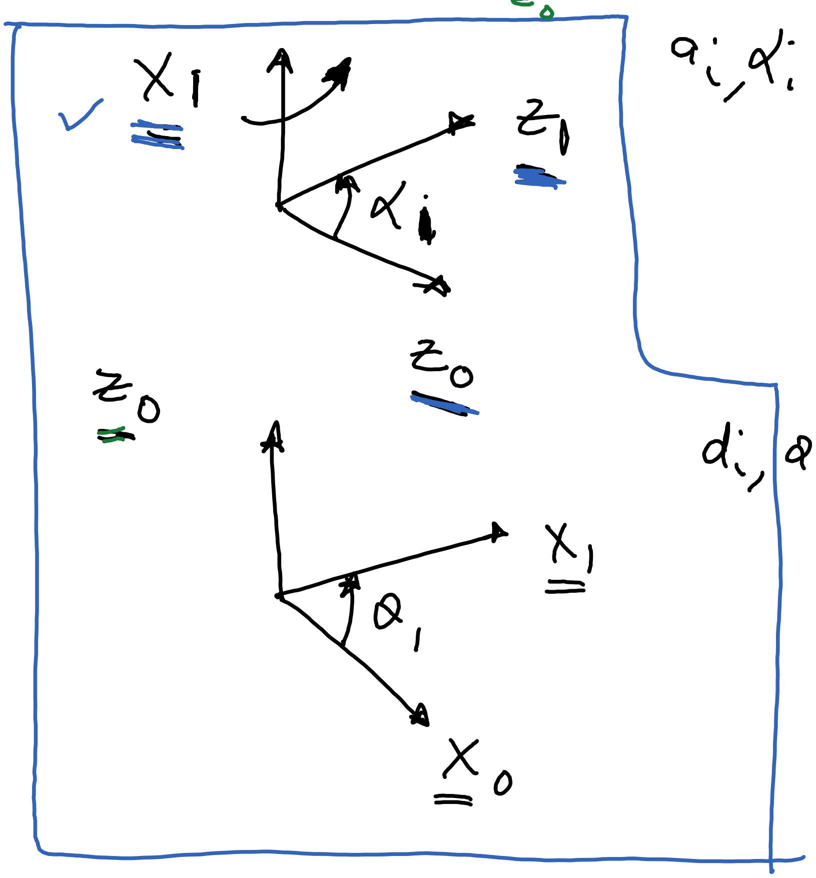


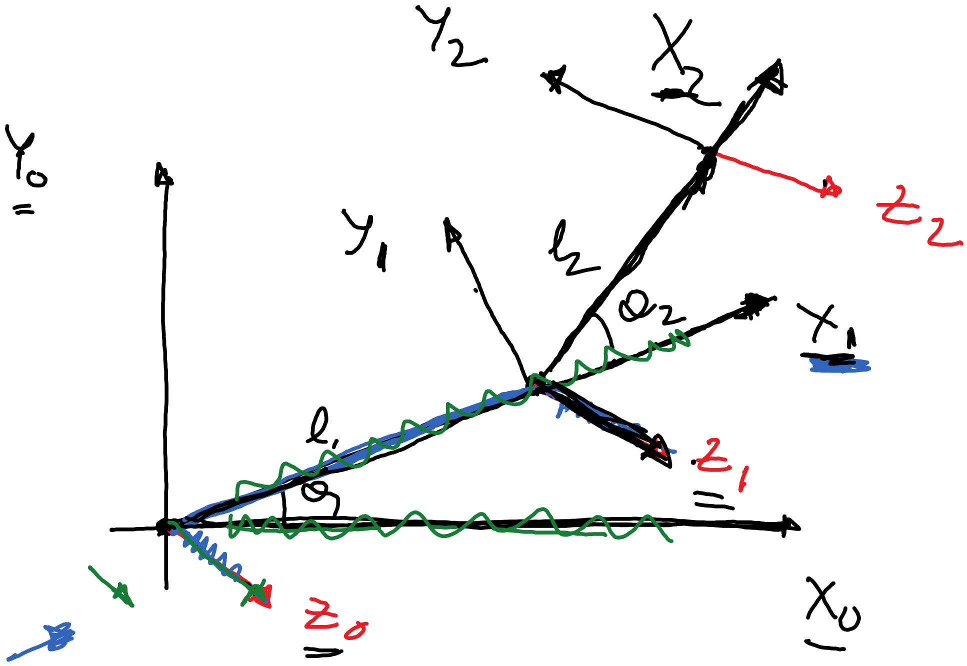
(2) $\underline{a_1} = l_1$

$\alpha_1 = 0$

$d_1 = 0$

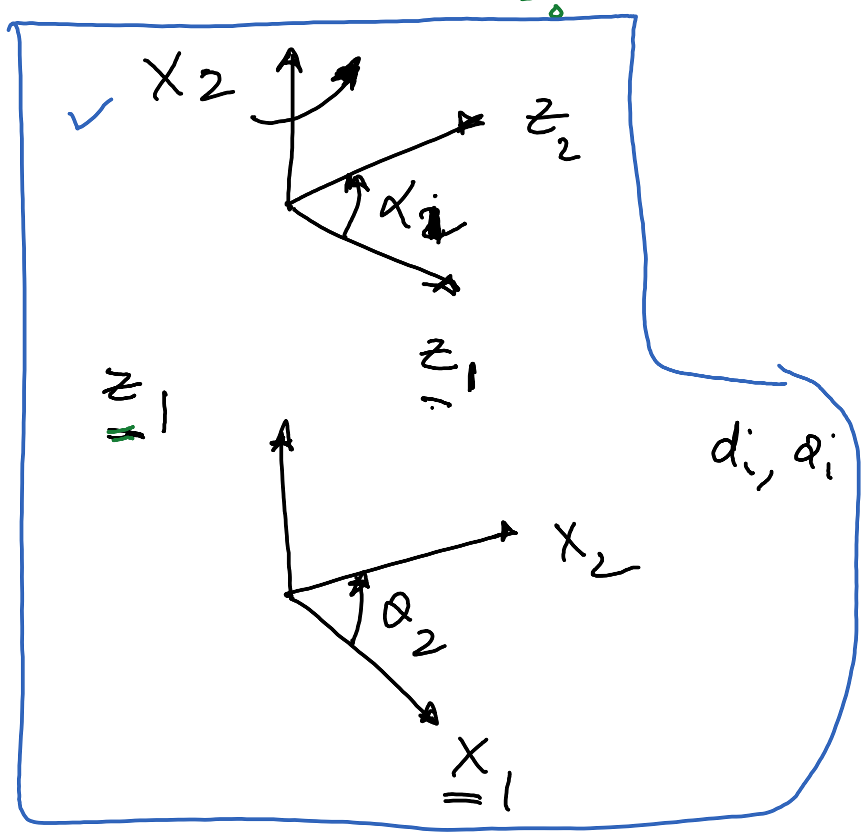
$\theta_1 = \theta_1$





(2)

$$a_2 = l_2$$
$$\alpha_2 = 0$$
$$d_2 = 0$$
$$\theta_2 = \theta_2$$



Link i	a_i	α_i	d_i	θ_i
1	l_1	0	0	θ_1
2	l_2	0	0	θ_2

3) $H_1^0 = \begin{bmatrix} c_1 & -s_1 & 0 & l_1 c_1 \\ s_1 & c_1 & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\quad c_1 = \cos\theta_1$, $s_1 = \sin\theta_1$

$H_2^1 = \begin{bmatrix} c_2 & -s_2 & 0 & l_2 c_2 \\ s_2 & c_2 & 0 & l_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\quad c_2 = \cos\theta_2$, $s_2 = \sin\theta_2$

$$H_2^0 = H_1^0 H_2^1 = \left.\begin{bmatrix} c_{12} & -s_{12} & 0 & l_1 c_1 + l_2 c_{12} \\ s_{12} & c_{12} & 0 & l_1 s_1 + l_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right\}$$

$c_{12} = \cos(\theta_1 + \theta_2)$; $s_{12} = \sin(\theta_1 + \theta_2)$

Position: $\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{\text{end-effector}} = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ 0 \end{bmatrix}$

Orientation $= \begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow (\theta_1 + \theta_2)$ along z_2 axis