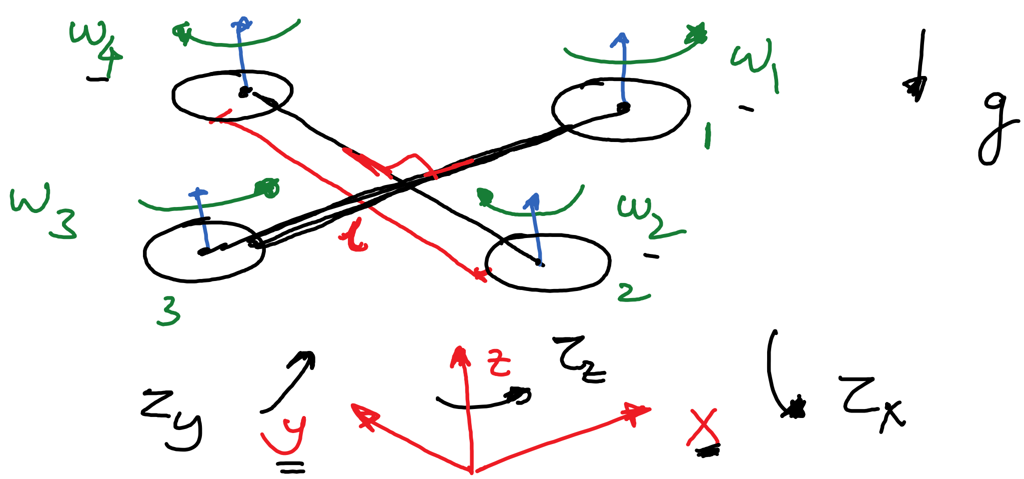
↓ view

Quadcopters / drones



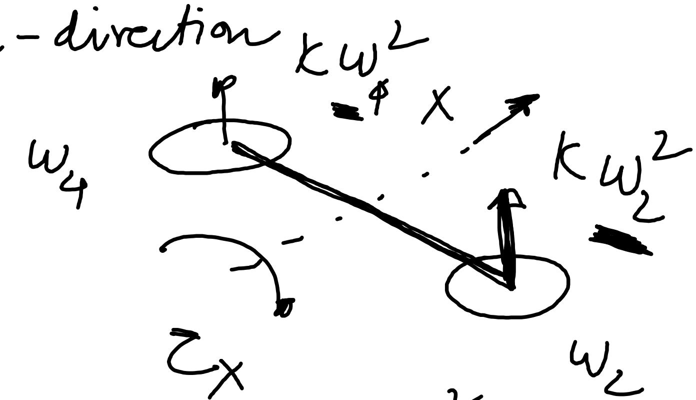
Each motor provides an upward thrust (or thrust in the z-direction) thrust $\propto \omega^2$

trust $= k\omega^2$

↑ Lift constant

→ $F_z = k(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)$ [balancing mg]

F_z → force in the z-direction

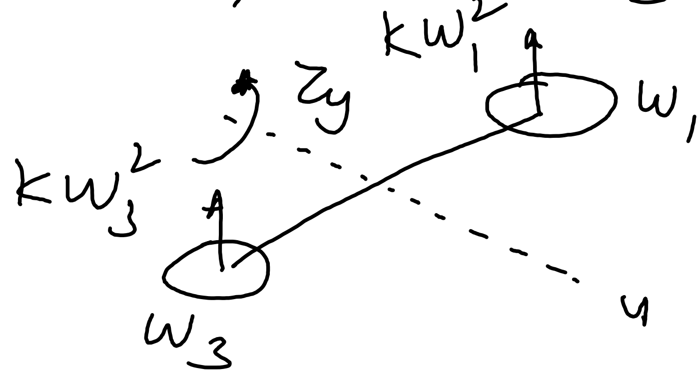


→ $\tau_x = ?$

$\tau_x = lk(\omega_4^2 - \omega_2^2)$

→ $\tau_y = ?$

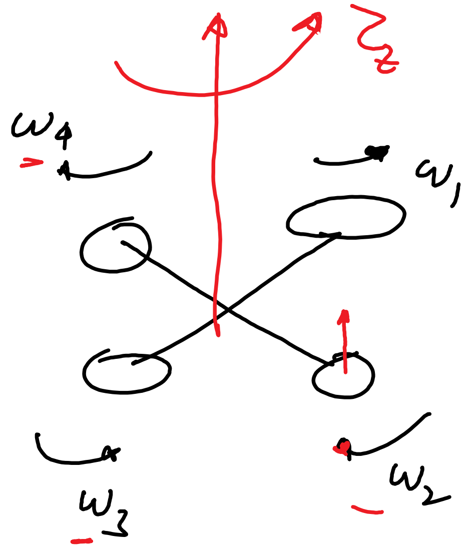
$\tau_y = lk(\omega_3^2 - \omega_1^2)$



⇒ $\tau_z = ?$

$$\tau_z = b\,(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2)$$

drag constant



$z \uparrow \quad \downarrow$

Summarise:

① F_z — force in z

②, ③, ④ τ_x, τ_y, τ_z → torques in x, y, z direction.

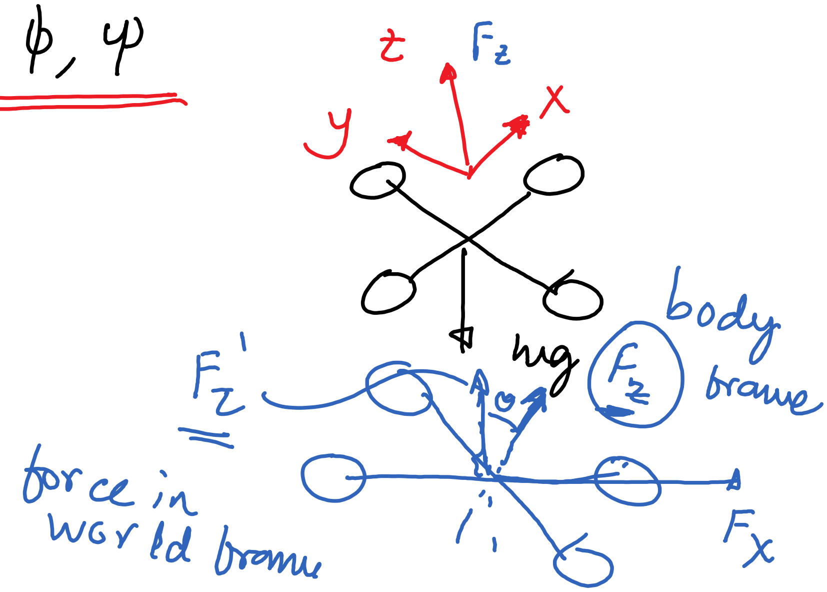
You cannot induce F_y, F_x direction

Another way to think of this is:

⇒ We have 4 control variables: $\omega_1, \omega_2, \omega_3, \omega_4$

⇒ But we have to control 6 numbers: $x, y, z, \theta, \phi, \psi$

How to get F_y, F_x



Equations of motion of a quadcopter

1) Positions : x, y, z

ϕ, θ, ψ

Velocities : $\dot{x}, \dot{y}, \dot{z}$

$\dot{\phi}, \dot{\theta}, \dot{\psi}$

$$\underset{\substack{\uparrow \\ \text{body} \\ \text{frame}}}{\omega_b} = \begin{bmatrix} \omega_{bx} \\ \omega_{by} \\ \omega_{bz} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta \sin\phi \\ 0 & -\sin\phi & \cos\phi \cos\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

2) $T = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{1}{2}(I_x \omega_{bx}^2 + I_y \omega_{by}^2 + I_z \omega_{bz}^2)$

$V = mgz$

$\mathcal{L} = T - V$

3) $$\underline{\underline{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right)}} - \frac{\partial \mathcal{L}}{\partial q_j} = \underset{\substack{\uparrow \\ \text{external forces/torques}}}{\Gamma_j}$$

$\rightarrow$ $q = \{x, y, z, \phi, \theta, \psi\}$

$$\underline{\Gamma} = \begin{bmatrix} f_{ext} \\ \tau_{ext} \end{bmatrix}$$

$$\tau_{ext} = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} k\,l\,(\omega_4^2 - \omega_2^2) \\ k\,l\,(\omega_3^2 - \omega_1^2) \\ b(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \end{bmatrix}$$

k — lift constant

b — drag constant

$F_{ext} = R\,\text{Thrust} - \underline{\text{Drag}}$ $\qquad$ $vec_{fixed} = R\ vec_{body}$

similiar force we saw in the projectile

$$F_{ext} = \underline{R} \begin{bmatrix} 0 \\ 0 \\ k(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \end{bmatrix} - \begin{bmatrix} A_x V_x \\ A_y V_y \\ A_z V_z \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} F_{ext} \\ \tau_{ext} \end{bmatrix}$$

4) Simplify the equations to this form

$$\underset{6\times 6}{A}\ \underset{6\times 1}{X} = \overset{6\times 1}{b}$$

→ centripetal force, gravity, external force, coriolis force, drage

$$X = [\ddot{x}\ \ \ddot{y}\ \ \ddot{z}\ \ \ddot{\phi}\ \ \ddot{\theta}\ \ \ddot{\psi}]$$