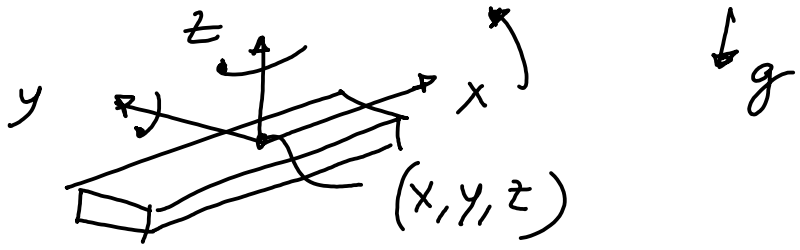
3D Dynamics



Given an initial linear speed and angular speed describe the motion of the object

- Derive the equations
- Simulate
- Animate

Equations of motion using Euler-Lagrange method

1) Position of the center of mass : x, y, z
Angular position : Euler angle 3-2-1 $\rightarrow \psi - \theta - \phi$

Velocity : $\dot{x}, \dot{y}, \dot{z}$
Angular velocity

$$\omega_b = \begin{bmatrix} \omega_{bx} \\ \omega_{by} \\ \omega_{bz} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta \sin\phi \\ 0 & -\sin\phi & \cos\theta \cos\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

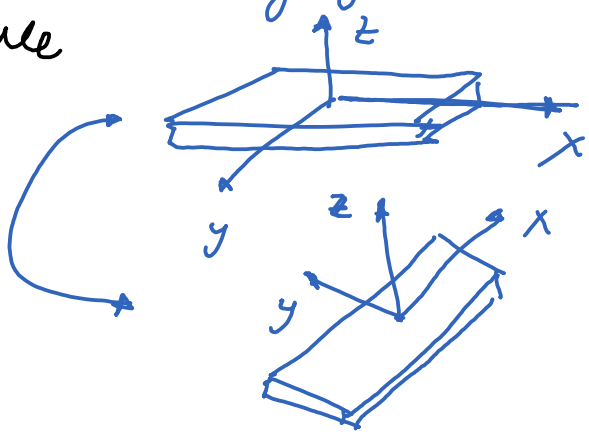
Derived in the last class

2) $T = 0.5\, m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + 0.5\left(I_x \omega_{bx}^2 + I_y \omega_{by}^2 + I_z \omega_{bz}^2\right)$

Inertia in the body frame

$V = mgz$

$x, y, z \rightarrow$ position in the fixed / world frame



$\mathcal{L} = T - V$

3) Euler-Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = 0 \qquad q_j = \{x, y, z, \phi, \theta, \psi\}$$

6 numbers

There will be 6 equations

4) Simplify the equations to look like this:

$$A X = b \quad \text{(6x1)}$$

(6x6) (6x1)

$$X = [\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\phi}, \ddot{\theta}, \ddot{\psi}]$$