$$\omega = \dot{\psi}\begin{bmatrix}0\\0\\1\end{bmatrix} + R_z\dot{\theta}\begin{bmatrix}0\\1\\0\end{bmatrix} + R_zR_y\dot{\phi}\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$\boxed{\omega = \begin{bmatrix}\cos\psi\cos\theta & \sin\psi & 0\\ \cos\theta\sin\psi & \cos\psi & 0\\ -\sin\theta & 0 & 1\end{bmatrix}\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}} \quad -(1)$$

$$\omega_b = \dot{\phi}\begin{bmatrix}1\\0\\0\end{bmatrix} + R_x^T\dot{\theta}\begin{bmatrix}0\\1\\0\end{bmatrix} + R_x^TR_y^T\dot{\psi}\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\boxed{\omega_b = \begin{bmatrix}1 & 0 & -\sin\theta\\ 0 & \cos\phi & \cos\theta\sin\phi\\ 0 & -\sin\phi & \cos\phi\cos\theta\end{bmatrix}\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}} \quad -(2)$$

→ $\omega = A\dot{\Theta}$ — same as (1) → $\dot{\Theta} = \underline{A}^{-1}\omega$

→ $\omega_b = A_b\dot{\Theta}$ — same as (2) → $\underline{\dot{\Theta}} = \underline{A_b}^{-1}\omega_b$

$|\underline{A}| = 0$ $|\underline{A_b}| = 0$ when $\underline{\cos\theta} = 0$ singularity

or

$\theta = \pi/2$

This is a problem with the use of <u>Euler angles / rates</u>

Fix: <u>Quarternions</u>

to represent 3D rotations

3D angular velocity (final)

Last class:

$$\Rightarrow \quad \boxed{\underline{\omega} = \dot{\psi}\,\hat{k} + R_z\,\dot{\theta}\,\hat{j} + R_z R_y\,\dot{\phi}\,\hat{i}}$$

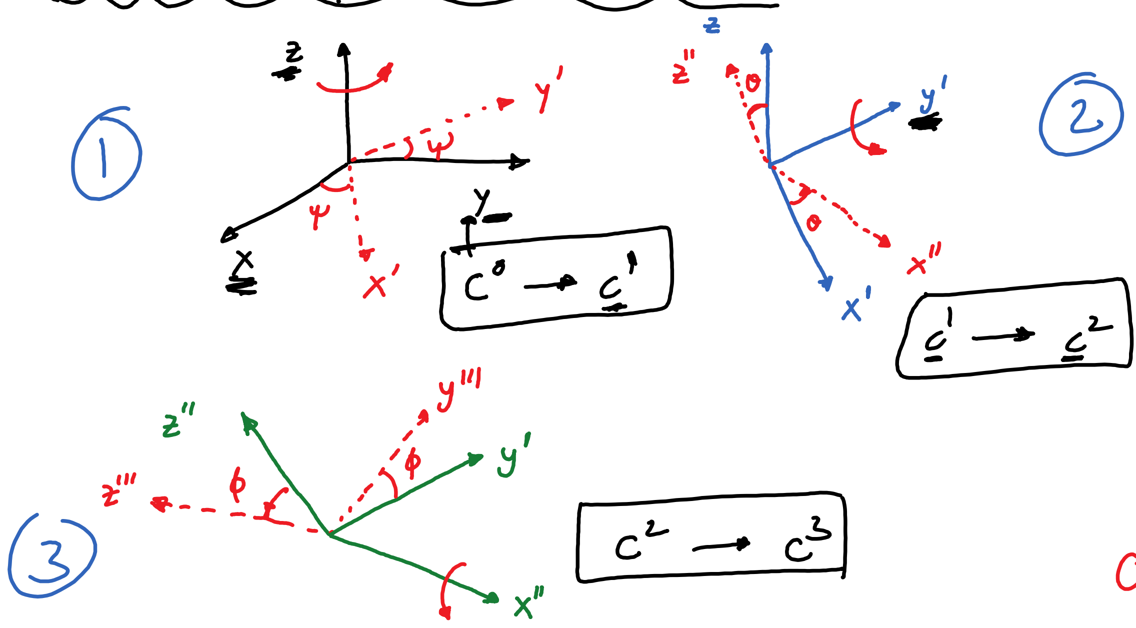
where $R_z \Rightarrow R_z(\psi)$

$R_y \Rightarrow R_y(\theta)$

Derivation from simplifying $\underline{\underline{S}}(\omega) = \dot{R}R^T$

$R \rightarrow R_z R_y R_x$

Shorter way to derive ω



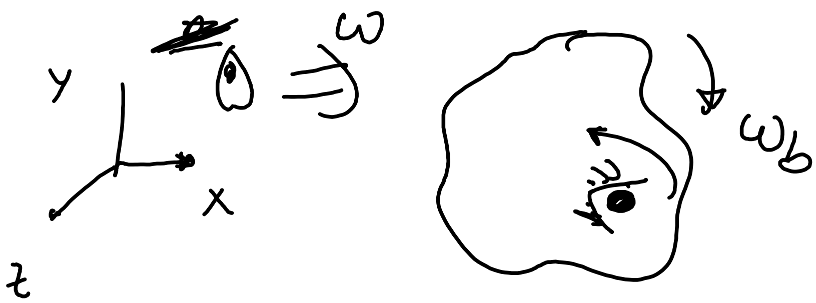
Checks Out

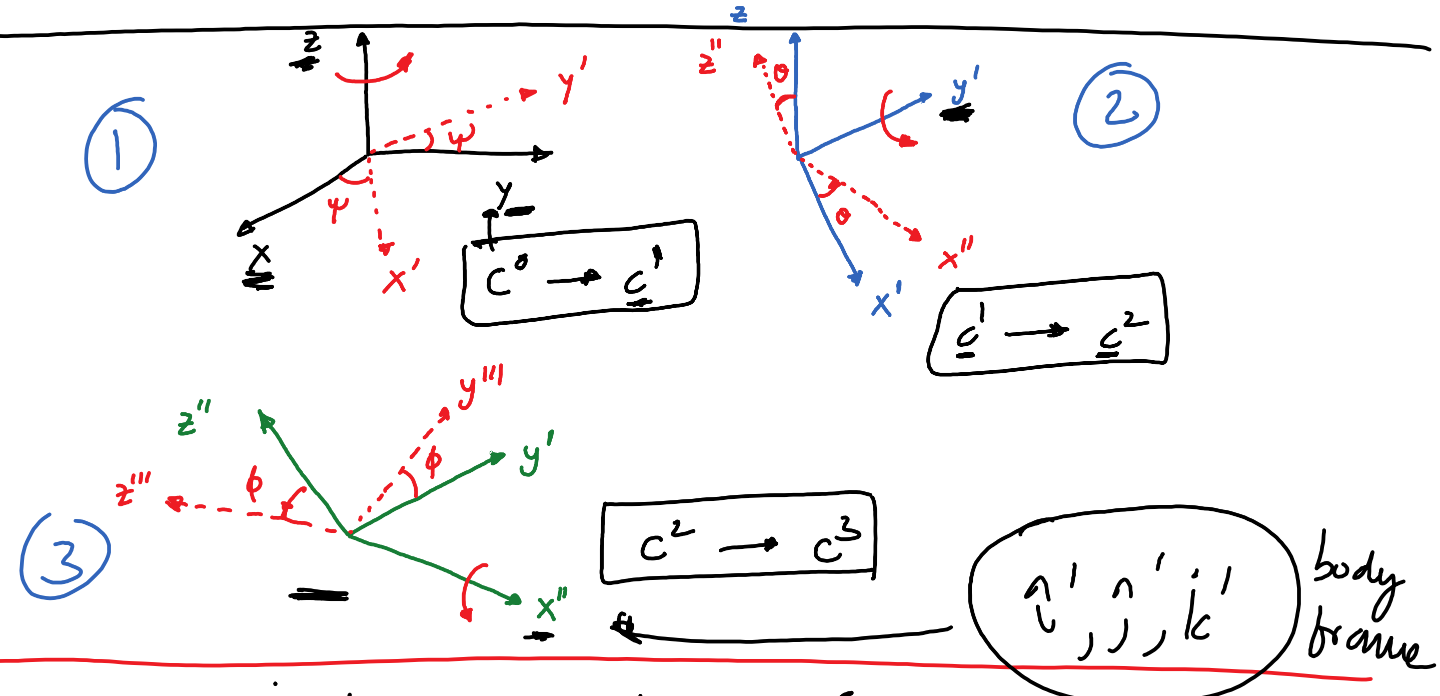
$$\omega = \underbrace{\dot{\psi}\, z}_{1} + \underbrace{\dot{\theta}\, y'}_{2} + \underbrace{\dot{\phi}\, x''}_{3}$$

$$\rightarrow \boxed{\omega = \dot{\psi}\,\hat{k} + \dot{\theta}\,R_z\,\hat{j} + \dot{\phi}\,R_z R_y\,\hat{i}}$$

ω — fixed frame angular velocity

ω_b — body frame angular velocity





$$\omega_b = \dot{\phi}\, x'' + \dot{\theta}\, y' + \dot{\psi}\, \hat{z}$$

$$= \dot{\phi}\, \hat{i}' + \dot{\theta}\, R_x^{-1} \hat{j}' + \dot{\psi}\, R_x^{-1} R_y^{-1} \hat{k}'$$

But $RR^T = I \to R^T = R^{-1}$

$$* \quad \boxed{\omega_b = \dot{\phi}\, \hat{i}' + \dot{\theta}\, R_x^T \hat{j}' + \dot{\psi}\, R_x^T R_y^T \hat{k}'}$$