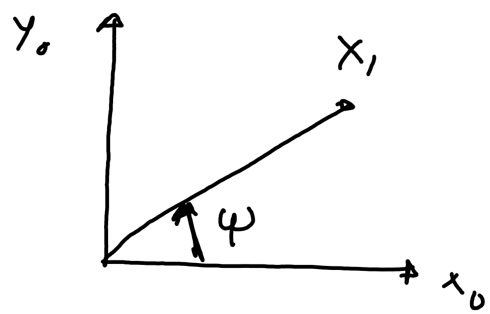
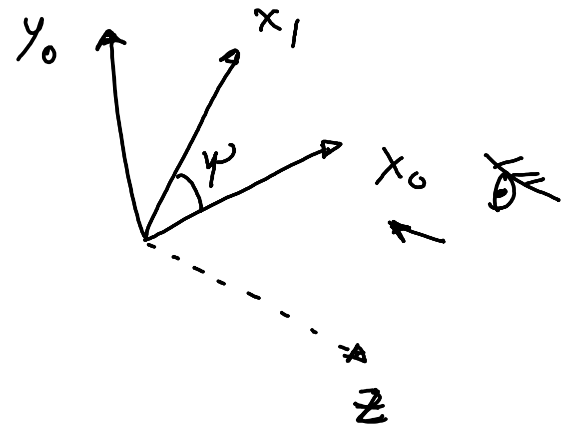
3D rotations & velocity



$$R_1^0 = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}$$

2-D rotation

3D $\Longrightarrow$



$$\checkmark\ R_z(\psi) = R_1^0 = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3D rotation.

$$\checkmark\ R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\checkmark\ R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

In general rotation in 3D

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

9 - numbers in R

$R^T R = I$ (I = 3x3 identity matrix)

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1 — $\sum_{i=1,2,3} r_{i1}^2 = 1 = r_{11}^2 + r_{21}^2 + r_{31}^2 = 1$

2 — $\sum_{i=1,2,3} r_{i2}^2 = 1$

3 — $\sum_{i=1,2,3} r_{i3}^2 = 1$

4 — $\sum_{i=1,2,3} r_{i1} r_{i2} = 0$

5 — $\sum_{i=1,2,3} r_{i2} r_{i3} = 0$

6 — $\sum_{i=1,2,3} r_{i3} r_{i1} = 0$

9 numbers $r_{11}, r_{12}, r_{13} \ldots$

6 equations or constraints

Free variables $9-6 = 3$

means we can describe rotations using only 3 numbers → angles

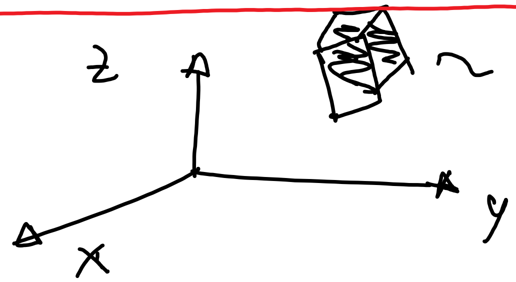
Euler angles $\to$ 3 angles

– Bryant angles

1-2-3 $\to$ x – y – z	y – x – z	z – x – y
x – z – y	y – z – x	z – y – x
x – y – x	y – z – y	$\boxed{z - y - z}$
x – z – x	y – x – y	z – x – z

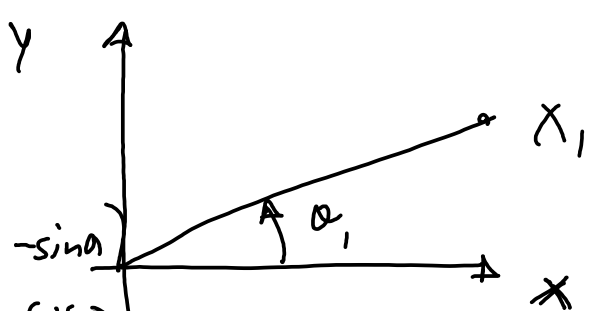
$4 + 4 + 4 = 12$

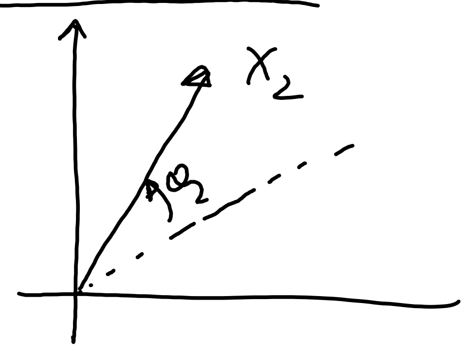
we will use in the course or 3-2-1



~ orientation? 12 ways.

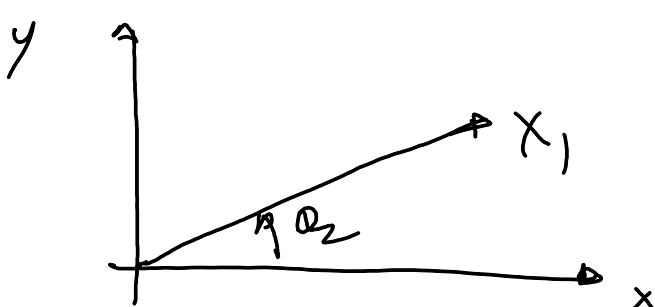
Rotations in 2D are commutative

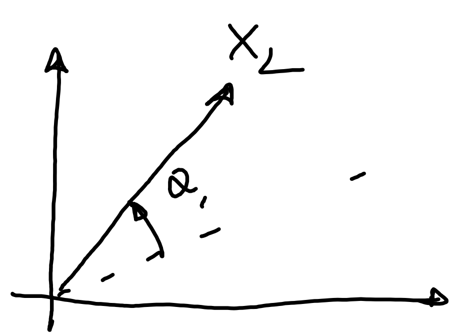




$$R_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$\theta_1 \to \theta_2$





$R = R_{\theta_1} R_{\theta_2} = R_{\theta_1+\theta_2}$ $\qquad$ $R = R_{\theta_2} R_{\theta_1} = R_{\theta_1+\theta_2}$

Rotations in 3D are NOT commutative