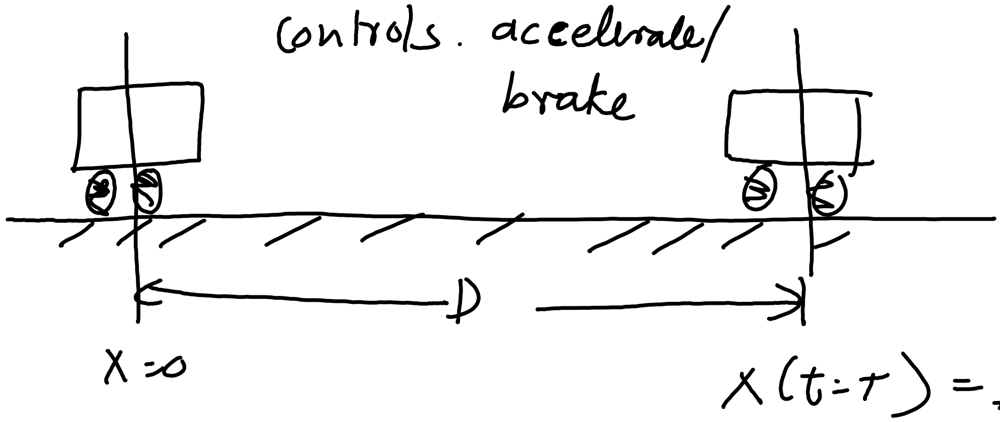
Trajectory optimization



$a = u$ where $a = \ddot{x}$ (acceleration)
u = control $\quad -5 < u < 5$

$\Rightarrow \quad \underline{\underline{\ddot{x} = u}}$ System dynamics

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Formulation

Go from start ($x=0$) to the goal ($x=D$) in minimum time

Formulation

$$\min_{T,u} \int_{t=0}^{t=T} dt$$

$$\dot{x}_1 = x_2 \qquad x_1 = \text{position}$$
$$\dot{x}_2 = u \qquad x_2 = \text{velocity}$$

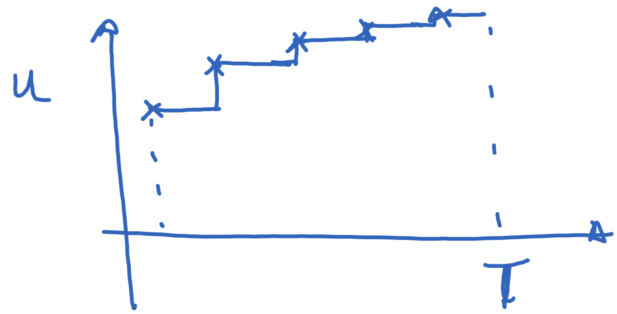
$$-5 \le u \le 5$$

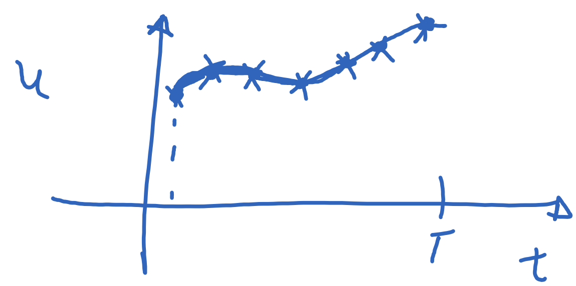
$$x_1(t=0) = 0 \;, \quad x_1(t=T) = D = 5$$
$$x_2(t=0) = 0 \;; \quad x_2(t=T) = 0$$

This is an infinite dimensional problem because t has infinite values between 0 & T and hence $u(t)$ is infinite dimensional

Convert this into a finite dimensional problem for implementation in MATLAB.

PARAMETER OPTIMIZATION





2 ways of solving the car problem in MATLAB

1. Collocation method
2. Shooting method

① collocation method

satisfy the differential equations at distinct point.

a) Optimization variable : T, $u(i)$, $x_1(i)$, $x_2(i)$

? $x_2(1)$ $x_2(2), x_2(3)$ . . . $x_2(N)$

? $x_1(1)$ $x_1(2), x_1(3)$ . . $x_1(N)$

? $u(1)$ $u(2)$ $u(3)$ $u(N)$

$i=1$ $i=2$ $i=3$ $i=4$ $i=5$. $i=NT$

t $t+dt$ $t+2dt$ . . . .

b) Optimization objective : minimize T

c) Optimization constraints:

$$\boxed{\begin{aligned} &\dot{x}_1 = x_2 \quad - ⑥\\ &\dot{x}_2 = u \quad - ⑦\\ &-5 \le u \le 5 \quad - ①\\ &x_1(t=0) = 0 \quad - ②\\ &x_2(t=0) = 0 \quad - ③\\ &x_1(t=T) = D = 5 \quad - ④\\ &x_2(t=T) = 0 \quad - ⑤ \end{aligned}}$$

① $-5 \le u(i) \le 5$ N

② $x_1(1) = 0$ 1

③ $x_2(1) = 0$ 1

④ $x_1(N) = 5$ 1

⑤ $x_2(N) = 0$ 1

⑥ $\dot{x}_1 = \dfrac{x_1(t+\Delta t) - x_1(t)}{\Delta t} = x_2$

$N \rightarrow$ $\boxed{x_1(i+1) = x_2(i)\Delta t + x_1(i)}$

$3N+4$

$3N + 4$

equation

$N \rightarrow \boxed{x_1(i+1) = x_2(i)\Delta t + x_1(i)}$

N 7 $\boxed{x_2(i+1) = u(i)\Delta t + x_2(i)}$

T, u, x_1, x_2

fmincon

$F(x)$, $lb \le x \le ub$, $A_{eq}x = b_{eq}$, $Ax < b$, c, c_{eq}.
(i) (ii) (iii) (iv) (v) (vi)

$A_{eq} = [\,]$, $b_{eq} = [\,]$

(i) $-T$

(ii) $-5 \le u(i) \le 5$ $\quad 0 \le x_1 \le 5$ $\quad -\infty \le x_2 \le \infty$

(iii) $\boxed{2, 3, 4, 5, 6, 7}$ — (vi) c_{eq} To make it easy to code

(iv) $A = [\,]$ $\quad b = [\,]$

(v) $c = [\,]$

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1 $-5 \le u(i) \le 5$ ✓

2 $x_1(1) = 0$; ✗

3 $x_2(1) = 0$; ✗ } not need if you take care of x_1, x_2 in $lb < x_1 < ub$, $lb < x_2 < ub$

4 $x_1(N) = 5$;

5 $x_2(N) = 0$;

4 6 $\dot{x}_1 = \dfrac{x_1(t+\Delta t) - x_1(t)}{\Delta t} = x_2$

$$\boxed{x_1(i+1) = x_2(i)\Delta t + x_1(i)}$$

7
$$\boxed{x_2(i+1) = u(i)\Delta t + x_2(i)}$$