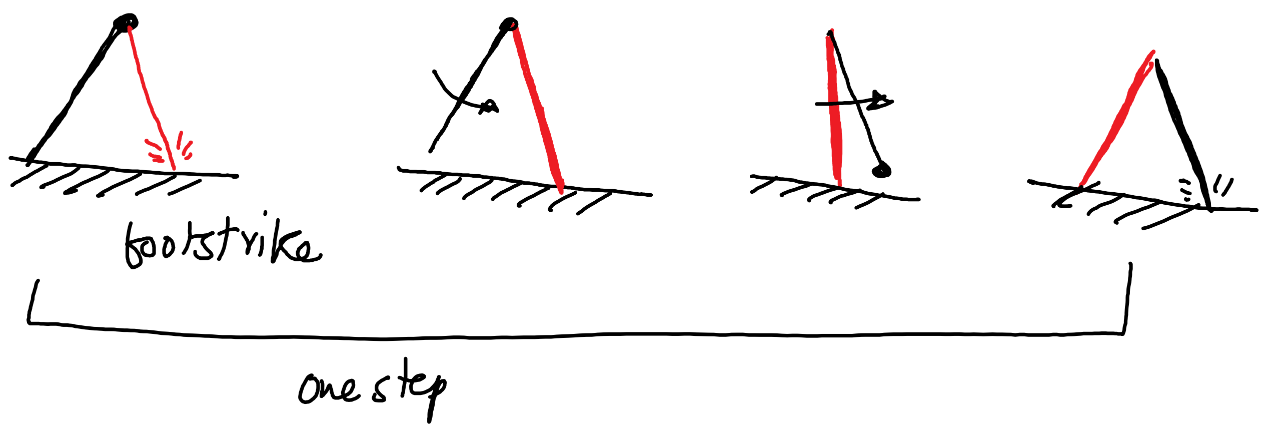
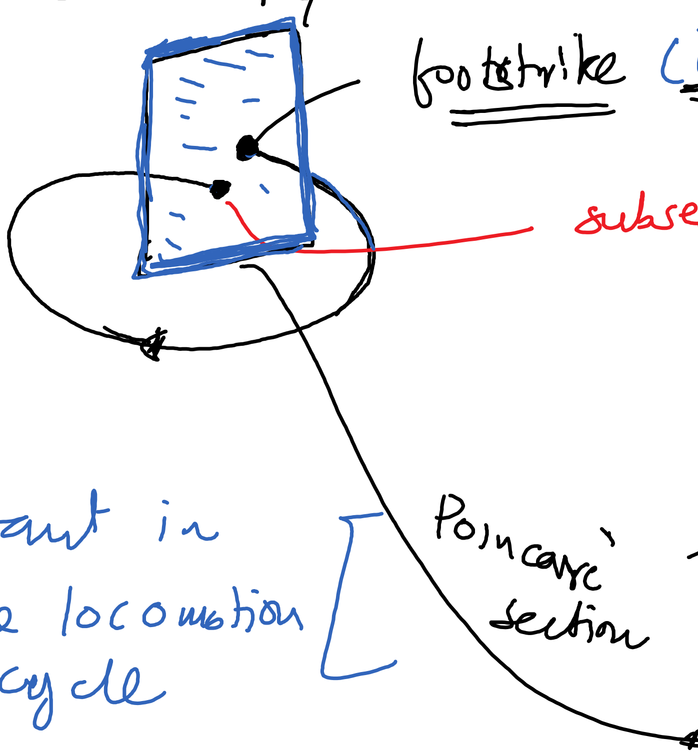
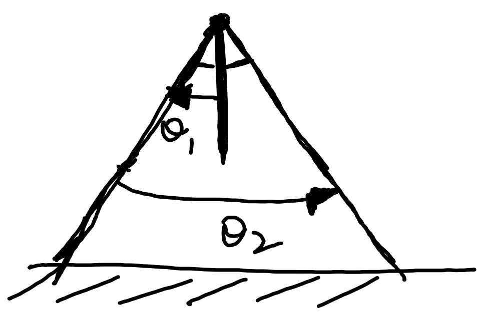
Analyze walking/legged system



Poincaré map / Poincaré section



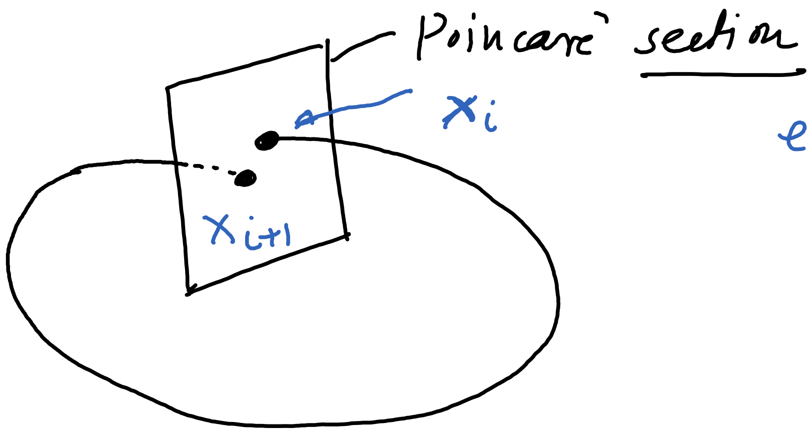
Instant in the locomotion cycle



$\theta_2 = -2\theta_1$

$\theta_2 + 2\theta_1$

Poincaré map: $\rightarrow$ mapping of the system dynamics from one step to the next.



e.g. $x_i = \{\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2\}$

$$\boxed{x_{i+1} = F(\underline{\underline{x_i}})}$$

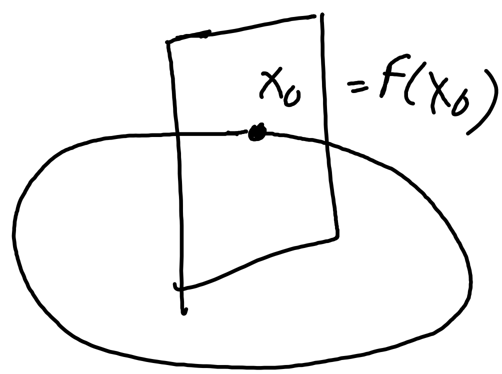
Poincaré map

Find x_0 such that

$$\boxed{\underbrace{x_0 = F(x_0)}}$$

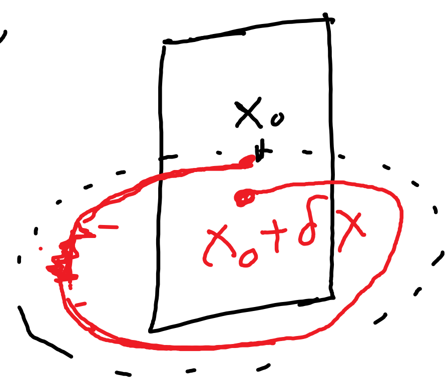
periodic/ steady state walking
gait

Limit cycle walking



Stability of the limit cycle

If δx perturbation decreases every step then the limit cycle is stable else it is unstable.



Test for stability

$$J = \left.\frac{\partial F}{\partial x}\right|_{x=x_0}$$

J = Jacobian of the Poincaré map

If the largest eigenvalue of J is less that 1 then the system is stable else it is unstable

$$\boxed{\begin{array}{lll} \max(\text{eig}(J)) & < 1 & \text{stable} \\ & > 1 & \text{unstable} \\ & = 1 & \text{neutrally stable} \end{array}}$$

— F . there is no analytical formula for F

$J = \frac{\partial F}{\partial x}$: need to do this numerically; central difference

Example:

$$x_{i+1} = f(x_i) = x_i^2$$

Find: ① Find x_0 such that $x_0 = f(x_0)$
$x_0 \longrightarrow$ fixed point

② Find the stability of the fixed point.

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calculator:
given an initial x_i & see what happens to x_{i+1} when f is applied repeatedly.

$0 \leq x_i < 1$ degrades to

$x_{i+1} \to 0$ as $i \to \infty$

$x_i > 1$ then $x_{i+1} \to \infty$. $i \to \infty$.

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① Find fixed point $x_0 = f(x_0)$

$$x_0 = x_0^2$$

$$x_0 - x_0^2 = 0$$

$$x_0(1 - x_0) = 0$$

$\Rightarrow$ $x_0 = 0$ $\quad x_0 = 1$ (2 fixed points)

② $x_{i+1} = F(x_i) = x_i^2$

$$J = \frac{\partial F(x_i)}{\partial x_i} = \frac{\partial (x_i)^2}{\partial x_i} = 2x_i$$

$$J\big|_{x_0=0} = 2x_i\big|_{x_0=0} = 0 < 1 \quad \text{stable}$$

$$J\big|_{x_0=1} = 2x_i\big|_{x_0=1} = 2 > 1 \quad \text{unstable}$$