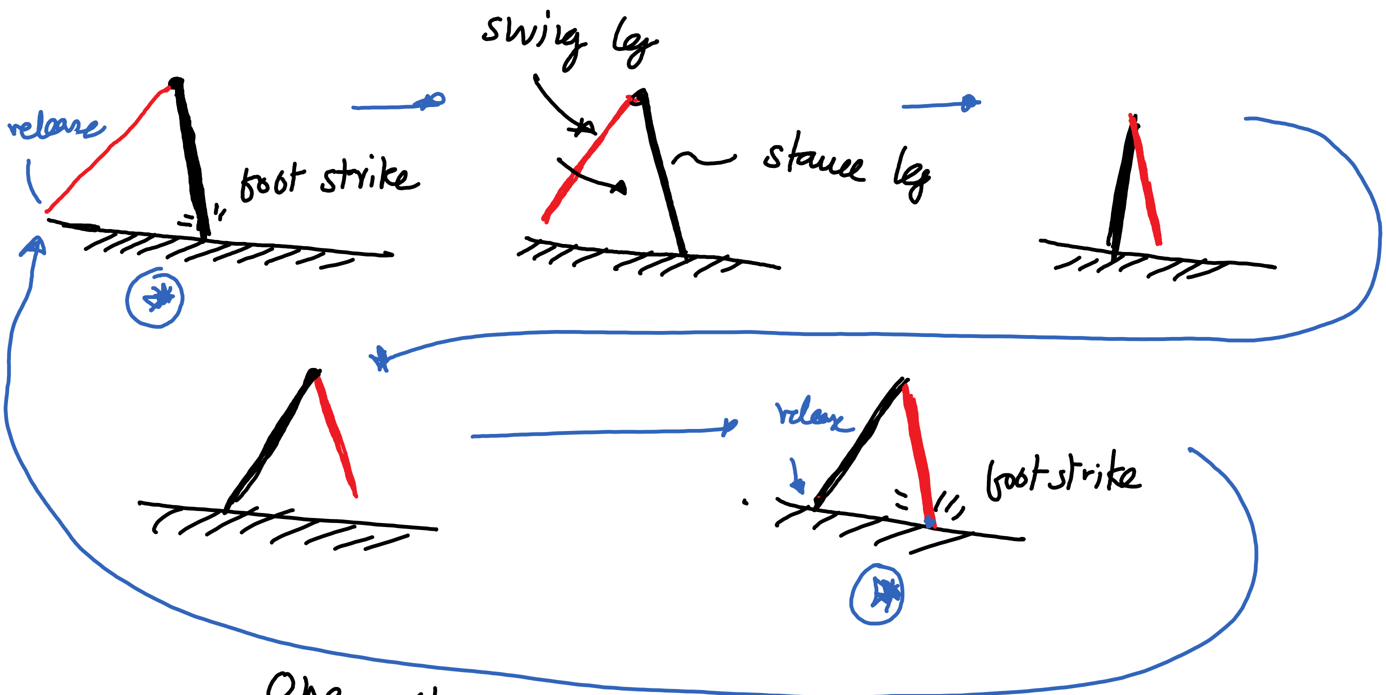
Introduction to passive dynamic walking



One-step

$\text{single-stance} \xrightarrow[\text{ground}]{\text{detect}} \text{footstrike} \longrightarrow \text{single stance} \rightarrow \text{foot strike}$

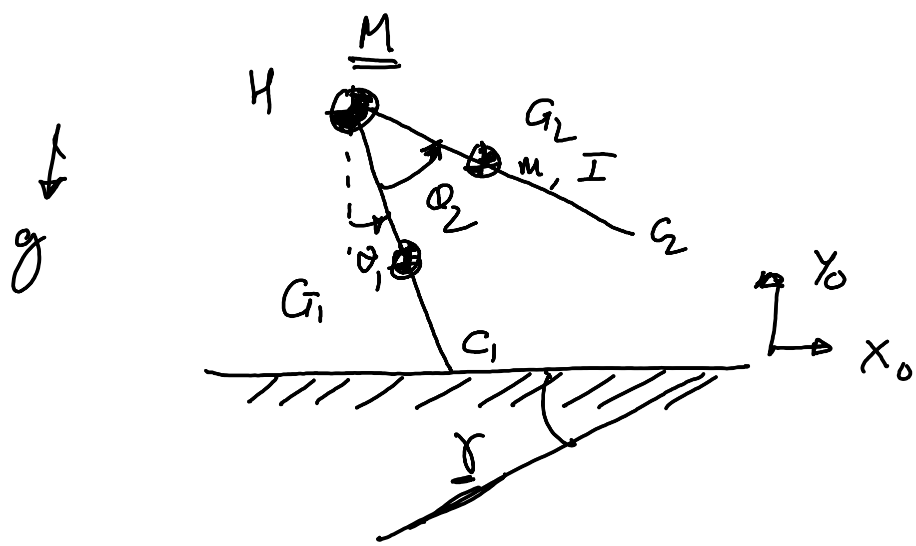
one-step (repeating unit)

To simulate one step we need

① Equations for single stance

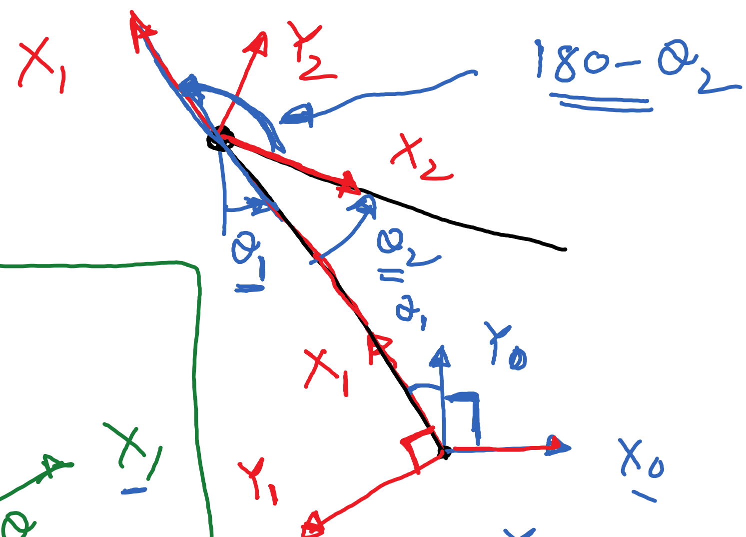
② Equation for footstrike.

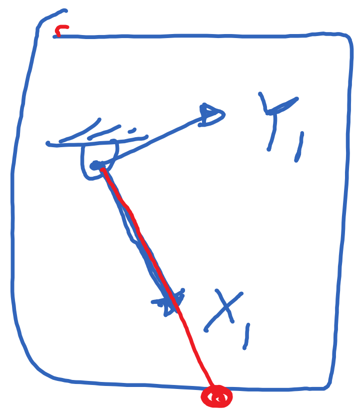
①

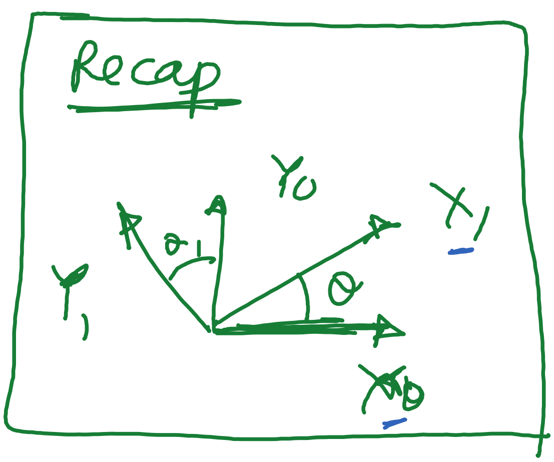


Mass parameters / geometry

②

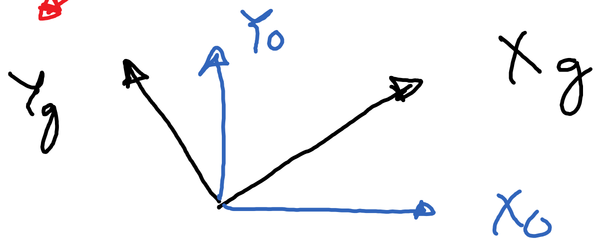






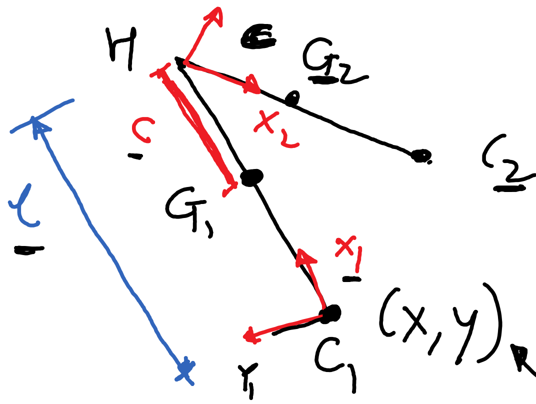
$X_0 - X_1 \quad \pi/2 + \theta_1$

$X_1 - X_2 = \pi - \theta_2$



$$R_1^0 = \begin{bmatrix} \cos(\pi/2 + \theta_1) & -\sin(\pi/2 + \theta_1) \\ \sin(\pi/2 + \theta_1) & \cos(\pi/2 + \theta_1) \end{bmatrix}$$

$$R_2^1 = \begin{bmatrix} \cos(\pi - \theta_2) & -\sin(\pi - \theta_2) \\ \sin(\pi - \theta_2) & \cos(\pi - \theta_2) \end{bmatrix}$$



Floating base model

$$r^0_{C_1} = \begin{bmatrix} x^0_{C_1} \\ y^0_{C_1} \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$r^0_{G_1} = r^0_{C_1} + R^0_1 \begin{bmatrix} \ell - c \\ 0 \end{bmatrix}$$

distance along frame $x_1 - y_1$

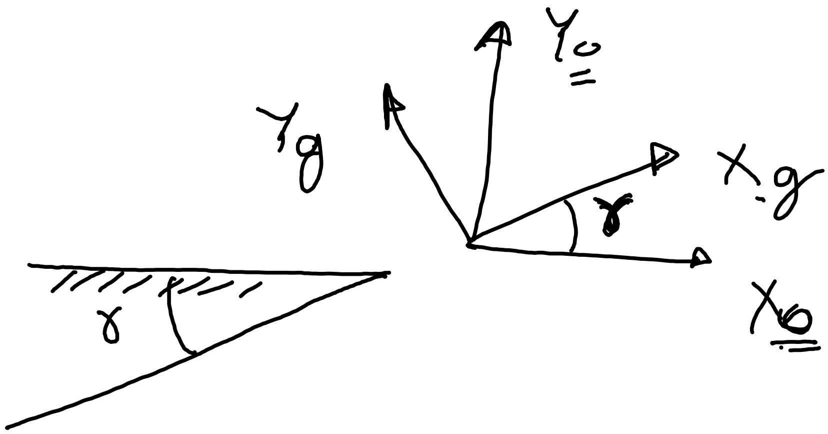
$$r^0_H = r^0_{C_1} + R^0_1 \begin{bmatrix} \ell \\ 0 \end{bmatrix}$$

$$r^0_{G_2} = r^0_H + R^0_1 \, r^1_{G_2}$$

$$r^0_{G_2} = r^0_H + R^0_1 \, R^1_2 \, r^2_{G_2}$$

$$= r^0_H + R^0_1 \, R^1_2 \begin{bmatrix} c \\ 0 \end{bmatrix}$$

$$r^0_{C_2} = r^0_H + R^0_1 \, R^1_2 \begin{bmatrix} \ell \\ 0 \end{bmatrix}$$



$$r^o = R_\gamma \, r^g$$

$$R_\gamma = \begin{bmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{bmatrix}$$

$$\boxed{r^g = \underline{R_\gamma^{-1}} \, r^o}$$

inverse

$$R_\gamma R_\gamma^{-1} = I \quad \Rightarrow \quad R_\gamma^{-1} = \begin{bmatrix} \cos(-\gamma) & -\sin(-\gamma) \\ \sin(-\gamma) & \cos(-\gamma) \end{bmatrix}$$

$\Rightarrow$ so for $\left.\begin{array}{l} \underline{r_{G_1}^o, \; r_{G_2}^o, \; r_H^o} \\ \underline{r_{G_1}^g, \; r_{G_2}^g, \; r_H^g} \end{array}\right\}$ Positions

Velocity

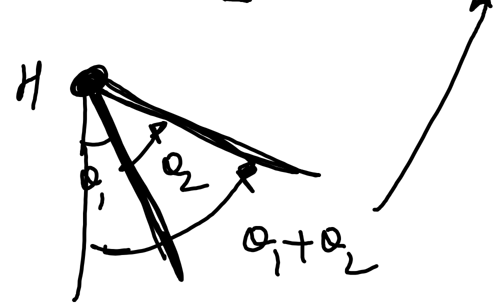
$$v_{G_1} = \underline{\dot{r}_{G_1}^o} = \frac{\partial J_{G_1}}{\underline{\partial q}} \dot{q}$$

$v_H, \; v_{G_2}$

② Euler-Lagrange

$$T = \frac{1}{2} M\left(v_{H_x}^2 + v_{H_y}^2\right) + \frac{1}{2} m\left(v_{G_{1x}}^2 + v_{G_{1y}}^2\right) + \frac{1}{2} m\left(v_{G_{2x}}^2 + v_{G_{2y}}^2\right) + \frac{1}{2} I \omega_1^2 + \frac{1}{2} I (\omega_1 + \omega_2)^2$$

$$\omega_1 = \dot{\theta}_1 \quad ; \quad \omega_2 = \dot{\theta}_2$$



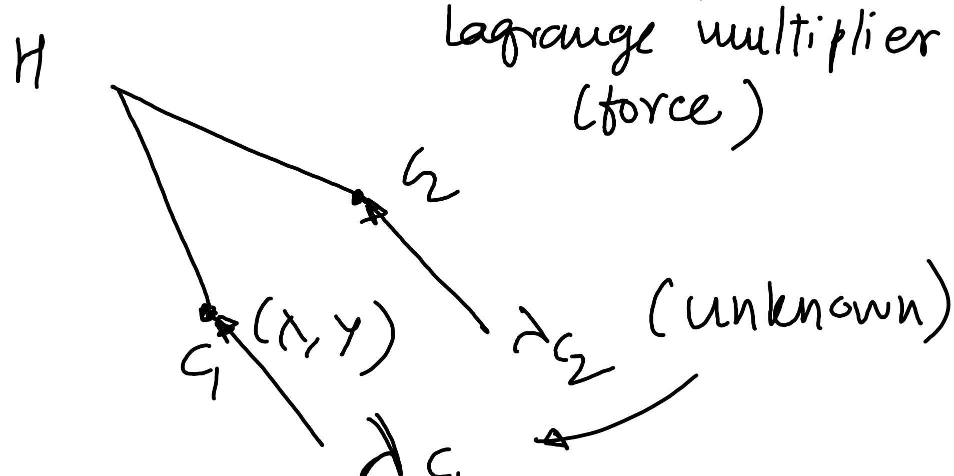
$$V = M g y_H^g + m g y_{G_1}^g + m g y_{G_2}^g$$

$$\mathcal{L} = T - V$$

③ <u>Euler-Lagrange equations</u>

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = \lambda_{C_1} \overbrace{\left(\frac{\partial r_{C_1}^o}{\partial q}\right)}^{= J_{C_1}} + \lambda_{C_2} \overbrace{\left(\frac{\partial r_{C_2}^o}{\partial q}\right)}^{= J_{C_2}}$$

Lagrange multiplier (force)



$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right) - \frac{\partial L}{\partial q} = J_{c_1}^T F_{c_1} + \cancel{J_{c_2}^T F_{c_2}}$$

$H \to c_2$

$H \to c_1$

c_1 (x, y) = anchore $\ddot x = \ddot y = 0$

since c_1 is anchored & c_2 is free

$F_{c_2} = 0$

$$\underbrace{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right) - \frac{\partial L}{\partial q}} = J_{c_1}' \overset{?}{F_{c_1}}$$

In MATLAB

$(\theta_1, \theta_2, \dot\theta_1, \dot\theta_2)$

$$\underset{4\times 4}{M}\, \ddot{\underline{z}} = \underset{4\times 1}{B(\theta, \dot\theta)} + \underset{4\times 2}{J_{c_1}'}\, \underset{2\times 1}{F_{c_1}}$$

$\ddot{\underline{z}} = \begin{Bmatrix} x \\ y \\ \theta_1 \\ \theta_2 \end{Bmatrix}$ $\quad F_{c_1} = \begin{Bmatrix} F_{c_1 x} \\ F_{c_1 y} \end{Bmatrix}$

MATLAB: we ~~deo~~ not solve for $F_{c_1 x}$ & $F_{c_1 y}$

simplify

$$\boxed{\underset{2\times 2}{A} \underset{2\times 1}{\begin{bmatrix} \ddot\theta_1 \\ \ddot\theta_2 \end{bmatrix}} = \underset{2\times 1}{b}}$$

MATLAB

$\ddot z = [\ddot x, \ddot y, \ddot\theta_1, \ddot\theta_2) = [0, 0, \ddot\theta_1, \ddot\theta_2]$