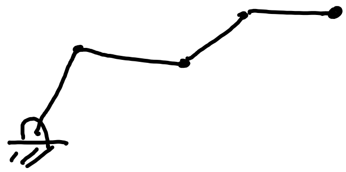
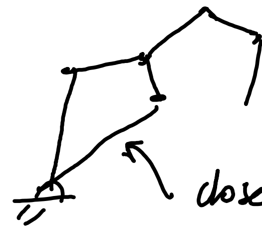
Manipulator Kinematics in 3D

Open link manipulator



Only study these manipulators in this course

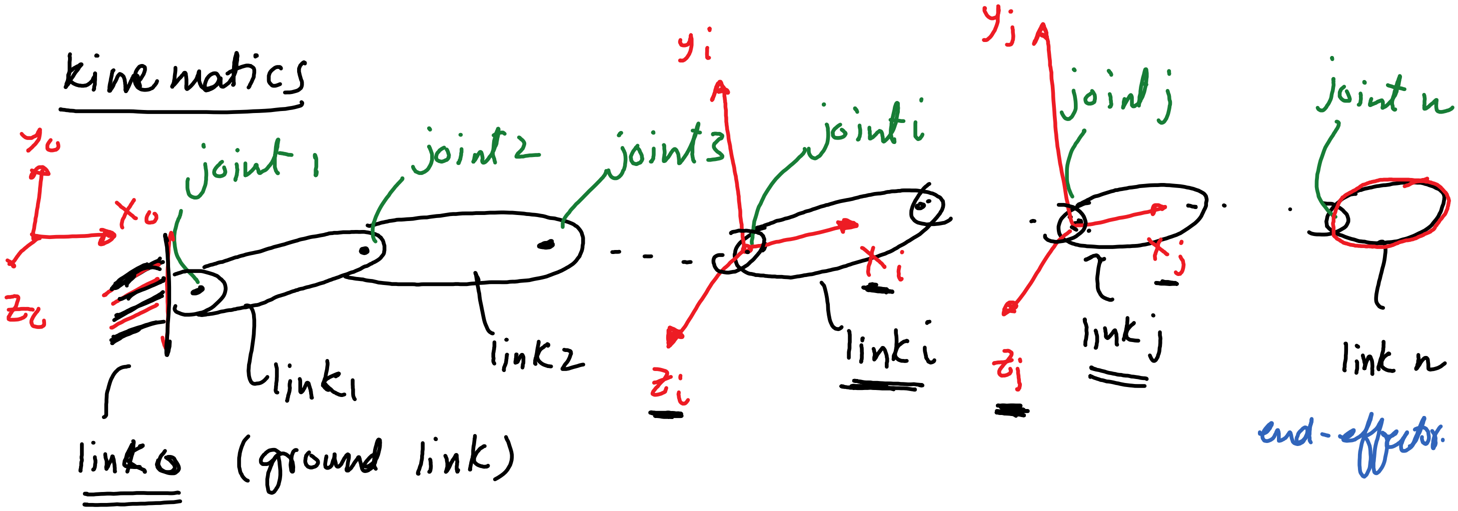
Closed link manipulator



closed chain

e.g. Stewart Platform.

kinematics



$$H^i_j = \begin{cases} H^i_{i+1}\, H^{i+1}_{i+2} \cdots H^{j-1}_j & i<j \\ I & i=j \\ \left(H^j_i\right)^{-1} & i>j \end{cases}$$

Homogeneous transformation

I = 4×4 identity matrix

Denavit - Hartenberg convention (DH)

$$H_i^{i-1} = H_z(\theta_i)\, H_z(d_i)\, H_x(a_i)\, H_x(\alpha_i)$$

$$= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$c\theta_i = \cos\theta_i$; $s\theta_i = \sin\theta_i$. $c\alpha_i = \cos\alpha_i$; $s\alpha_i = \sin\alpha_i$

$\theta_i, d_i, \alpha_i, a_i$ → DH parameters

4 numbers to describe each link. However, we know that it takes 6 number $(x, y, z, \theta, \phi, \psi)$ to describe position/orientation of each link. We get away with only 4 number (as opposed to 6) because DH uses a special way of defining the axis of each link

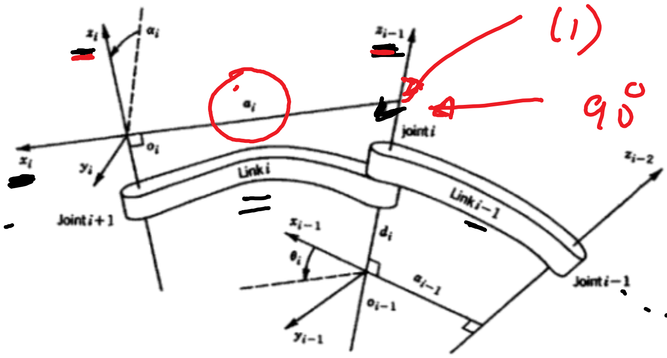
1. axis x_i is perpendicular to z_{i-1}
2. axis x_i intersecting z_{i-1}

1. a_i is the distance between z_i and z_{i-1} along x_i.
2. α_i is the angle between z_i and z_{i-1} along x_i.
3. d_i is the distance between x_{i-1} and x_i along z_{i-1}.
4. θ_i is the angle between x_{i-1} and x_i along z_{i-1}.

From the book by Spong

intersection (1)

90° (2)



end-effector

joint 0

Algorithm for using DH for forward kinematics There are three steps.

1. **Assign coordinate frames:**

   (a) Assign z_i along the axis of actuation for each link, where $i = 0, 1, 2, ...(n-1)$.

   (b) Assign the base frame $o_0 - x_0 - y_0 - z_0$. The z_0 has already been assigned. Assign x_0 arbitrarily. Assign y_0 based on x_0 and z_0 using right hand rule.

   (c) Now assign coordinate frames $o_i - x_i - y_i - z_i$ for $i = 1, 2, ..., n-1$. z_i is already attached in first step. Next we assign x_i using these rules.

      i. **z_{i-1} and z_i are not coplanar:** In this case, there is a unique shorted distance segment that is perpendicular to z_{i-1} and z_i. Choose this as x_i axis. The origin o_i is where x_i intersects z_i. The y_i is found from right hand rules.

      ii. **z_{i-1} and z_i parallel:** In this case, there infinitely many perpendiculars. Choose any of these perpendiculars for x_i. Furthermore, where x_i intersects z_i we draw the origin x_i. Finally, y_i is found from the right hand rule. To make equations simpler, choose x_i such that is passes through o_{i-1}. This will make $d_i = 0$. Also, since z_{i-1} is parallel to z_i, $\alpha_i = 0$.

      iii. **z_{i-1} and z_i intersect:** In this case, x_i is chosen to be normal to the plane formed by z_{i-1} and z_i. There will be two possible directions for x_i, one of them is chosen arbitrarily and o_i is obtained by the intersection of $z - i$ and x_i. Finally y_i is obtained from right hand rule. Also, since z_{i-1} intersects z_i, $a_i = 0$.

   (d) Finally we need to attach an end effector frame, $o_n - x_n - y_n - z_n$. Attach z_n to be the same direction as z_{n-1}. Now depending on the relation between z_n and z_{n-1}, attach frame x_n. Finally, attach y_n using the right hand rule.

2. **Generate a table for DH parameter:** Now generate the DH table as follows.

Link	a_i	α_i	d_i	θ_i
1				
2				
.				
.				
.				
n				

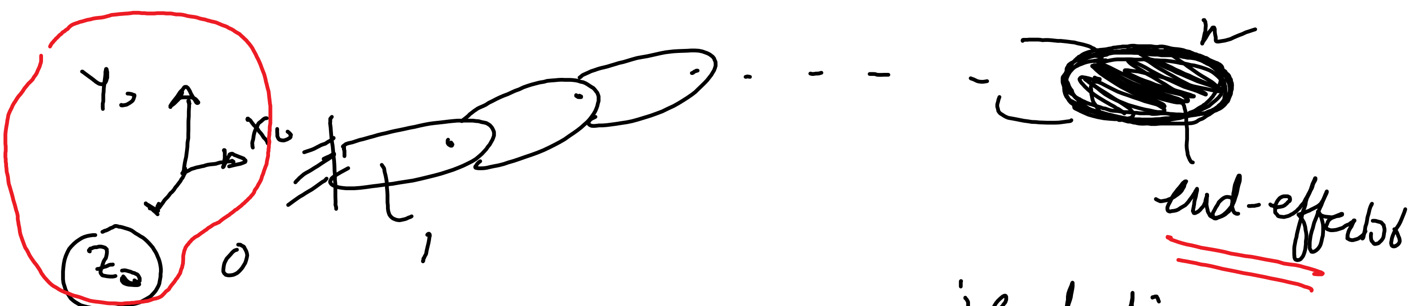
3. **Apply DH transformation to evaluate forward kinematics:** Finally, use the DH formulate to link two adjacent frames

$$\mathbf{H}_i^{i-1} = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} = H_z(\theta_i)\, H_z(d_i) \cdots H_x(a_i)\, H_x(\alpha_i)$$

The position and orientation of the end-effector is found using the formula

$$\mathbf{H}_n^0 = \underline{\mathbf{H}_1^0 \mathbf{H}_2^1 \mathbf{H}_3^2 \ldots \mathbf{H}_n^{n-1}} = \begin{bmatrix} \mathbf{R}_n^0 & \mathbf{d}_n^0 \\ \mathbf{0} & 1 \end{bmatrix} \quad 4 \times 4$$

The position of the end-effector is $\mathbf{d}_n^0$ and the orientation is $\mathbf{R}_n^0$. From $\mathbf{R}_n^0$, we can recover the Euler angles for the end-effector frame.



fixed frame

orientation R_n^0

position d_n^0