Versatile and Energy Efficient Walking for the Simplest Biped

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***Abstract*—This report presents a methodology to generate energy efficient, steady state and non-steady state walking motions for a simple biped robot, and analyzes how the Mechanical Cost Of Transport (MCOT) changes with step length and step velocity. For simplicity, we ignore the mechanical energy of swing leg which is a small potation of total mechanical energy. Simulation results show that for steady state walking, MCOT increases with step velocity and step length. However, for non-steady state walking, MCOT increases with step velocity but decreases with step length. Next, multiple-step walking is generated to compare its MCOT with that of one-step walking for the same distance travelled. Results show that MCOT and total step time respectively decreases and increases as the number of steps increases.**

***Index Terms*—Passive dynamic walking, energy efficiency, multi-step walking, versatility, walking speed.**

I. Introduction

PASSIVE dynamic walking has been developed as a possible explanation for energy-efficient human walking. McGeer [1] showed that a dynamic walker without any actuation or control can walk stably on downhill shallow slopes powered by gravity. Garcia et al. [2] proposed the simplest stable walking model of previously developed passive dynamic walkers but allows one to take advantage of analytical methods to analyze its dynamics. Some researchers have added actuators to passive dynamic walkers to enable them to walk on level ground while talking advantage of their high energy efficiency [3]. Particularly, applying an impulse at toe-off immediately before touch down or a torque on the stance leg are two general methods of actuation [1].

Though passive dynamic walkers are well-known for their energy efficiency, they mostly suffer from versatility. Changing speed is one of the aspects that can increase versatility of dynamic walkers. There has been little research on changing speed of walkers while keeping the energy cost low in the literature. Mandersloot et al. [4] used a dynamic programming approach to develop control laws for each desired velocity. Hobbelen and Wisse [5] controlled the walking speed using a feedforward and feedback control, but the desired speed was achieved after several steps. Van Zijl [6] proposed a strategy to make walking speed transitions in a single step. Seethapathi and Srinivasan [7] showed that the cost of changing walking speeds between steps is quite high, and predicted that subjects prefer lower speeds for shorter distances.

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In this study, energy efficient walking motions are realized for a simple biped walking on level ground for both steady state and non-steady state cases. For simplicity, we ignore the energy cost of swing leg in our optimization. Furthermore, we develop multiple step planning motions in which the walker takes several short steps instead of a single long step.

This report is set up as follows. In Section II, the model of the walker is introduced. Section III presents the methods used to develop energy-efficient, versatile walking. Simulation results are given in Section IV. Section V gives discussion. The report is finished with a conclusion in Section VI.

II. Model

A. Model Description

Fig. 1 (a) shows a cartoon of the simplest walker. The model has a mass M at the hip and point mass m at each of the feet. Each leg has length. Gravity g points downwards. The leg in contact with the ramp is called the stance leg while the other leg is called the swing leg. The angle made by the stance leg with the normal to the ramp is θ and the angle made by the swing leg with the stance leg is φ. The hip torque is T. There is a torsional spring with spring constant K between the two legs (not shown). The rest length of the spring is zero and corresponds to the position when both legs are parallel. Fig. 1 (b) describes a typical step of the simplest walker. In the mid-stance position shown in (iii), the gravity is along the stance leg direction. Between (iii) and (iv), the swing leg penetrates the ground leading to foot scuffing. We ignore this but a physical robot needs a mechanism to lift the leg. At (v) or the instance just before the foot strike, an impulsive push-off, P, is applied by the trailing leg. In (vi), the swing leg collides with the ground and becomes the new stance leg.

B. Equations of Motion

A single step of the walker is given below:

$$\underbrace{\text{Single Stance} \overset{\text{collision}}{\overbrace{\rightarrow}} \text{Heel}-\text{strike} \rightarrow}_{\text{one step/ period}-\text{one limit cycle}}\text{Single Stance} \tag{1}$$

We describe the equations for single stance, heel-strike and

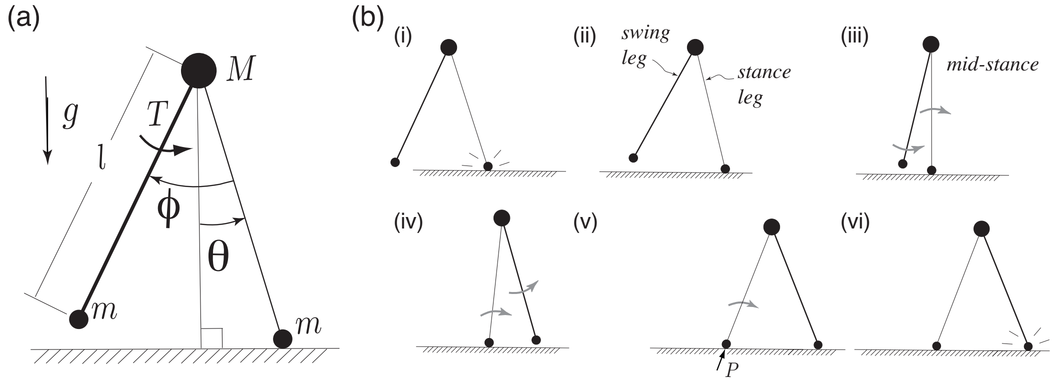

Fig. 1. Simplest walking model.

the switching surface called collision in the above equation.

C. Single Stance Phase (Continuous Dynamics):

In this phase of motion, the stance leg pivots and rotates about the stationary foot; while the swing leg pivots and rotates about the hinge connecting the two legs. We assume that the stance leg does not slip, there is no hip hinge friction, and ignore foot scuffing. We obtain (2) and (3) defined below by taking moments about stance foot contact point and hip hinge respectively, and non-dimensionalizing time with $\sqrt{\ell / g}$ and applying the limit, $m / M \rightarrow 0$. In (3), τ is the non-dimensional torque obtained by dividing the torque, T, by Mg. The non-dimensional spring constant is k and is obtained by dividing K by Mg*l*. The equations are

$$\ddot{\theta} = \sin(\theta) \tag{2}$$

$$\ddot{\phi} = \sin(\theta) + \{\dot{\theta}^2 - \cos(\theta)\}\sin(\phi) - k\phi + \tau \tag{3}$$

We suppose that there is no flight phase during locomotion. This is guaranteed through applying the following condition:

$$\dot{\theta}^2 - \cos 2\theta \le 0 \tag{4}$$

D. Collision Event (Switching Surface):

We integrate the single stance equations given above till the foot-strike event, wherein the swing leg is about to impact the ground. Using super-script and to denote the instant just before and just after foot-strike respectively, we can write the foot-strike event as

$$\cos(\theta^-) - \cos(\phi^- - \theta^-) = 0 \tag{5}$$

E. Heel-strike Phase (Discontinuous Dynamics):

In this phase of motion, the legs exchange their roles, that is, the current swing leg becomes the new stance leg and the current stance leg becomes the new swing leg. We assume that the stance leg applies an impulse P. This is followed by an instantaneous plastic collision (no slip and no bounce) of the swing leg. The swapping of legs is expressed by (6) and (7). The angular rates of the legs after support exchange are given by (8) and (9) and are obtained by applying conservation of angular momentum about stance foot contact point and hip hinge respectively, followed by non-dimensionalizing time with $\sqrt{\ell / g}$ and applying the limit, $m / M \rightarrow 0$. The non-dimensional impulse is p and is obtained by dividing the impulse, P by $M\sqrt{g\ell}$.

$$\theta^+ = -\theta^- \tag{6}$$

$$\phi^+ = -\phi^- = -2\theta^- \tag{7}$$

$$\dot{\theta}^+ = \dot{\theta}^- \cos 2\theta^- + p \sin 2\theta^- \tag{8}$$

$$\dot{\phi}^+ = (1 - \cos 2\theta^-)(p \sin 2\theta^- + \dot{\theta}^- \cos 2\theta^-) \tag{9}$$

III. METHODS

A. Single-Step Walking Motions

We develop a control strategy that enables the walker to change its speed in a single step. To this end, we define the Pointcare map as

$$\mathbf{x}_{n+1} = S(\mathbf{x}_n, \mathbf{u}_n) \tag{10}$$

where $\mathbf{x}_n$, $\mathbf{X}_{n+1}$, and $\mathbf{u}_n$ are the vector of states at the beginning of the step, the vector of states at the beginning of the next step, and the vector of control inputs, respectively. The function *S* is obtained through equations of motions. In case of steady state walking where the states at the beginning of the step are the same as the states at the beginning of the next step, Eq. (10) changes to

$$\mathbf{x}^* = S(\mathbf{x}^*, \mathbf{u}^*) \tag{11}$$

where $\mathbf{x}^*$ is called the vector of fixed points and $\mathbf{u}^*$ is the corresponding vector of control inputs. Eq. (10) is a set of nonlinear equations where the number of unknown variables (control inputs and some joint angles) is more than that of equations. To solve these equations, we use optimization.

A mathematical optimization problem has the form

$$\begin{aligned}&\text{minimize } f_0(\mathbf{x})\\&\text{subject to } f(x) \le \mathbf{b}\end{aligned} \quad (12)$$

Where **x** is the vector of optimization parameters, the function f_0 is the cost function, **f** is the vector of the constraint functions, and **b** is the vector of constant bounds for the constraints.

In our optimization problem we define Mechanical Cost of Transport (MCOT) as the cost function. MCOT is defined as the total mechanical energy used per weight per distance traveled.

In order for the walker to be able to walk on level ground, some mechanical energy should be added to the system to overcome the loss of energy during heel strike. This can be done by applying a push-off force P to the stance leg and/or torque τ to the swing leg. The mechanical work of force P and torque τ done on the system can be calculated as [3]

$$W_P = \frac{1}{2}P^2 \quad (13)$$

$$W_\tau = \int_0^T \tau\dot{\phi}\,dt \quad (14)$$

MCOT can be then defined as a weighted sum of the two mechanical works per weight and distance traveled,

$$MCOT = \frac{0.5P^2 + \beta\int_0^T \tau\dot{\phi}\,dt}{m_{tot}gd} \quad (15)$$

where β, $m_{tot}g$, and d are weighted factor, total weight, and distance traveled, respectively.

In this report, for simplicity we assume that β=0. In other words, we suppose that both swing and stance legs are massless and there is only one mass M at the hip. Thus, MCOT is reduced to

$$MCOT = \frac{P^2}{2m_{tot}gd} \quad (16)$$

We consider all (initial, before and after heel-strike, and final) joint variables of the stance leg and P as optimization variables. The constraints of the optimization problem are

$$\begin{aligned}
&\theta^+ - (-\theta^-) = 0\\
&\dot{\theta}^+ - \left(\dot{\theta}^- \cos 2\theta^- + p \sin 2\theta^-\right) = 0\\
&d - \left(|\sin\theta^-| + |\sin\theta^+|\right) = 0\\
&\dot{\theta}^2 - \cos 2\theta \le 0\\
&\theta(0) - \theta(T) = 0\\
&\dot{\theta}_1 - \dot{\theta}(0) = 0\\
&\dot{\theta}_2 - \dot{\theta}(T) = 0\\
&P_{min} \le P \le P_{max}\\
&\theta_{min} \le \theta \le \theta_{max}\\
&\dot{\theta}_{min} \le \dot{\theta} \le \dot{\theta}_{max}
\end{aligned} \quad (17)$$

To summarize, first we fix the step length, the initial velocity and final velocity. Next, we divide the single step walking into three phases (Eq. (1)): 1- single stance, 2- heel-strike, 3- single stance. The first single-stance phase is continuous in time and the walking motion in this phase is computed by forward integration of Eq. (2) over the time interval from the beginning of the walking to just before heel-strike. The heel-strike phase occurs at an infinitesimal time interval where there is a jump in joint velocities. This phase is discontinuous in time and the walking motion in this phase is computed by solving algebraic equations (Eqs. (6) and (8)). The second single stance is similar to the first single stance, but with a difference, the walking motion in this phase is computed over the time interval from just after heel-strike to the end of walking. Finally, we solve Eq. (16) subject to the constraints expressed in Eq. (17) using SNOPT [8].

B. *Multiple Steps Walking Motions*

Multiple steps walking motions consist of N single-step walking motions where the continuity of joint variables between steps should be maintained. Thus the procedure is similar to that mentioned in the previous section but, the cost. function is modified as

$$MCOT = \sum_{i=1}^{N} \frac{P_i^2}{2m_{tot}\,g\,d_i} \quad (18)$$

Where P_i is the push-off force applied at step i and d_i is the step length at step i.

In order to compare the energy efficiency of single-step walking motion and multiple steps walking motions, we apply the following constraint (Eq. (19)) where d^* is the total distance traveled by the walker. In other words, we are interested in determining whether it is more energy efficient to take only one long step with the length d^* or to take N short steps d_i.

$$d^* - \sum_{i=1}^{N} d_i = 0 \quad (19)$$

IV. Results

In order to determine how MCOT changes with step length and step velocity for steady state walking, we solve the optimization problem for each particular step length and the same initial and final velocities. Fig. 2 shows the contour of MCOT as a function of step length and step velocity. As can be seen from the figure, MCOT increases as the step length increases and MCOT increases as the step velocity increases, and vice versa. The minimum value of MCOT is 1.566 (10^{-4}) which is corresponding to the step length of 0.1 and step velocity of 0.1. The maximum value of MCOT is 0.09338 which is corresponding to the step length of 0.7 and step velocity of 0.9.

For Non-steady state walking, we fix the initial velocity at

$\dot{\theta}_1 = 0.5$ and solve the optimization problem for different values of step length and final step velocity. Fig. 3(a) demonstrates the plot of MCOT as a function of final step velocity $\dot{\theta}_2$ for $\dot{\theta}_1 = 0.5$ and different values of step length. As seen from the figure, when the final step velocity increases for a fixed value of step length, MCOT also increases. This means that in order for the walker to have higher agility, a larger push-off force should be applied to the stance leg just before heel-strike. Furthermore, when the final step velocity is $\dot{\theta}_2 = 0.5$ (steady state walking), we see that MCOT increases as the step length increases, similar to Fig. (2). However, when the final step velocity is higher than the initial step velocity, MCOT decreases as the step length increases. This is due to the fact that first MCOT is defined as energy usage per weight per distance traveled (step length) so large step length means smaller MCOT, and second larger step length requires longer time assuming the velocity is constant (Fig. 3(b)). When the step length increases, the walker has longer time to reach the specified final velocity. This implies that the amount of push-off force as the control input can decrease, leading to less energy usage.

Fig. 4 depicts MCOT versus step length when the walker speed decreases from $\dot{\theta}_1 = 0.8$ to $\dot{\theta}_2 = 0.7$. From figure, it is clear that MCOT increases as step length increases. It should be noted that the walker is unable to decrease the speed considerably. For decreasing speed, the walker needs to dissipate energy. Since there is no control input to remove energy from the system, the required amount of energy can only be dissipated through heel-strike impact and taking longer step length. Since there is a bound on the step length the walker can take, reaching to much slower speed is not always possible.

Fig. 5(a) shows MCOT versus number of steps for two cases, the blue dashed line for steady state walking $\dot{\theta}_1 = \dot{\theta}_2 = 0.5$, and the red dashed line for non-steady state walking, $\dot{\theta}_1 = 0.5$ and $\dot{\theta}_2 = 0.6$. For both cases, the total distance that the walker needs to travel is d^*=0.9. As we can see MCOT decreases as the number of steps increases for both cases. Thus taking several short steps is more energy efficient than talking one long step. Fig. 5(b) illustrates corresponding total step time versus number of steps. From the figure, we see the total step time increases as the number of steps increases. Since in our simulation mechanical cost of transport is minimized, time is a parameter that chosen by optimization automatically. We see that there is a trade-off between MCOT and total step time.

V. Discussion

We have presented a methodology that enables the walker to walk on level ground with steady state motion and non-steady state motion while keeping the energy cost minimum. The results confirm the previous observations that the cost of changing walking speeds between steps is considerably high.

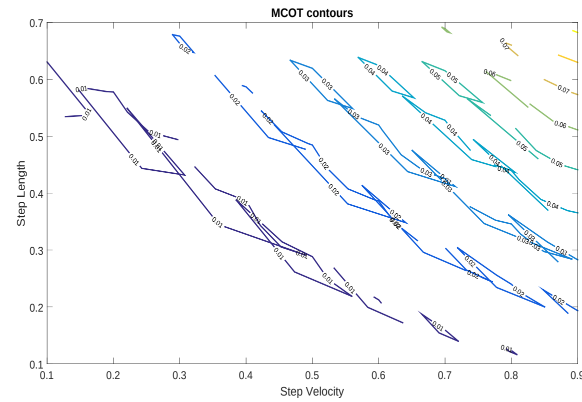

Fig. 2. The contour of MCOT as a function of step length and step velocity.

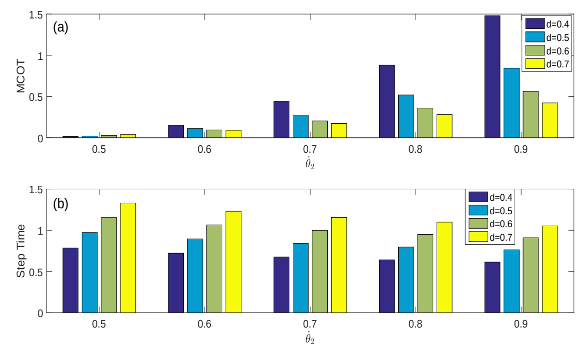

Fig. 3. (a) MCOT versus $\dot{\theta}_2$ for different values of step length and $\dot{\theta}_1 = 0.5$, (b) Step time versus $\dot{\theta}_2$ for different values of step length and $\dot{\theta}_1 = 0.5$.

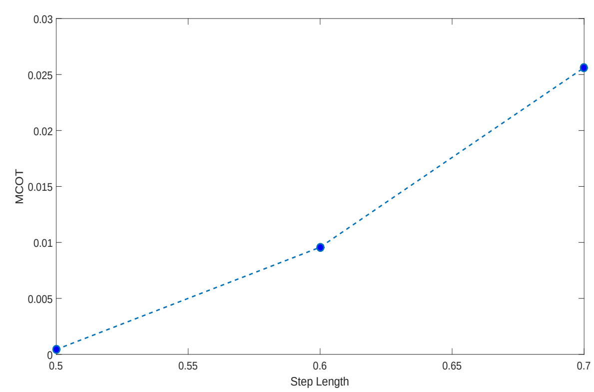

Fig. 4. MCOT versus step length for $\dot{\theta}_1 = 0.8$ to $\dot{\theta}_2 = 0.7$

Moreover, multiple-steps walking motions are showed to be more energy efficient than a single step walking motion for a particular traveled distance.

Our results imply that there is a trade-off between energy efficiency and versatility. When energy efficient is the main priority, the walker should take step length and step velocity corresponding to minimum energy cost and stick to the steady state walking. However, when the agility is important or the walker desires to take different step lengths in order to pass

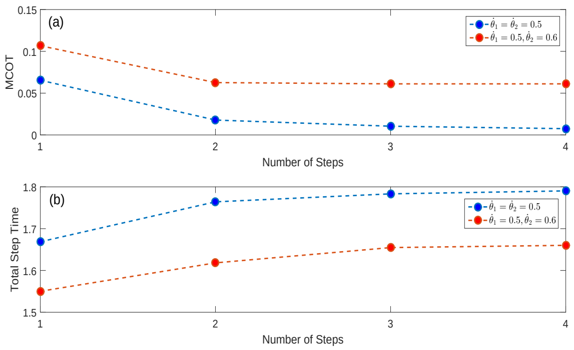

Fig 5. (a) MCOT versus number of step for both steady state walking and non-steady state walking. (b) Total step time versus number of step for both steady state walking and non-steady state walking.

over ditches, non-steady state walking is preferred. In addition, we can see a trade-off between MCOT and total step time for multiple steps walking motions. Selection of appropriate number of steps (N) should be made based on task specifications.

While our optimization algorithm improves energy efficient and versatility of bipedal robots, it also demonstrates several shortcomings. First, we only considered the stance leg in the optimization. Though the energy usage of the swing leg is not considerable compared to that of the stance leg, the more accurate method would be to include the cost of swinging leg in the mechanical cost of transport. Second, stability and disturbance rejection are big issues for bipedal robots. We suppose that the biped is stable and no disturbance is applied to the robot. However, many kinds of disturbances exit in reality. Thus formulating the stability and disturbance rejection problems and imposing them as constraints in the optimization problem can make the system viable in real environments. Third, our algorithm is valid for only level ground. However, the most advantage of legged robots over other mobile robots is their ability to move on any terrain. Our approach does not consider the flight phase which happens when the speed of the robot exceeds a particular speed.

VI. Conclusion and Future work

This report presented an optimization algorithm to improve energy efficiency and versatility of bipedal robots. Simulation results demonstrated that for steady state walking, MCOT increases with step velocity and step length. However, for non-steady state walking, MCOT increases with step velocity but decreases with step length. Then, multiple-step walking was generated to compare its MCOT with that of one-step walking for the same distance travelled. Results showed that MCOT and total step time respectively decreases and increases as the number of steps increases. Future work would be considering swing leg cost in the optimization, taking external disturbances into account and including flight phase in locomotion.

References

[1] T. Mcgeer, "Passive Dynamic Walking," *International Journal of Robotics Research,* vol. 9, pp. 62-82, Apr 1990.

[2] M. Garcia, A. Chatterjee, A. Ruina, and M. Coleman, "The simplest walking model: stability, complexity, and scaling," *Journal of biomechanical engineering,* vol. 120, pp. 281-288, 1998.

[3] A. D. Kuo, "Energetics of actively powered locomotion using the simplest walking model," *Journal of biomechanical engineering,* vol. 124, pp. 113-120, 2002.

[4] T. Mandersloot, M. Wisse, and C. G. Atkeson, "Controlling velocity in bipedal walking: A dynamic programming approach," in *2006 6th IEEE-RAS International Conference on Humanoid Robots*, 2006, pp. 124-130.

[5] D. G. Hobbelen and M. Wisse, "Controlling the walking speed in limit cycle walking," *The International Journal of Robotics Research,* vol. 27, pp. 989-1005, 2008.

[6] L. H. F. Van Zijl, "Changing Speed in a Simple Walker with Dynamic One-Step Transitions," Delft University of Technology, 2009.

[7] N. Seethapathi and M. Srinivasan, "The metabolic cost of changing walking speeds is significant, implies lower optimal speeds for shorter distances, and increases daily energy estimates," *Biology letters,* vol. 11, p. 20150486, 2015.

[8] P. E. Gill, W. Murray, and M. A. Saunders, "SNOPT: An SQP algorithm for large-scale constrained optimization," *SIAM review,* vol. 47, pp. 99-131, 2005.