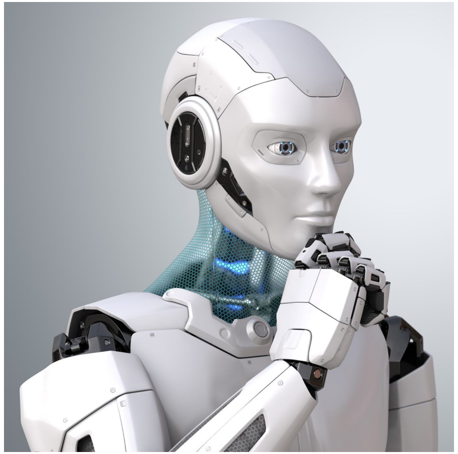
Trajectory Generation for Digit

By Tanmay Mittal

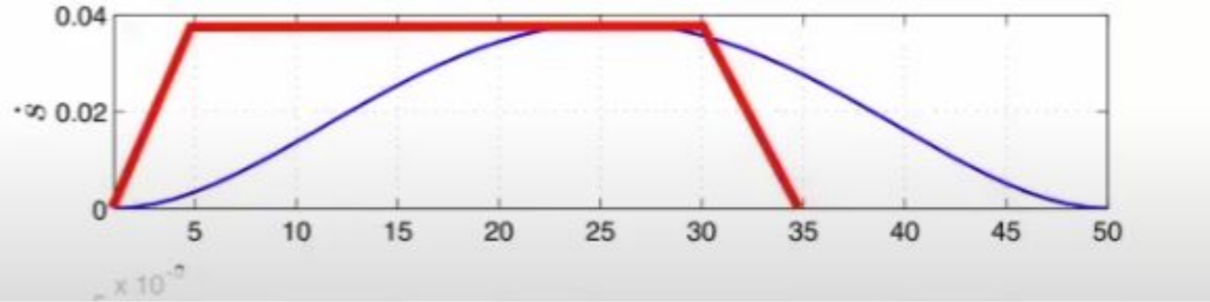
Introduction

The goal is to make the robot move by computationally calculating joint angles according to generated trajectory to achieve a desired distance. I worked on the trajectory generation part.



Trapezoidal trajectory

Trapezoidal vs polynomial comparison



Research paper method

JERK LIMITED VELOCITY PROFILE GENERATION FOR HIGH SPEED INDUSTRIAL ROBOT TRAJECTORIES

Proposes a trapezoidal trajectory

Generates a time optimal trajectory with the given jerk value (assuming possible max acceleration, max deceleration, max velocity)

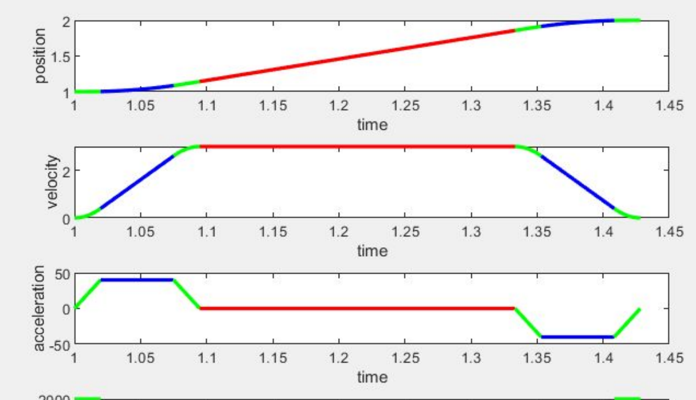
Modifies the jerk value to fit the proposed final time generating a time fixed trajectory(assuming possible max acceleration, max deceleration, max velocity)

Generate 3 time segments

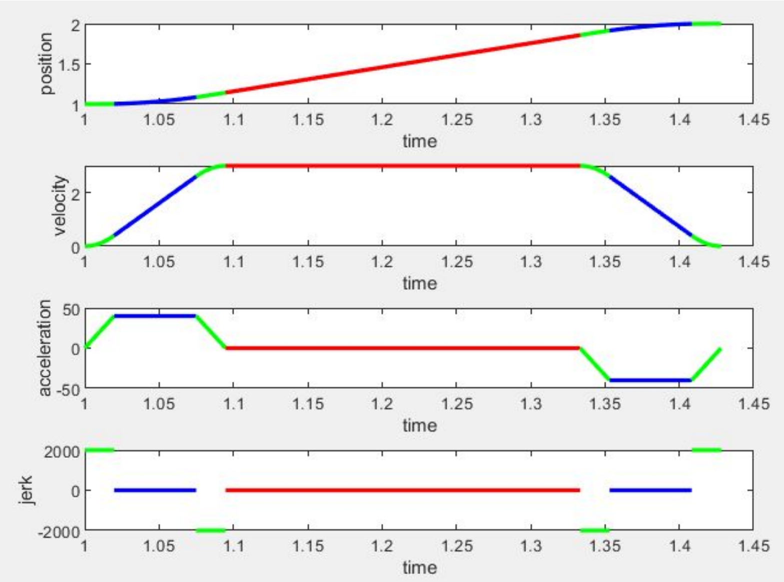
- Acceleration (x) - jerk(green) + constant acceleration(blue) + jerk(green)
- Constant velocity (x_hat) - constant velocity(red)
- Deceleration (x_line) - jerk(green) + deceleration(blue) + jerk(green)

Use piecewise functions to generate trajectory

https://www.sciencedirect.com/science/article/pii/S1474667016373815



Time optimal traveling period



Calculates the best time regions

X(acceleration)(green blue green)left side

X_hat (constant velocity)(red)

X_line (deceleration)(green blue green) right side

Does not change the green input jerk values

Decides to delete or keep the constant velocity(red) and/or acceleration(blue) and/or deceleration(blue).

And/or it decides to adjusts the max and min acceleration and deceleration value according to the equation below-

$$
\begin{aligned}
A &\leftarrow 0.5.J x, \text{ if } v_p - v_0 > 0.25.J x^2 (\text{no const. acc.}) \\
D &\leftarrow 0.5.J \bar{x}, \text{ if } v_p - v_f > 0.25.J \bar{x}^2 (\text{no const. dec.})
\end{aligned} \quad (7)
$$

Time fixed trajectories

Checks if input tf-t0 is larger than time optimal solution. If not it gives an error message and asks to set the optimal time value or a larger value to be the input tf-t0 difference.

Calculates time periods of acceleration deceleration and constant acceleration

1. X(acceleration)(green blue green)left side
2. X_hat (constant velocity)(red)
3. X_line (deceleration)(green blue green) right side

Adjusts the jerk value and stretch the graph to reach tf

Adjust A or D if necessary (explain the γ constant later)

$$A \leftarrow 0.5Jx, \text{ if } x < \frac{2A}{\gamma J} \text{ (no const. acc.)} \quad (33a)$$

$$D \leftarrow 0.5Jx, \text{ if } x < \frac{2D}{\gamma J} \text{ (no const. dec.)} \quad (33b)$$

$$J = J_e \leftarrow \gamma J \quad (33c)$$

Decides to delete the constant velocity(red) and/or acceleration(blue) and/or deceleration(blue) if the trajectory is going back in time accommodate the max velocity.

Jerk readjustment

A jerk minimizing constant y is calculated (different for each condition depending on whether constant acceleration and/or constant deceleration and/or constant velocity is removed).

Value between 0 & 1 to decrease the original jerk value

Jerk value decreased to make the graph continuous and respect the tf value while achieving accurate distance.

Acceleration and deceleration readjustments (rare)

Time optimal

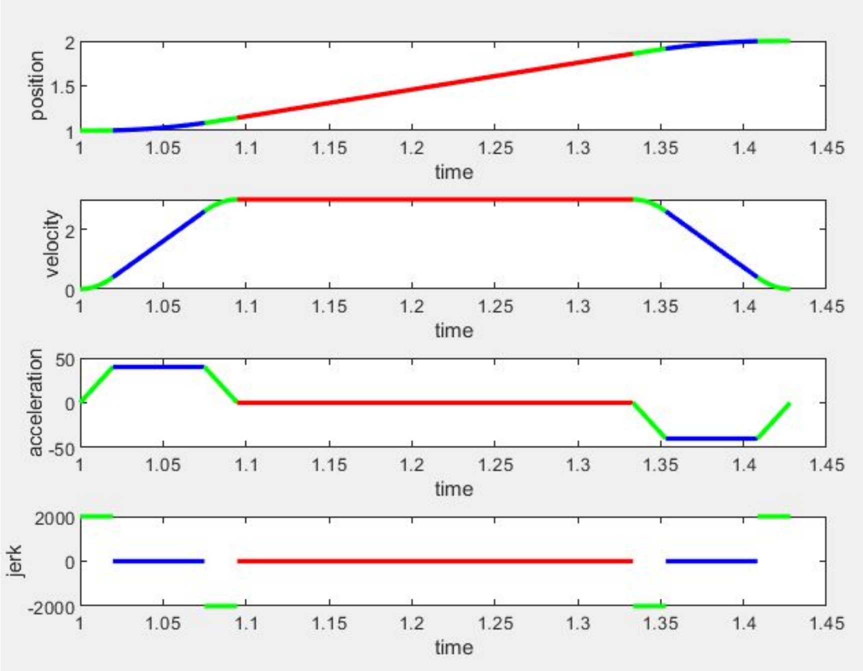
$$A \leftarrow 0.5Jx, \text{ if } v_p - v_0 > 0.25Jx^2 (\text{no const. acc.})$$
$$D \leftarrow 0.5Jx, \text{ if } v_p - v_f > 0.25Jx^2 (\text{no const. dec.}) \quad (7)$$

Time fixed

$$A \leftarrow 0.5Jx, \text{ if } x < \frac{2A}{\gamma J} \ (\text{no const. acc.}) \quad (33a)$$
$$D \leftarrow 0.5Jx, \text{ if } x < \frac{2D}{\gamma J} \ (\text{no const. dec.}) \quad (33b)$$
$$J = J_e \leftarrow \gamma J \quad (33c)$$

Normal trajectory

t4= constant velocity(red)
t2= constant acceleration(1st blue)
t6= constant deceleration(2nd blue



CASE ($T_4 > 0, T_2 > 0, T_6 > 0, v_0 \neq V, v_f \neq V$)

$$V = v_0 - \frac{A^2}{\gamma J} + Ax = v_f - \frac{D^2}{\gamma J} + Dx \qquad (46)$$

$$\left(v_0 - V + \frac{D}{A}(v_f - V)\right)x + 2VT - 2L + \frac{1}{D}(v_f - V)\left(\frac{D^2}{A^2}(v_0 - V) - v_f + V\right) = 0 \qquad (47)$$

$$\gamma = \frac{A^2}{J(Ax + v_0 - V)} \qquad (48)$$

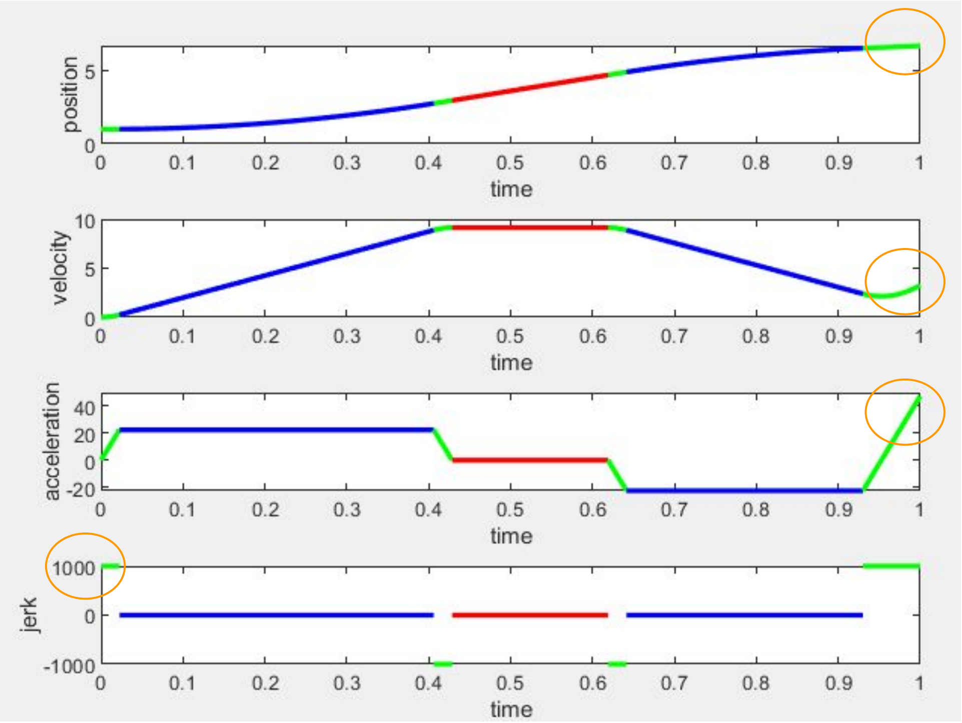
$$x = \frac{V - v_f}{D} + \frac{D}{J}$$

Jerk readjustment

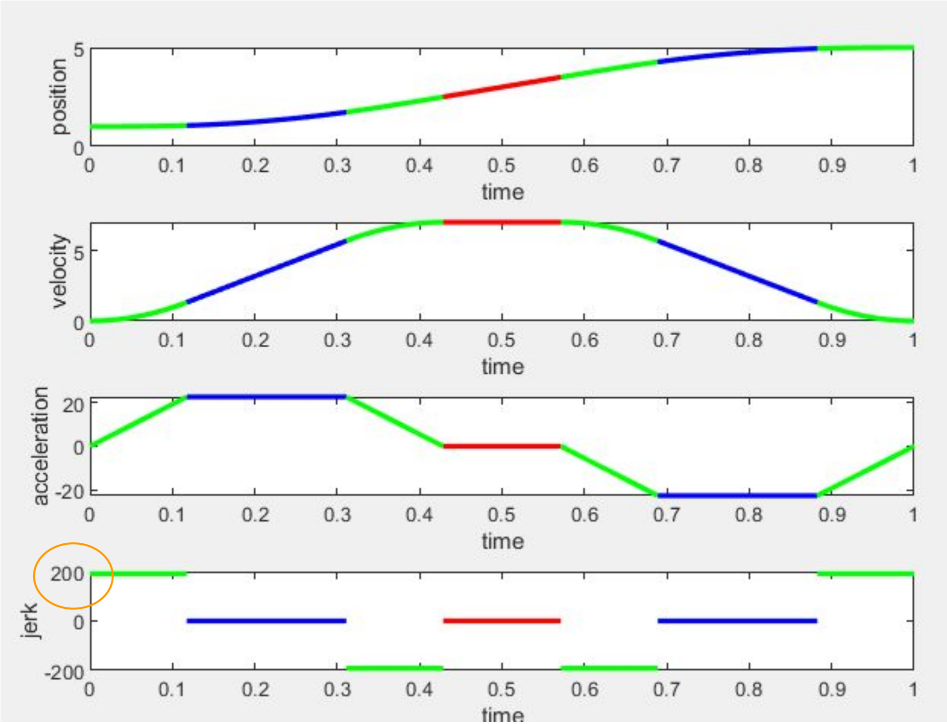
Different equation for each condition

$$\gamma = \frac{A^2}{J(Ax + v_0 - V)}$$

Without correction



With correction



Constant deceleration region removed

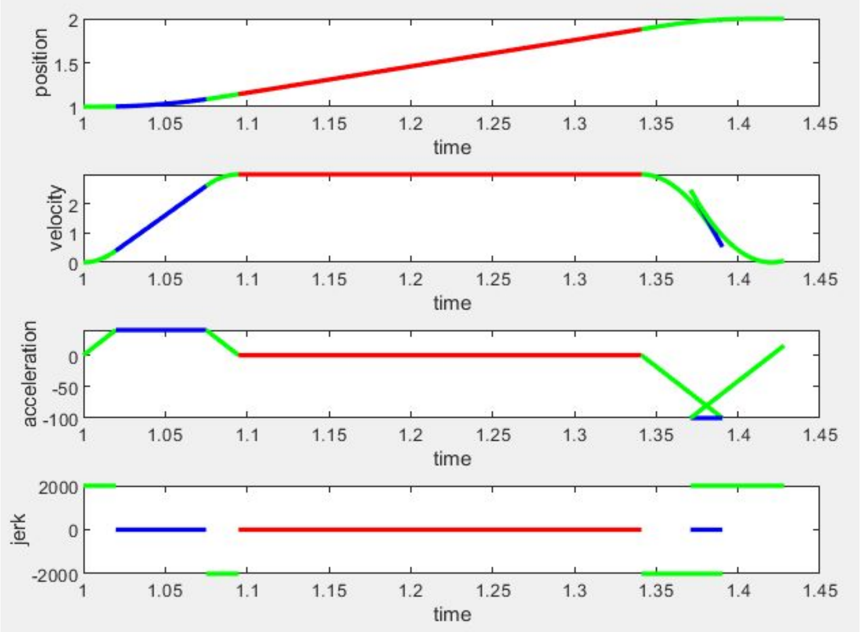
CASE ($T_4 > 0, T_2 > 0, T_6 = 0, v_0 \neq V, v_f \neq V$)

$$V = v_0 - \frac{A^2}{\gamma J} + Ax = v_f + \frac{1}{4}\gamma J x^2 \tag{52}$$

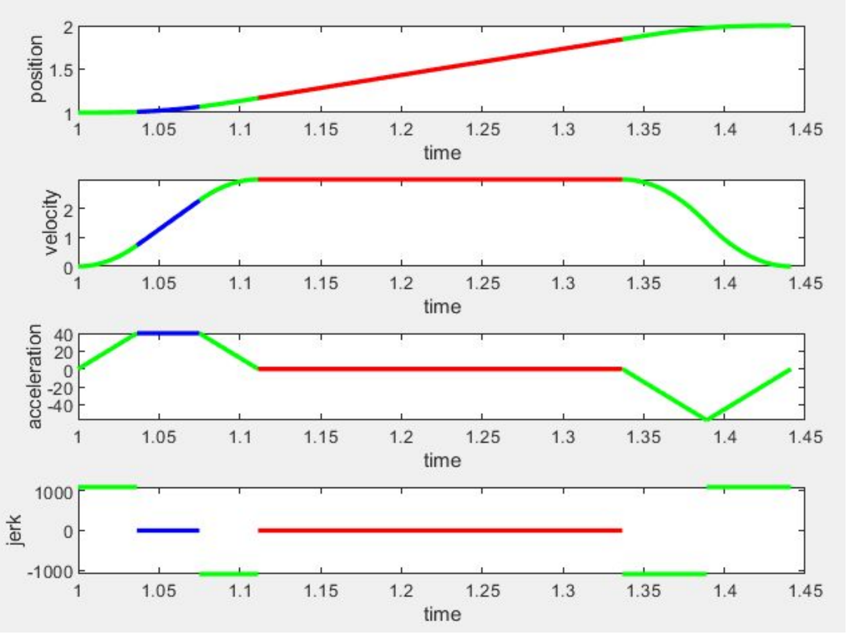
$$\frac{A(v_0 - V)}{4(V - v_f)}x^2 + (v_f - V)x + 2VT - 2L - \frac{1}{A}(v_0 - V)^2 = 0 \tag{53}$$

$$\gamma = \frac{4(V - v_f)}{Jx^2} \tag{54}$$

Without correction



With correction



CASE ($T_4 > 0, T_2 = 0, T_6 > 0, v_0 \neq V, v_f \neq V$)

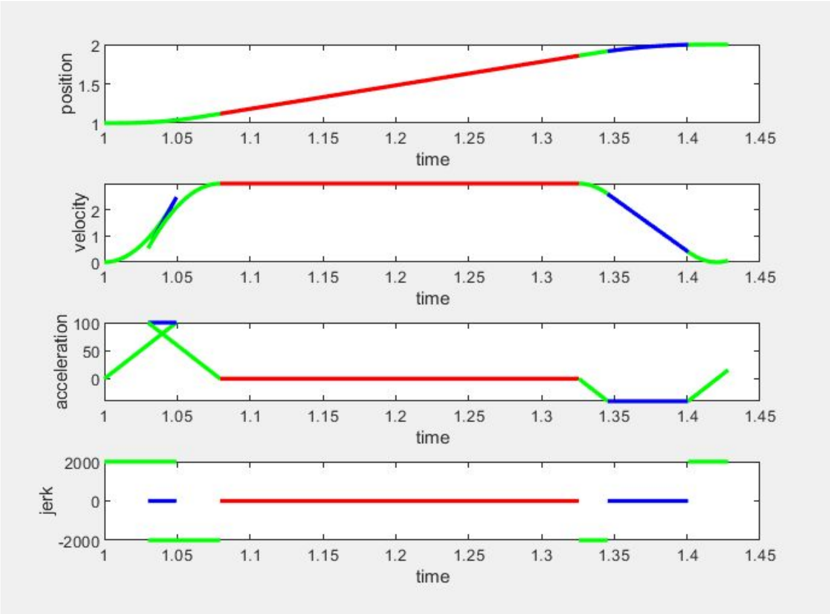
$$V = v_0 + \frac{1}{4}\gamma J x^2 = v_f - \frac{D^2}{\gamma J} + Dx \tag{49}$$

$$\frac{D(v_f - V)}{4(V - v_0)}x^2 + (v_0 - V)x + 2VT - 2L - \frac{1}{D}(v_f - V)^2 = 0 \tag{50}$$

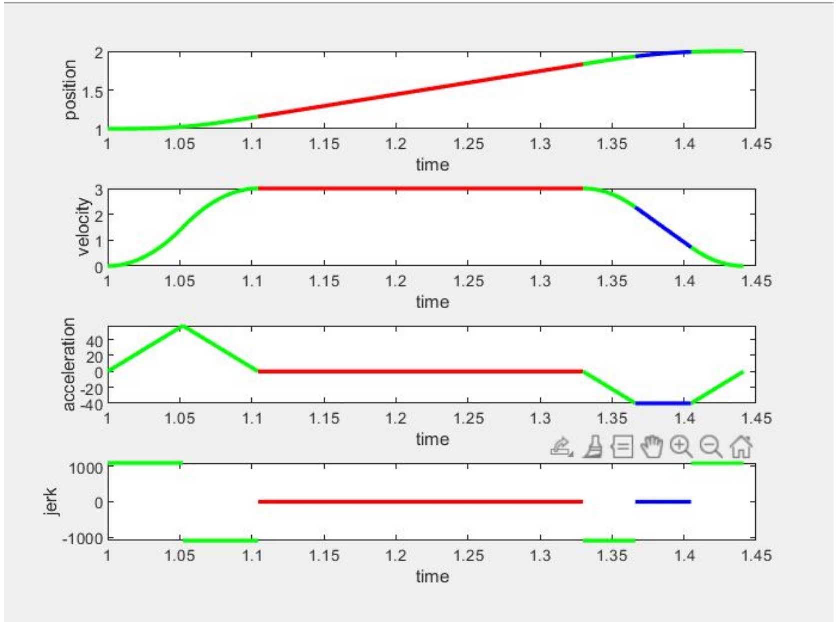
$$\gamma = \frac{4(V - v_0)}{Jx^2} \tag{51}$$

Constant Acceleration region removed

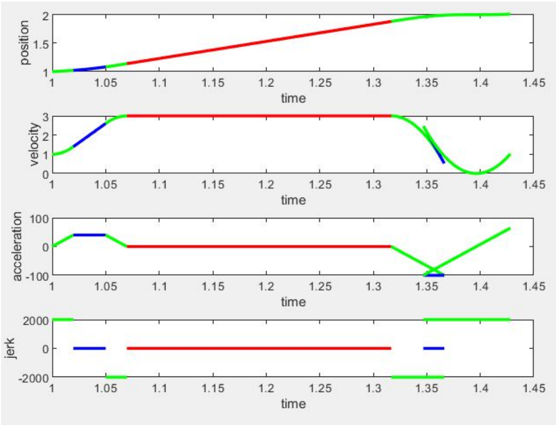
Without correction

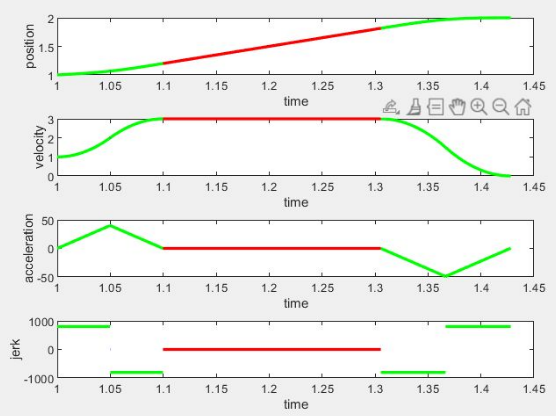


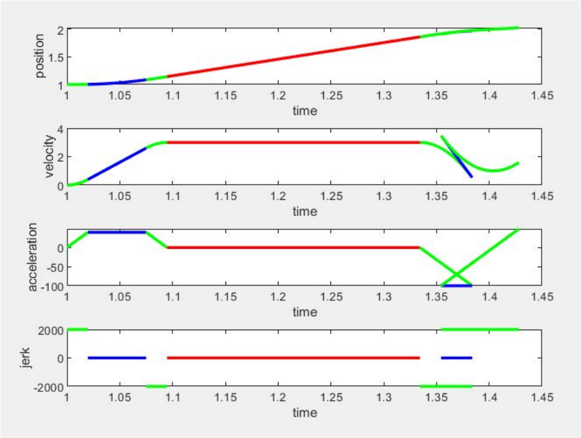
With correction

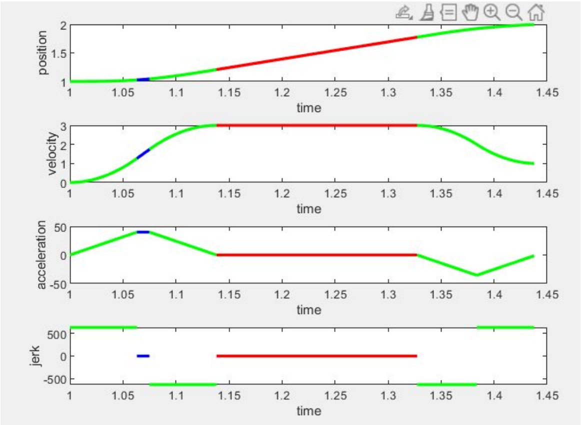


Non zero initial and final velocities with constant deceleration removed









Constant acceleration and deceleration regions removed

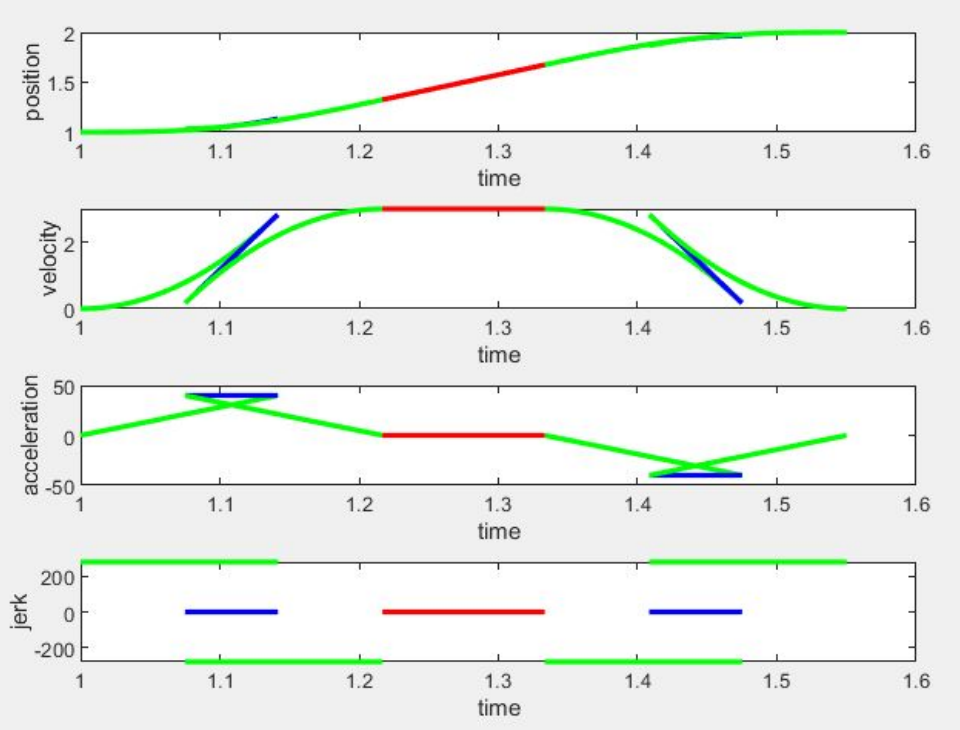
CASE ($T_4 > 0, T_2 = 0, T_6 = 0, v_0 \neq V, v_f \neq V$)

$$v = v_0 + \frac{1}{4}\gamma J x^2 = v_f + \frac{1}{4}\gamma J x^2$$

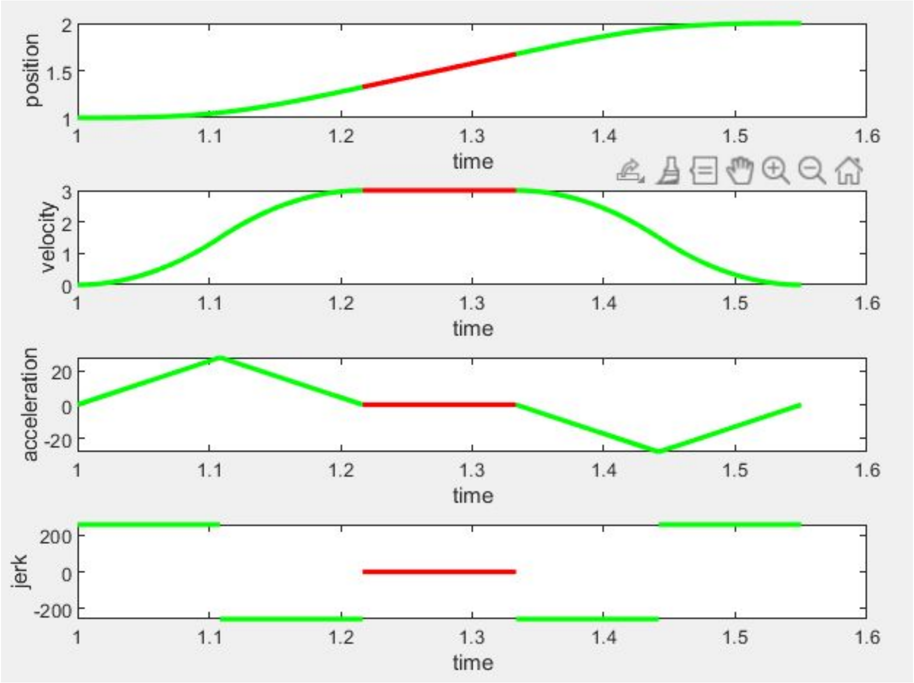
$$\left((v_0 - V)^2 - \frac{(v_f - V)^3}{(v_0 - V)}\right)x^2$$
$$-4(L - VT)(v_0 - V)x + 4(L - VT)^2 = 0$$

$$\gamma = \frac{4(V - v_0)}{Jx^2}$$

Without correction



With correction



Constant velocity region removed

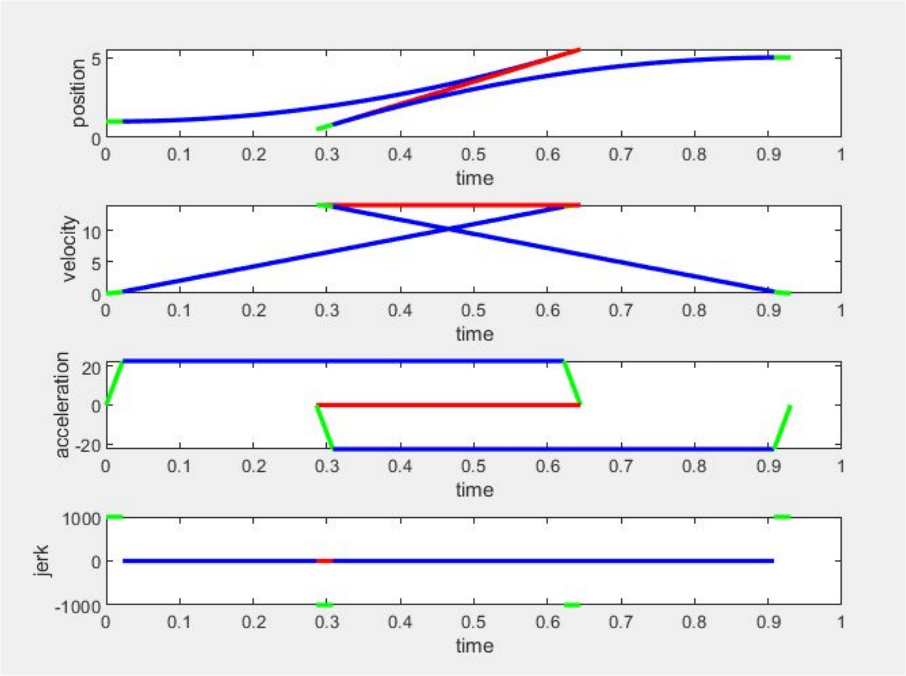
CASE ($T_4 = 0, T_2 > 0, T_6 > 0$)

$$v_p = v_0 - \frac{A^2}{\gamma J} + Ax = v_f - \frac{D^2}{\gamma J} + Dx \tag{34}$$

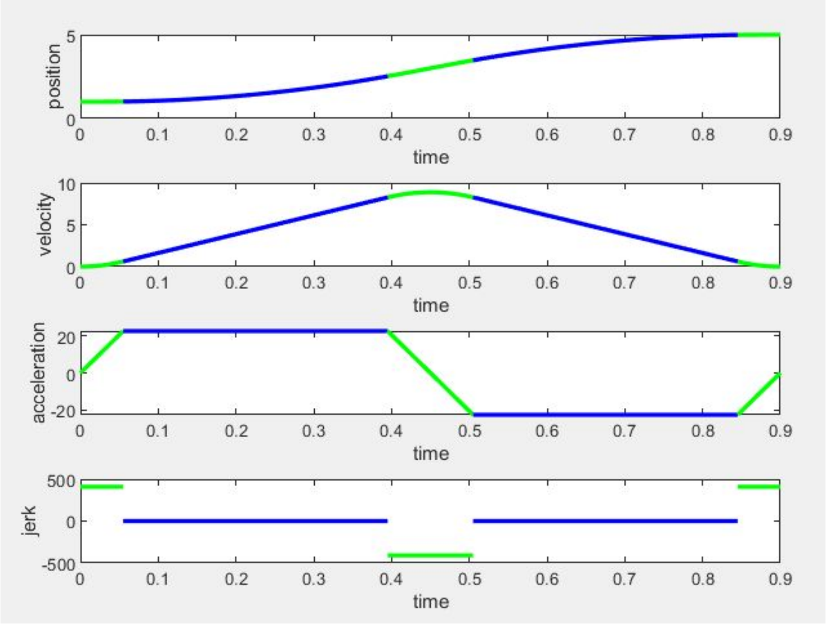
$$x = \begin{cases} \dfrac{\left(\frac{A^2(v_0 - v_f - DT)}{D^2 - A^2} + v_0 + v_f\right)T - 2L}{\left(\frac{A^2}{D-A} + A\right)T + v_0 - v_f} & , A \neq D \\ \dfrac{1}{2A}(v_f - v_0 + AT) & , A = D \end{cases} \tag{35}$$

$$\gamma = \begin{cases} -\dfrac{D^2 - A^2}{J\{(A+D)x + v_0 - v_f - DT\}} & , A \neq D \\ \dfrac{A^2 T}{J\{(v_0 + v_f)T + (v_0 - v_f + AT)x - 2L\}} & , A = D \end{cases} \tag{36}$$

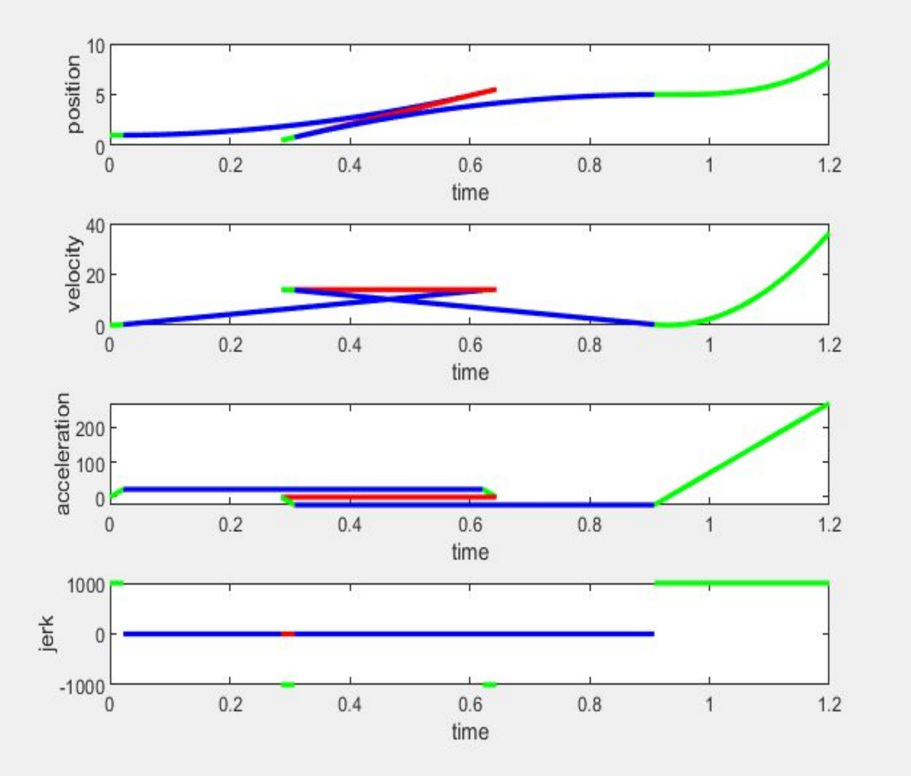
Without correction

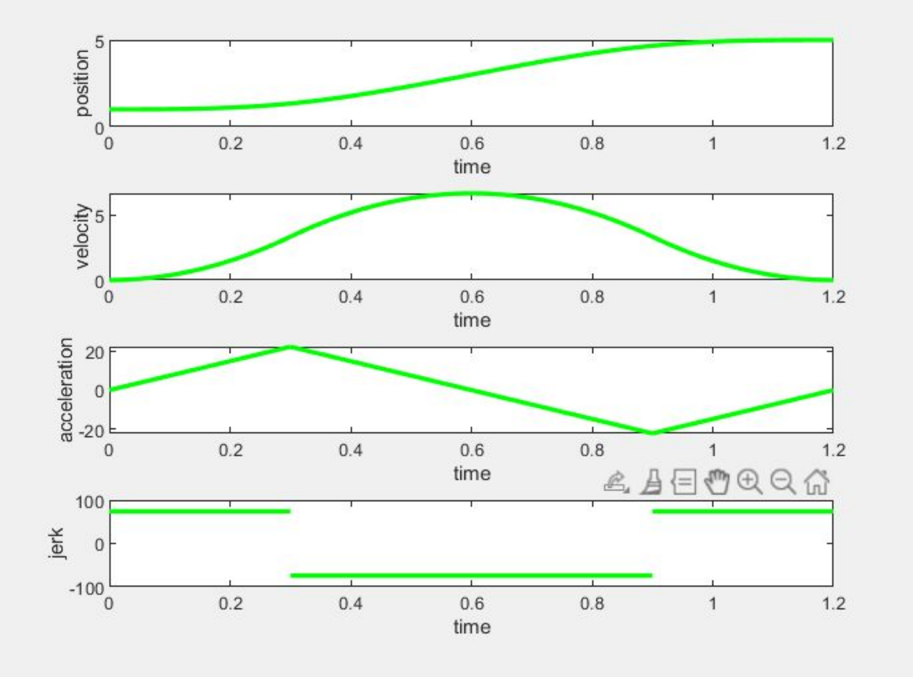


With correction



Constant velocity, acceleration and deceleration region removed





Time optimal equations t4>0 constant velocity region(red)

$$\hat{x} = \frac{2L - (v_0 + V)x - (V + v_f)x}{2V} \geq 0 \qquad (23)$$

CASE ($T_4 > 0, T_2 > 0, T_6 > 0$)

$$V = v_0 - \frac{A^2}{J} + Ax = v_f - \frac{D^2}{J} + Dx \qquad (24)$$

$$x = \frac{V - v_0}{A} + \frac{A}{J}, \; x = \frac{V - v_f}{D} + \frac{D}{J} \qquad (25)$$

CASE ($T_4 > 0, T_2 = 0, T_6 > 0$)

$$V = v_0 + \frac{1}{4}Jx^2 = v_f - \frac{D^2}{J} + Dx \qquad (26)$$

$$x = 2\sqrt{\frac{V - v_0}{J}}, \; x = \frac{V - v_f}{D} + \frac{D}{J} \qquad (27)$$

CASE ($T_4 > 0, T_2 > 0, T_6 = 0$)

$$V = v_0 - \frac{A^2}{J} + Ax = v_f + \frac{1}{4}Jx^2 \qquad (28)$$

$$x = \frac{V - v_0}{A} + \frac{A}{J}, \; x = 2\sqrt{\frac{V - v_f}{J}} \qquad (29)$$

CASE ($T_4 > 0, T_2 = 0, T_6 = 0$)

$$V = v_0 + \frac{1}{4}Jx^2 = v_f + \frac{1}{4}Jx^2 \qquad (30)$$

$$x = 2\sqrt{\frac{V - v_0}{J}}, \; x = 2\sqrt{\frac{V - v_f}{J}} \qquad (31)$$

Time optimal equations t4=0 constant velocity region(red)

CASE ($T_4 = 0, T_2 > 0, T_6 > 0$)

$$v_p = v_0 - \frac{A^2}{J} + Ax = v_f - \frac{D^2}{J} + Dx \tag{11}$$

$$A(\frac{A}{D} + 1)x^2 + \frac{1}{JD}(A + D)(AD - 2A^2 + 2v_0 J)x$$
$$-2L - \frac{1}{D}(v_0 + v_f - \frac{A^2}{J})(v_f - v_0 + \frac{A^2 - D^2}{J}) = 0 \tag{12}$$

$$x \geq \frac{2A}{J}, x \geq \frac{2D}{J} \tag{13}$$

CASE ($T_4 = 0, T_2 = 0, T_6 > 0$)

$$v_p = v_0 + \frac{1}{4}Jx^2 = v_f - \frac{D^2}{J} + Dx \tag{14}$$

$$\frac{J^2}{16D}x^4 + \frac{1}{4}Jx^3 + \frac{1}{4}(2\frac{Jv_0}{D} + D)x^2 +$$
$$2v_0 x - 2L + \frac{1}{D}(v_0 + v_f)(v_0 - v_f + \frac{D^2}{J}) = 0 \tag{15}$$

$$0 \leq x < \frac{2A}{J}, x \geq \frac{2D}{J} \tag{16}$$

CASE ($T_4 = 0, T_2 > 0, T_6 = 0$)

$$v_p = v_0 - \frac{A^2}{J} + Ax = v_f + \frac{1}{4}Jx^2 \tag{17}$$

$$\frac{J^2}{16A}x^4 + \frac{1}{4}Jx^3 + \frac{1}{4}(2\frac{Jv_f}{A} + A)x^2 +$$
$$2v_f x - 2L + \frac{1}{A}(v_f + v_0)(v_f - v_0 + \frac{A^2}{J}) = 0 \tag{18}$$

$$x \geq \frac{2A}{J}, 0 \leq x < \frac{2D}{J} \tag{19}$$

CASE ($T_4 = 0, T_2 = 0, T_6 = 0$)

$$v_p = v_0 + \frac{1}{4}Jx^2 = v_f + \frac{1}{4}Jx^2 \tag{20}$$

$$\frac{1}{4}(v_f - v_0)Jx^4 + JLx^3 - (v_f - v_0)^2 x^2$$
$$+8v_0 Lx - 4\{L^2 + \frac{1}{J}(v_0 + v_f)^2(v_f - v_0)\} = 0 \tag{21}$$

$$0 \leq x < \frac{2A}{J}, 0 \leq x < \frac{2D}{J} \tag{22}$$

Time fixed equations t4>0 constant velocity region(red)

CASE ($T_4 > 0, T_2 > 0, T_6 > 0, v_0 \neq V, v_f \neq V$)

$$V = v_0 - \frac{A^2}{\gamma J} + Ax = v_f - \frac{D^2}{\gamma J} + Dx \quad (46$$

$$\left(v_0 - V + \frac{D}{A}(v_f - V)\right) x + 2VT - 2L + \frac{1}{D}(v_f - V)\left(\frac{D^2}{A^2}(v_0 - V) - v_f + V\right) = 0 \quad (47)$$

$$\gamma = \frac{A^2}{J(Ax + v_0 - V)} \quad (48)$$

CASE ($T_4 > 0, T_2 = 0, T_6 > 0, v_0 \neq V, v_f \neq V$)

$$V = v_0 + \frac{1}{4}\gamma J x^2 = v_f - \frac{D^2}{\gamma J} + Dx \quad (49)$$

$$\frac{D(v_f - V)}{4(V - v_0)} x^2 + (v_0 - V)x + 2VT - 2L - \frac{1}{D}(v_f - V)^2 = 0 \quad (50)$$

$$\gamma = \frac{4(V - v_0)}{Jx^2} \quad (51)$$

CASE ($T_4 > 0, T_2 > 0, T_6 = 0, v_0 \neq V, v_f \neq V$)

$$V = v_0 - \frac{A^2}{\gamma J} + Ax = v_f + \frac{1}{4}\gamma J x^2 \quad (52)$$

$$\frac{A(v_0 - V)}{4(V - v_f)} x^2 + (v_f - V)x + 2VT - 2L - \frac{1}{A}(v_0 - V)^2 = 0 \quad (53)$$

$$\gamma = \frac{4(V - v_f)}{Jx^2} \quad (54)$$

CASE ($T_4 > 0, T_2 = 0, T_6 = 0, v_0 \neq V, v_f \neq V$)

$$v = v_0 + \frac{1}{4}\gamma J x^2 = v_f + \frac{1}{4}\gamma J x^2 \quad (55)$$

$$\left((v_0 - V)^2 - \frac{(v_f - V)^3}{(v_0 - V)}\right) x^2 - 4(L - VT)(v_0 - V)x + 4(L - VT)^2 = 0 \quad (56)$$

$$\gamma = \frac{4(V - v_0)}{Jx^2} \quad (57)$$

CASE ($T_4 > 0, v_0 = v_f = V$)
Since there are no acceleration and deceleration regions, the jerk scaling γ is meaningless, and we have the following.

$$x = x = 0, \hat{x} = T \quad (58)$$

CASE ($T_4 > 0, v_0 = V, v_f \neq V$)
Since there is no acceleration region, $x = 0$ and x can be found from the displacement condition (6).

$$x = \frac{2(L - VT)}{V - v_f} \quad (59)$$

We find the equations for γ by dividing the cases depending on whether or not constant acceleration region exists.

$$\gamma = \begin{cases} \dfrac{4(V - v_f)}{Jx^2} & , \dfrac{2(V - v_f)}{Jx} \leq \dfrac{D}{J} \\ \dfrac{D^2}{J(Dx - V + v_f)} & , \text{otherwise} \end{cases} \quad (60)$$

CASE ($T_4 > 0, v_0 \neq V, v_f = V$)
With similar reasoning used in the previous case, the following conditions.

$$x = \frac{2(L - VT)}{V - v_0}$$

$$\gamma = \begin{cases} \dfrac{4(V - v_0)}{Jx^2} & , \dfrac{2(V - v_0)}{Jx} \leq \dfrac{A}{J} \\ \dfrac{A^2}{J(Ax - V + v_0)} & , \text{otherwise} \end{cases}$$

Use x or x_line equations from time optimal solution if not specified

Time fixed equations t4=0 constant velocity region(red)

CASE ($T_4 = 0, T_2 > 0, T_6 > 0$)

$$v_p = v_0 - \frac{A^2}{\gamma J} + Ax = v_f - \frac{D^2}{\gamma J} + Dx \tag{34}$$

$$x = \begin{cases} \dfrac{\left(\frac{A^2(v_0 - v_f - DT)}{D^2 - A^2} + v_0 + v_f\right) T - 2L}{\left(\frac{A^2}{D-A} + A\right) T + v_0 - v_f} & , A \neq D \\ \dfrac{1}{2A}(v_f - v_0 + AT) & , A = D \end{cases} \tag{35}$$

$$\gamma = \begin{cases} -\dfrac{D^2 - A^2}{J\{(A+D)x + v_0 - v_f - DT\}} & , A \neq D \\ \dfrac{A^2 T}{J\{(v_0 + v_f)T + (v_0 - v_f + AT)x - 2L\}} & , A = D \end{cases} \tag{36}$$

CASE ($T_4 = 0, T_2 = 0, T_6 > 0$)

$$v_p = v_0 + \frac{1}{4}\gamma J x^2 = v_f - \frac{D^2}{\gamma J} + Dx \tag{37}$$

$$\begin{aligned} &C_2 x^2 + C_1 x + C_0 = 0, \text{ where} \\ &C_2 = (v_0 - v_f - \frac{DT}{2})^2 \\ &C_1 = -2v_f T(DT + 2v_f) + T(v_0 + v_f)^2 + 4L(v_f - v_0) + 2LDT \\ &C_0 = \{2L - (v_0 + v_f)T\}\{2L - T(2v_f + DT)\} \end{aligned} \tag{38}$$

$$\frac{1}{4}Jx^2\gamma^2 + (v_0 - v_f - Dx)\gamma + \frac{D^2}{J} = 0 \tag{39}$$

CASE ($T_4 = 0, T_2 > 0, T_6 = 0$)

$$v_p = v_0 - \frac{A^2}{\gamma J} + Ax = v_f + \frac{1}{4}\gamma J x^2 \tag{40}$$

$$\begin{aligned} &C_2 x^2 + C_1 x + C_0 = 0, \text{ where} \\ &C_2 = (v_f - v_0 - \frac{AT}{2})^2 \\ &C_1 = -2v_0 T(AT + 2v_0) + T(v_0 + v_f)^2 + 4L(v_0 - v_f) + 2LAT \\ &C_0 = \{2L - (v_0 + v_f)T\}\{2L - T(2v_0 + AT)\} \end{aligned} \tag{41}$$

$$\frac{1}{4}Jx^2\gamma^2 + (v_f - v_0 - Ax)\gamma + \frac{A^2}{J} = 0 \tag{42}$$

CASE ($T_4 = 0, T_2 = 0, T_6 = 0$)

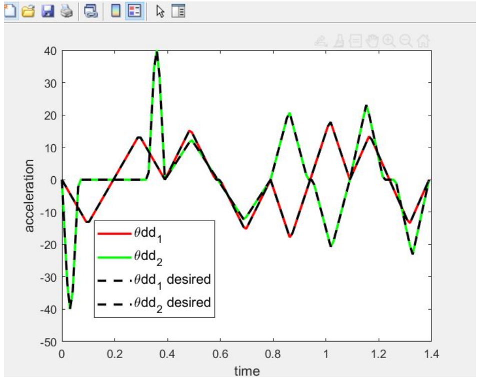
$$v_p = v_0 + \frac{1}{4}\gamma J x^2 = v_f + \frac{1}{4}\gamma J x^2 \tag{43}$$

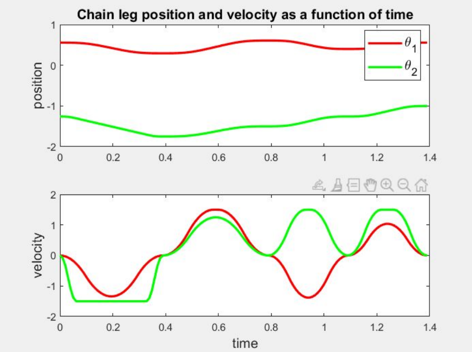
$$(v_0 - v_f)x^2 + \{(3v_f + v_0)T - 4L\}x + \{2L - (v_0 + v_f)T\}T = 0 \tag{44}$$

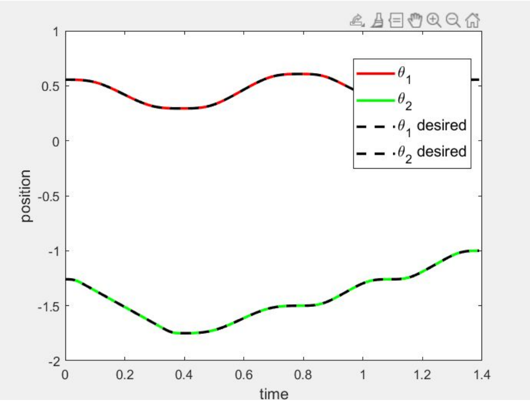
$$\gamma = \begin{cases} \dfrac{4(v_f - v_0)}{J(x^2 - x^2)}, & v_f \neq v_0 \\ \dfrac{4(L/x - 2v_0)}{Jx^2}, & v_f = v_0 \end{cases} \tag{45}$$

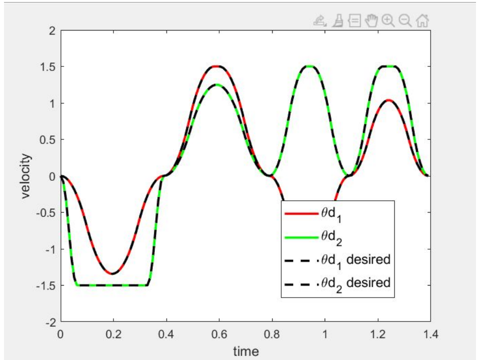
Use x or x_line equations from time optimal solution if not specified

4 pose animation









C conversion of matlab code

Matlab coder-assigned sizes to all the variable and arrays

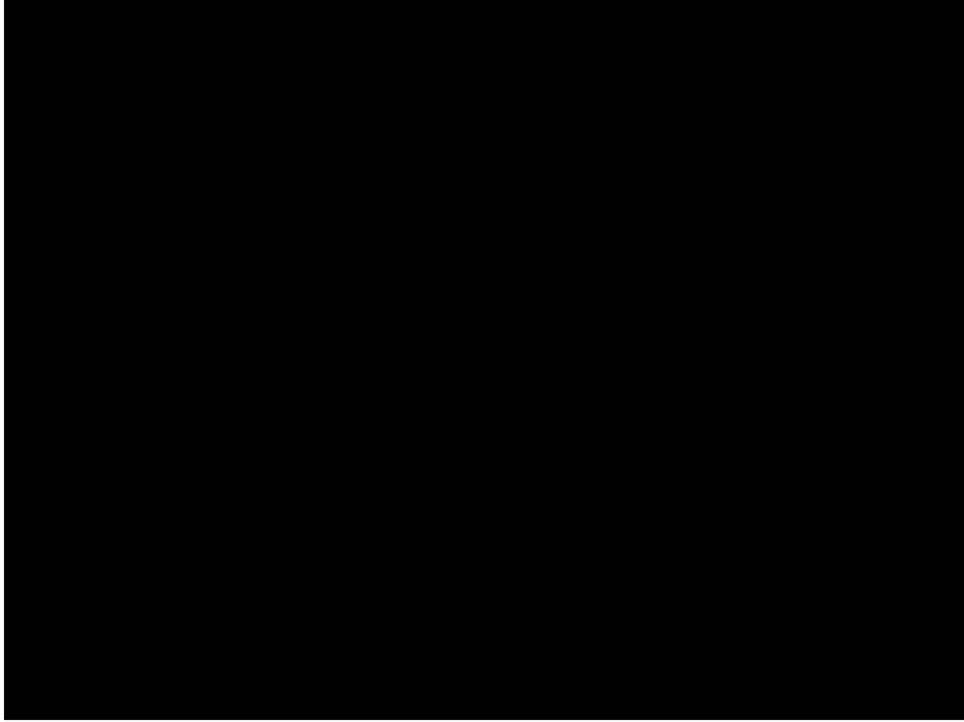
Cygwin

Replit to test the generated files

Robot simulation

Generated angles applied on a simulation

Show simulation



Thank You